NON-PERTURBATIVE PHENOMENA IN
QCD VACUUM, HADRONS, AND QUARK–GLUON PLASMA

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Lectures given in the
Academic Training Programme of CERN
1981–1982

GENEVA
1983
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ABSTRACT

These lectures provide a brief review of recent progress in non-perturbative quantum chromodynamics (QCD). They are intended for non-specialists, mainly experimentalists.

The main object of discussion, the QCD vacuum, is a rather complicated medium. It may be studied either by infinitesimal probes producing microscopic excitations (= hadrons), or by finite excitations (say, heating some volume to a given temperature $T$). In the latter case, some qualitative changes (phase transitions) should take place. A summary is given of the extent to which such phenomena can be observed in the laboratory by proton-proton, proton-nucleus, and nucleus-nucleus collisions.
INTRODUCTION

The many experimental facts explained so far by quantum chromodynamics (QCD), as well as the great beauty of its first principles, convince us that this is the true fundamental theory of strong interactions.

The essential difference between QCD and quantum electrodynamics (QED) shows up already at the perturbative level. The renormalization of the charge is such that it is decreasing at small distances $r^{-1}$, which is called asymptotic freedom. The expansion in powers of $\alpha_s(r)$ - the perturbative method - is now well developed, owing to the efforts of many theoreticians, and it should work for the description of hard processes taking place at small distances.

However, our topic is non-perturbative QCD, which is much less understood. Nevertheless, during the last few years some empirical information about the structure of non-perturbative QCD vacuum became available, supplied also by the recent development of numerical Monte Carlo simulations on the lattice. A qualitative picture concerning these effects in vacuum and hadronic structure seems to have been created. It contains several surprising facts, e.g. the colour confinement turns out not to be the strongest non-perturbative effect and the confinement length $R \approx 1$ fm is not the typical scale of non-perturbative fluctuations. It now seems reasonable to discuss the consequences of these findings in more general terms and for a more general audience, because it is very possible that they are relevant to a wider range of phenomena. In particular, the existence of some substructure inside hadrons, in the form of "constituent" quarks, becomes natural in such a framework.

Another point is to convince experimentalists that not only microscopic excitations (= hadrons) are of interest, but macroscopic ones as well. Moreover, some applications of such an idea are rather successful for the explanation of already existing data, a fact not yet recognized by the physical community.

The difficulties in presenting such a complicated topic, which is not very well understood so far, are quite evident. However, the attempt to present it in simple terms is interesting in itself. For example, the author discovered for himself that all methods used for the studies of QCD vacuum have their analogues in the studies of ordinary matter.

The author also apologizes for not giving a complete reference list and for discussing mainly those topics that are more familiar to him. Many specialists do not share all his conclusions, and this should be kept in mind when reading these lectures.
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### CONCLUSIONS
Lecture 1

THE STRUCTURE OF THE QCD VACUUM AND HADRONS

1.1 WHERE DO THE NON-PERTURBATIVE EFFECTS SHOW UP?

Let us start with a brief discussion of this general question. It will lead us directly to the most striking and unexpected findings of the non-perturbative QCD of the last few years.

Looking at perturbative series in powers of $\alpha_s$, calculated for many phenomena by numerous theoreticians, one may expect that the non-perturbative physics sets in at distances at which $\alpha_s$ becomes of the order of unity. Using the known value of $\Lambda$ and asymptotic freedom relation

$$\alpha_s(r) = 2\pi/[b \ln(1/r\Lambda)]; \quad b = 11 - 2/3 N_{\text{flavours}} \approx 9, \quad \Lambda \approx 100 \text{ MeV}$$

(1.1)

we can then calculate this distance to be about $R \approx (1/2\Lambda) \approx 1$ fm. It is known experimentally that all hadrons are indeed of this size, so it seems natural enough that non-perturbative confining effects really take place at such distances.

This philosophy is expressed in many models of hadrons, e.g. the MIT bag model, to be discussed later. It assumes quarks to be free up to distances $R \sim 1$ fm, where confinement effects sharply set in.

However, as we are going to argue below, such a simple scenario is very far from being true and much stronger non-perturbative phenomena are in fact observed. They start to be essential already at distances of about 0.1–0.4 fm (depending on the particular quantity under consideration), where perturbative series still look good enough.

On the other hand, these strong effects seem to be not directly relevant to confinement — at least we do not see how they are connected. It also turns out that the total vacuum energy density due to non-perturbative fluctuations is about one order of magnitude larger than that connected with confinement forces. So, the non-perturbative QCD turns out to be more complicated than just a smooth transition to some strong coupling regime.

Now, let us discuss in greater detail how all this became realized.

1.2 CONFINEMENT

The confinement of coloured objects is the most non-trivial property of the strong interactions. Although recent Monte Carlo lattice calculations give rather convincing evidence that this phenomenon indeed follows from the QCD Lagrangian, we still do not understand its physical origin.

A popular phenomenological way of describing the effect of confinement is that of the MIT bag model, which associates some positive volume energy

$$E_{\text{bag}} = B_{\text{bag}} \cdot V_{\text{bag}}$$

(1.2)

with the volume $V_{\text{bag}}$ occupied by the coloured objects. In other words, the bag is under the constant pressure $B_{\text{bag}}$ from the vacuum, trying to expel coloured objects from it.

A qualitative suggestion to explain the physical nature of this pressure $B_{\text{bag}}$ was made by Callan, Dashen and Gross in 1978. It can be useful to invoke the analogy with superconductivity to explain this idea.

It is well known that Cooper-pairing of electrons with opposite spins lowers the ground-state energy of a metal. An external magnetic field tries to make such spins parallel and thus suppresses the pairing. As a result, some extra positive energy is associated with a field which is therefore expelled from superconductors. Incidentally, if magnetic monopoles exist, their pairs are confined inside the superconductors by the string with finite tension, going from one monopole to another with one unit of magnetic flux. This is exactly what happens with quarks in the QCD vacuum.

We are now able to determine the energy density of the QCD vacuum; compared to the “empty” perturbative vacuum it is negative.

Less general considerations for some types of fluctuations (instantons) to be discussed below show that the quark density, external field, etc., suppress such fluctuations, and again negative energy is lost. In this case, such matter can also be expelled from the vacuum.

Unfortunately, this idea was not demonstrated to work in a sufficiently wide context (weak fields, etc.). It has also some weak points on the phenomenological side (see below).

1.3 CHIRAL SYMMETRY BREAKING

This phenomenon is less famous than confinement, although the theoretical papers on this subject are very numerous.

It is probably desirable to explain first what is the chiral symmetry. If we have $N_f$ quarks of the same mass we can arbitrarily mix them. So, for $u, d, s$ quarks we have approximate U(3) symmetry. If they not only have the same mass
but are actually massless we can rotate right-handed and left-handed quarks separately, which is the chiral symmetry. This symmetry then demands that all physical states should be parity degenerate.

It is known that this symmetry is spontaneously broken in the QCD vacuum (it is asymmetric). The direct consequence is the fact that the pion is massless, so it can be added to any state, changing its parity and without changing its energy.\(^9\)

To give an idea of why it happens, let us again use the analogy with superconductors. The Cooper-pairing rearranges the surface of the Fermi sphere because electrons have attractive interaction due to phonon exchange.

In a similar way, the attraction in the scalar \(q\bar{q}\) channel is so great that it destroys the well-defined Dirac sea and produces a finite gap (= effective mass) between negative and positive energy states.

Another way to express this is to say that the anomalous average

\[
\langle 0 | \bar{\psi}(x)\psi(x) | 0 \rangle \neq 0
\]

(1.3)

(the so-called quark condensate) is developing, also being analogous to that in the superconductors.

The value of quantity (1.3) and its physical nature will be discussed below. Before we come to it, let us briefly mention how we obtain information about the QCD vacuum at all.

### 1.4 PHENOMENOLOGY OF THE QCD VACUUM

There are essentially two main sources of information at the moment:

i) The QCD sum rules;

ii) Monte Carlo calculations on the lattice.

The first source connects the QCD vacuum with experimentally known properties of hadrons, whilst the second one calculates them directly from the Lagrangian. Both are now developing at high speed, and share many technical points.

The main object which bridges the gap between the problems of vacuum and hadronic structure is the correlator of some infinitesimal currents (probes, if you like) at different space–time points:

\[
\Pi(x) = \langle 0 | T \{ j(x)j(0) \} | 0 \rangle, \quad x^2 < 0 .
\]

(1.4)

On the one hand, one may consider the quark–gluon language of QCD and consider propagation of fundamental constituents through the complicated QCD vacuum full of fluctuating fields. On the other hand, one may consider the propagation of hadrons (with the quantum numbers of the current \(j\), of course).

Evidently, the former language is most simple at small \(x\) owing to asymptotic freedom. The latter is simpler at large space-like \(x\), where \(\Pi(x) \sim \exp(-mx)\), \(m\) being the lowest mass. At intermediate \(x\), both languages somehow become more complicated, but in any case they should give identical results. This statement is known as the quark–hadron duality.

Without going into details, we will just say that a new field of investigations has appeared in our science: the phenomenology of the QCD vacuum. Let us mention two important numbers known so far\(^6\):

\[
\begin{align*}
\langle 0 | (gG^a_{\mu\nu})^2 | 0 \rangle & \approx 0.5 \text{ GeV}^4 \\
\langle 0 | \bar{\psi}\psi | 0 \rangle & \approx -1.6 \times 10^{-2} \text{ GeV}^3
\end{align*}
\]

(1.5)

where \(G^a_{\mu\nu}\) is the gluon field strength, and \(\psi\) is a quark field.

With such input we are able to calculate masses and couplings of such hadrons as charmonium levels\(^6,7\), vector and axial mesons\(^8\), upsilon levels\(^9\), baryons\(^9\), mesons made of heavy and light quarks\(^10\), and so on. Using the three-current correlators, in principle we are able to calculate form factors\(^11\), decay constants, etc.

So a lot of experimental information is explained in terms of essentially two numbers: the gluon and quark condensates!

This is, of course, the great success of the theory. However, it may be even more important that in certain cases these two numbers were found to be completely insufficient\(^12\). We will return to these questions later.

Now, the problem is to understand these numbers [formulae (1.5)] (and, of course, many other properties of the vacuum). The potentially most powerful way to calculate them is by means of lattice simulations. However, at the moment they are rather limited in their accuracy because of i) the small number of lattice points available and ii) complications in accounting of virtual quark fields. Still they produce values of a reasonable order of magnitude; see, for example, Banks et al.\(^13\) for the gluon condensates and Hamber et al.\(^14\) for the quark condensates.

\(^9\) Of course, the real pion has small non-zero mass, but this is due to non-zero masses of the quarks, or approximate chiral symmetry from the start.
1.5 INSTANTONS

For qualitative orientation it is always useful to try first some approximate but analytic method, and only then carry out heavy numerical calculations. Such a role is played in QCD by the semiclassical methods.

The non-trivial non-perturbative fluctuation of the fields in vacuum was found in 1975 by Polyakov and collaborators\(^\text{[9]}\) — this is the famous “instanton” solution of the Yang–Mills equations (the QCD analog of the Maxwell ones). Beautiful mathematics is connected with this solution, and hopefully very interesting physics will follow.

Briefly, the physics is as follows. “Vacuum” at the classical level means that the field \(G_{\mu \nu}^a\) is zero. In QED it implies that up to gauge transformation \(A_\mu\) is also zero. In QCD this is not so; there are classes of \(A_\mu^2\) which cannot be continuously transformed into one another. So at the classical level we have many “vacua”.

Moreover, on the quantum level the barriers separating them turn out to be penetrable! The “tunnelling” process is described in semiclassical approximation by instantons. The motion under the barrier is a virtual process, so we have to consider an imaginary time \(\tau = it\), rather than a real one \(t\) \(^\text{[9]}\).

The solution is remarkably simple:

\[
A_\mu = (2/g) \eta_{\mu \nu} x_\nu [l(x - z)^2 + \rho^2] \tag{1.6}
\]

[where \(\eta_{\mu \nu}\) is some numerical tensor introduced by ’t Hooft\(^\text{[16]}\) and depends on such parameters as the instanton location \(z_\mu\) and its size \(\rho\). However, the quantum theory of interacting instantons turns out to be rather complicated, so the most interesting effects cannot be so far reliably calculated from first principles.]

It is possible to try a more model-dependent approach\(^\text{[17]}\), fixing the instanton parameters from data. I cannot go into details here, so I will just say that the main phenomenological input is the value of the gluon condensate \((1.5)\). The principal results are the estimated instanton density \(n_i\) in space-time and typical radius \(\rho_c\):

\[
n_i \approx 10^{-3} \text{GeV}^4; \quad \rho_c \approx 1/600 \text{MeV} \approx 1/3 \text{fm}. \tag{1.7}
\]

The important new feature of the QCD vacuum, suggested by these numbers, is the so-called “twinkling” picture of the QCD vacuum. This would mean that the vacuum consists mainly of rather strong fluctuations, which occupy only a small fraction \(f\) of space–time:

\[
f = \pi^2 n_i \rho_c^4 \approx 1/20 \ll 1. \tag{1.8}
\]

Such a picture also implies that the idea of an individual instanton makes sense. Being a little bit more specific, it could be mentioned here that the action of the typical instanton is about

\[
S_0 = 8\pi^2 g^2(\rho_c) \approx 15–20, \tag{1.9}
\]

whilst \(\Delta S\), the correction to action, because of instanton interaction, is only about

\[
\Delta S \approx 4–5 \ll S_0. \tag{1.10}
\]

However, \(\exp (\Delta S)\) is far from being small, and this means that we have not a dilute instanton gas [as suggested by Callan et al.\(^\text{[18]}\)] but a strongly interacting “instanton liquid”.

To give an example of the results obtained in this model, we return to the problem of chiral symmetry-breaking mentioned above. It was proposed by Callan, Dashen and Gross\(^\text{[9]}\) that the attraction between quark and antiquark can be generated by instantons. This idea was transferred into quantitative estimates in Ref. 17, with the results

\[
\langle \bar{\psi}\psi \rangle = -(3n_i)^{1/4}/\rho_c \approx -1 \times 10^{-3} \text{GeV}^3,
\]

\[
m_{\text{eff}} = 2\rho_c (n_i/3)^{1/2} \approx 200 \text{MeV}. \tag{1.11}
\]

Here \(m_{\text{eff}}\) is the quark effective mass generated by this mechanism. It is seen that both results are phenomenologically acceptable: \(\langle \bar{\psi}\psi \rangle\) is close enough to formulae \((1.5)\), and \(m_{\text{eff}}\) is close to what is usually accepted for “constituent” quark mass.

This information concerning the vacuum structure is sufficient for our purposes, so we now turn to the discussion of hadrons.

\[^{\text{[9]}\text{There are all applications of the instanton calculus imply in fact virtual, not real, processes: say the current correlator outside the light cone. The same is true for QCD sum rules and lattice formulation as well.}}\]
1.6 HADRONIC SUBSTRUCTURE. THE NATURE OF "CONSTITUENT" QUARKS

It will be useful to continue our discussion of the MIT bag model (started in Section 1.2) in somewhat greater detail.

The point is that with the gluon condensate at hand, we are able to calculate the energy density of non-perturbative fluctuations in QCD vacuum$^9$:

$$e_{\text{vac}} = -(b/128\pi^2)(|\langle G_{\mu\nu}^2 \rangle| |0\rangle) \approx -500 \text{ MeV/fm}^3.$$  \hspace{1cm} (1.12)

It can now be compared$^9$ with the phenomenological bag constant$^2$. It turns out that $B_{\text{bag}}$ is much smaller than $B_{\text{vac}}$:

$$B_{\text{bag}} \approx (1/10 \text{ to } 1/20) B_{\text{vac}}.$$  \hspace{1cm} (1.13)

So our qualitative explanation of the bag model by suppression of fluctuations inside hadrons does not look very good on a more quantitative level. There are also several other arguments which show that some important point is missing$^8$.

They all are naturally explained in the so-called two-component picture of a hadron$^1$. Assume that the vacuum contains two types of fluctuations of sizes $\rho_c \sim 1/3$ fm and $R \sim 1$ fm, connected with energy densities $\epsilon \sim B_{\text{vac}}$ and $B_{\text{bag}}$, respectively. Now, putting some quarks into such a vacuum we have, by the same suppression mechanism, two kinds of "cavities" or "bubbles" in vacuum (see Fig. 1.1). We identify them with "constituent" quarks and hadrons, respectively. So, the "twinkling" vacuum produces the "grained" hadrons.

The existence of a substructure inside hadrons was noticed long ago, but recently more convincing arguments were given for it really having a small scale of about 1/3 of that of a nucleon:

i) The oldest argument is the additivity of the "constituent" quark cross-section. In 1965 it was noticed that $\sigma(nN)/\sigma(NN) \approx 2/3$ $^{19}$, but it is only recently that this idea was checked much better by $\pi A$ collisions (see Lecture 3).

ii) Parton "intrinsic" transverse momentum observed in hard processes such as Drell–Yan pair production, high-$p_\perp$ hadrons, etc., is of the order of 1 GeV, suggesting localization of partons inside objects essentially smaller than the hadrons themselves.

iii) The study of higher-twist effects in deep-inelastic scattering$^{30}$, which suggests a very high probability that two quarks can be found in the same point inside the nucleon, about an order of magnitude higher than in the bag model. It also suggests the existence of some small clusters, the "constituent" quarks.

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Fig. 1.1 Schematic picture of the energy density distribution in the proton according to the "two-component model" (Ref. 17) along the diameter shown by the dashed curve in the upper part of the figure.

---

$^9$ For example, if the quark condensate and pionic waves are not allowed inside the bag, the cloud of virtual pions outside it produces a pressure much larger than $B_{\text{vac}}$. 

4
1.7 SHORT-RANGE EFFECTS IN HADRONS

In the preceding section we have concluded that when average quark separation in hadrons is larger than the typical scale of main vacuum fluctuations, quasi-independent “cavities” (identified with “constituent” quarks) are formed. We are now going to discuss what happens if two quarks are close enough, at a distance comparable to \( \rho_c \). Although such a configuration has relatively small probability, \( \sim (\rho_c/R)^3 \), it can generate rather important effects.

The methodological main tool is now the QCD sum rules. As already mentioned before, it is suitable just for discussion of short-range phenomena. The particular effect we look for is the instanton-induced corrections to short-range two-current correlator.

It can be shown that in first approximation all particular currents can be divided into two classes: non-zero and zero spin currents. The instanton-induced effects are much stronger in the second case\(^{13}\).

As an example, let us consider vector mesons. The current \( \bar{\psi} \gamma_\mu \psi \) produces a quark and an antiquark of the same chirality, whilst the 'Hooft instanton vertex needs the opposite chirality of \( q \) and \( \bar{q} \)\(^{19}\). The price for the chirality flip is the following small factor\(^{19}\):

\[
(m_{\pi} \rho_c)^2 \simeq 1/10 \ll 1. \tag{1.14}
\]

Therefore, the energy of two quarks at short distances \( \sim \rho_c \) is not very different from the sum of their effective masses. One may say that vector mesons are in this sense “normal”. Many observations support this — e.g. mixing between strange and non-strange mesons is small, etc.

The next example is the current \( \bar{\psi} \gamma_\mu \psi \), connected with pseudoscalar mesons \( \pi, K, \eta, \eta' \). The small factor (1.14) is now absent, so the quark-quark interaction at distances \( \sim \rho_c \) is very strong. In the case of the pion it is attraction, and this is so strong that it compensates the sum of effective masses and makes the pion massless in the chiral limit. It is instructive to see by direct calculation how it happens; but in any case it is the consequence of the quite general Goldstone theorem.

In a recent paper\(^{21}\), I have calculated what happens in this model for other members of the pseudoscalar nonet. It turns out that the instanton-induced attraction is twice as weak for \( K \), therefore its mass is larger. The most interesting case is the \( \eta' \), in which attraction is changed by strong repulsion. As a result, \( \eta' \) becomes heavy, \( m_{\eta'}^2 \) being essentially larger than the “normal” \( m_{\eta}^2 \) value. This important fact was first discussed by Weinberg\(^{22}\) and became known as the U(1) problem, and it was 't Hooft\(^{10}\) who first noticed that instanton-induced effects can in principle solve this problem. Now the effect is shown to work quantitatively\(^*\).

Another related point is that of the mixing. Strange and non-strange quarks are nearly ideally mixed, so that \( \eta \) is a member of an SU(3) octet and \( \eta' \) is an SU(3) singlet. Even the small mixing angle, \( \sim 10^\circ \), was found to be explained in this model\(^{23}\).

Another interesting point is the gluonic currents with zero spin. It is shown in Refs. 12 and 17 that asymptotic freedom in such channels is strongly violated already at \( Q^2 \) as large as 10 GeV\(^2\)! The interaction induced by instantons is respectively very strong; it is most attractive in the \( J^{PC} = 0^{++} \) channel, so the lowest state is rather light, about 1–1.4 GeV. Similar results came from Monte Carlo calculations\(^{14}\) and from a quite different approach by Shifman\(^{20}\).

1.8 CONCLUSIONS

Listed below are the main points of this lecture:

1. The non-perturbative effects show up at scales much smaller than \( R \sim 1/2\Lambda_{\text{QCD}} \) [where \( \alpha_s(R) \sim 1 \)] suggested by perturbative calculations.
2. There exist strong fluctuations of scale \( \rho_c \sim 1/3 \) fm separated by somewhat larger distances, \( \sim 1 \) fm.
3. Their suppression by quarks offers the natural explanation for the additive “constituent quark”, known from phenomenology, as some “bubbles” around quarks in vacuum.
4. Short-range interaction of quarks is strong for zero-spin channels, and much smaller for non-zero-spin ones.
5. The instanton model seems to put all these phenomena together in a quantitative and logically consistent way.

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\(^*\) It is highly non-trivial that the real vacuum possesses such a small parameter\(^{17}\); this is strongly related with the “twinkling” picture of the vacuum as well.

\(^*\) It is interesting that Monte Carlo calculations\(^{14}\) have difficulties with \( \eta' \) mass, mixing, etc., which is probably because their lattice is too coarse grained to account for small-scale structure of the vacuum under consideration.
Lecture 2
MACROSCOPIC EXCITATION OF THE QCD VACUUM

2.1 INTRODUCTION
As was already stressed in Lecture 1, the QCD vacuum is a complicated medium, and the methods used for understanding its structure are essentially the same as those used for studies of usual condensed matter. For example, a quite conventional method is to generate microscopic excitations (e.g. phonons in solids) by some external probes and study their correlators, which obviously is analogous to the spectroscopy of hadrons with QCD sum rules.

However, for ordinary matter much simpler methods of investigation are known, such as heating and looking for some qualitative changes (e.g. the melting of solids) in order to estimate the strength of the forces that are creating the structures under consideration. If there exists some kind of hierarchy of different effects, the sequence of phases along the temperature axis would tell us this directly. It is tempting to try a similar approach in the case of the QCD vacuum as well. Theoretically this is simple — for example it is evidently simpler to consider infinite and homogeneous matter with quark density about the same as that inside the nucleon, rather than the isolated nucleon with three quarks. Owing to a specific feature of QCD, namely the asymptotic freedom, this problem becomes even more simple if the excitation energy density is much larger than $\Lambda^4$.

The most important point of these considerations is that in macroscopic systems qualitative changes (phase transitions) take place. In particular, the main qualitative features of the QCD vacuum discussed in the first lecture, namely confinement and chiral symmetry-breaking, should disappear at a certain excitation level.

The actual observation of such phenomena at predicted temperatures would evidently be a very convincing argument. We discuss in this lecture whether one can reach this aim experimentally by means of high-energy collisions of hadrons and nuclei.

Unfortunately, strictly speaking even the heaviest nuclei are not big enough to consider the process of excitation in perfectly macroscopic language. This is essentially due to the “grained” structure of the nucleons considered above. In particular, the mean free path of the “constituent” quark in nuclear matter is comparable with dimensions even of uranium nuclei. However, in a certain fraction of collisions a sufficient amount of energy can be “thermalized”.

In the last lecture we will return to this question, considering it from the phenomenological point of view, and will discuss recent attempts to explain available data by some macroscopic considerations.

2.2 PHASES OF HIGH-DENSITY AND/OR TEMPERATURE MATTER
Before we come to macroscopic considerations, let us first recollect the main points concerning the hadronic structure presented in Lecture 1. This structure derives essentially from the existence of “constituent” quarks, the quasiparticles inside hadrons with dimensions several times smaller than the hadrons themselves. We have connected their size with that of a typical scale of fluctuations in the QCD vacuum.

Motivated by this picture, I have proposed\(^{24}\) that “melting” of hadrons and “constituent” quarks should take place subsequently, when the level of excitation is steadily increased. By “excitation” we mean any way of producing large energy density $\varepsilon$, either by compression (high density) or by heating (high $T$). Following the line of argument presented above, it is natural that such transitions take place when $\varepsilon$ reaches typical values such as

$$\varepsilon_1 \sim B_{\text{bag}}, \quad \varepsilon_2 \sim B_{\text{vac}},$$

(2.1)

with $\varepsilon_2$ being about one order of magnitude larger than $\varepsilon_1$.

We are used to the idea that some qualitative changes should take place in such transitions. The obvious candidates for these cases are the deconfinement of colour and restoration of chiral symmetry.

At least three phases of matter are predicted\(^{24}\):

i) A hadronic phase (e.g. the well-known nuclear matter) made of separate hadrons.

ii) A plasma of “constituent” quarks. Hadrons do not exist, but the chiral symmetry is still broken. Therefore, quarks possess a non-zero effective mass; and the long-range pseudoscalar modes (the pions) are still present.

iii) A “perturbative” quark–gluon plasma, being in many respects similar to that made from electrons and photons. The non-perturbative effects are mainly due to instanton/anti-instanton “molecules”, and they very soon become unessential as we proceed from the transition region to higher excitation energies.

Having outlined these main pictures, let us make the necessary historical remarks. The deconfinement transition was suggested by Polyakov and Susskind\(^{25}\). Its “observation” by means of Monte Carlo calculations at finite temperature in gluodynamics on the lattice was made in Ref. 26. For the hadronic phase side, it was emphasized by Hagedorn\(^{27}\) a long time ago that it loses its meaning at a temperature of about 160 MeV, which is also a rough estimate of the critical temperature $T_c$ of the deconfinement transition.
The second transition in QCD was considered simultaneously in my paper and that by Pisarsky in 1981. More detailed investigations within the instanton model framework were made in my recent papers. The important conclusion is that \( \langle \bar{\psi} \psi \rangle \) cannot go smoothly to zero, but should have a finite jump, so the transition is of first order. Several months ago it was found that MC with fermions indeed seems to produce two phase transitions, with the typical energy densities separated by one order of magnitude! Moreover, it is noted that there is evidence that chiral symmetry restoration is a first-order transition. The latest work has found strong changes at \( T \approx T_2 \) in pure gluodynamics, presumably indicating the instanton suppression. Such good agreement between rather different approaches is very encouraging, although all of them are in a rather preliminary phase of investigation.

### 2.3 Nuclear Stars and the Asymptotics of the Rosenfeld Tables

Different approximations for the equation of state of high-density matter at zero temperature are shown in Fig. 2.1. They are the “old-fashioned” nuclear matter calculations, the Fermi gas of “constituent” quarks with \( m_{\text{eff}} \approx 200 \text{ MeV} \) [low-density limit] and the Fermi gas of massless “current” quarks with subtracted \( B_{\text{vac}} \) (high-density limit). The transition region is very uncertain, and different transition regimes (I–III) are possible.

A particularly interesting case is that of III, which permits a mechanically stable \( (p = 0) \) configuration at high density \( n' \). Of course, this is not the absolute minimum of the energy per quark (which is known to be in Fe nuclei!) but some metastable state; in this case multibaryon resonances with arbitrary large baryonic number may exist!

It is very interesting that such a possibility turns out to be in contradiction to data on neutron star masses. The point is that the highest possible mass \( M_{\text{max}} \) of a stable neutron star depends strongly on the compressibility of the matter just at the densities we are now discussing, namely several units of baryonic charge per cubic fermi. My calculations give \( M_{\text{max}} \approx 1.7 \text{ M}_\odot \) in case I and only about 1.2–1.3 \( \text{ M}_\odot \) in case III. It was argued by Kislinger and Morley that taking into account the dynamics of neutron star creation makes these numbers even somewhat (~ 20%) smaller.

Experimentally, \( M(\text{Her X1}) = 1.33 \pm 0.20 \text{ M}_\odot \) and \( M(\text{Vela X1}) = 1.6 \pm 0.3 \text{ M}_\odot \). Unfortunately, I cannot say definitely what the errors mean in these data, which come from the observation of the rotation of close binary systems (pulsar + ordinary star), but taken literally they seem to contradict possibility III. If so, multiquark resonances made of ordinary quarks can hardly exist.

![Graph showing different approximations for the equation of state of high-density matter](image)

Fig. 2.1 Different approximations for the equation of state of zero-temperature high-density matter: nuclear matter; “constituent” quarks with mass 200 MeV; “current” quark gas with zero mass and subtracted “vacuum pressure” \( B_{\text{vac}} \). The dashed curves (I)–(III) show possible transition regimes.

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* Earlier considerations of the chiral symmetry restoration in the sigma-model context were reviewed by Lee. However, properties of the transition in this model are very different from the real QCD case; e.g. the high-temperature phase is the gas of massless nucleons, not quarks and gluons.

** Although he considered \( T_1 = T_2 \) as the most natural possibility, the case \( T_2 > T_1 \) was discussed as well, with some interesting considerations concerning the intermediate phase.
Another suggestion (also made in Ref. 32) is that heavy quarks can help here. The physical arguments are quite clear. First, they move more slowly, and obviously their pressure is reduced. The second (less trivial) point is that gluomagnetic repulsive effects which make the six-quark bag heavier than two nucleon masses\(^1\) are reduced too. The attractive gluoelectric forces are increased for heavy quarks. So all corrections act in the same sense, namely they reduce the matter pressure at given density.

The argument becomes more clear in the following limit. Let us consider some matter made exclusively of heavy (say, b-type) quarks. It is essentially the non-relativistic system with Coulomb attraction\(^3\) similar to matter made of e\(^-\) and e\(^+\) or exitons and anti-exitons in solids. It is interesting that the existence of macroscopic drops of such exiton atoms was recently verified experimentally. So we conclude that, for some fraction \(f\) of heavy quarks, bound states or resonances of arbitrary size should exist.

My estimate for charmed quarks\(^2\) is \(f_c \approx 20\%\), while an independent study by Chin and Kerman\(^5\) gave for strange quarks \(f_s \sim 70\%\). All this does not look very promising experimentally, but in any case the conclusion that the Rosenfeld tables are in principle infinite is essential, at least for people involved in discovering particles. Experimentally the trend is obviously right: the only well-defined states of \(Q\bar{Q}\) structure are those decaying into \(D\bar{D}\), etc. (with charmed quarks!); also, the S-wave dibaryons with strangeness seem to exist, in contrast to the non-strange case. In principle, macroscopic considerations can even provide some asymptotics of the properties of such multiquark states.

So concluding this section we may say that the spectrum of states containing heavy quarks seems to be much richer than that of ordinary hadrons, and it includes also completely new types of states containing arbitrary large numbers of quarks.

2.4 MACROSCOPIC THEORY OF HIGH-ENERGY COLLISIONS

It is probably reasonable to start with a brief discussion of the general aims of such studies, leading presumably to the most direct and transparent demonstration of the reality of non-perturbative phenomena in QCD vacuum.

Let us comment that, although we live in this vacuum, we have not so far noticed these phenomena, exactly as several centuries ago people hardly noticed the presence of air. Once it became technically possible to pump the air out of a volume, the reality of atmospheric pressure appeared to be obvious.

The phenomenon of confinement (at least as it is now discussed in bag-model-type considerations) is in principle of a similar type. If we take two quarks and pull them apart, it is assumed that we are working against the “vacuum pressure” in order to stretch the QCD string\(^*\). Unfortunately, such direct experiment is not feasible, although it is partly realized in high-spin hadronic states where the centrifugal force separates quarks.

Our idea is as follows: since it is not possible to “pump the vacuum out” of a volume, let us fill it with some other matter we understand better. As a good candidate the asymptotically dense quark–gluon plasma can be considered.

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\(^1\) It should be remembered that quarks of different colour are attracted to each other, although the attraction is weaker than for quark and antiquark of opposite colours. This has no analogs in QED, in which electrons are repulsed.

\(^*\) It is interesting that in every-day units the QCD string tension is of the order of 10–20 tons.
Now, let us start by considering this problem as an absolutely macroscopic one, e.g., consider the collision of $A \to \infty$ nuclei (neutron stars...) which in reality do not exist. The complexity of the realistic case is the subject of Section 3.2.

In the macroscopic case the theory is very well developed. Shock waves excite matter at the microscopic length $l$, which is much smaller than the system dimension $L$, so the general conservation laws push the equation of state to do the job. The schematic picture of the excitation curve in the phase plane is shown in Fig. 2.2. Any point on it corresponds to a certain velocity of the collision (but, of course, does not depend on the size of the colliding objects!).

The next stage is the expansion of the system and its cooling. The macroscopic approach is again well known: it is the hydrodynamical equations

$$\partial_\mu T_{\mu\nu} = 0 \quad ; \quad T_{\mu\nu} = (\varepsilon + p) u_\mu u_\nu - p g_{\mu\nu}$$

with $u_\mu(x)$ being the four-velocity and $T_{\mu\nu}$ the stress tensor containing $\varepsilon$ and $p$, energy density and pressure. As noted by Landau\textsuperscript{31} their important feature (again, due to $L \gg l$) is that entropy is conserved, so the cooling curve in Fig. 2.2 is an ordinary adiabatic one. Finally, as noted by Pomeranchuk\textsuperscript{30}, the system becomes so dilute that $l$ reaches $L$ and the system decays into independent secondaries.

### 2.5 MACROSCOPIC THEORY VERSUS REALITY. THEORETICAL CONSIDERATIONS

The applicability of the macroscopic theory to high-energy collisions of hadrons and nuclei has remained a matter of controversy during the last 30 years, and this question is still far from being understood. In this section we present some preliminary theoretical considerations which reflect the present status of the author's understanding of this complicated problem.

It is instructive to start with the unrealistic example of a really large system made of ordinary matter. Assume for a moment that we are able to accelerate charged drops (or something of the kind) up to the energies of interest. Can we use the macroscopic theory for such collisions?

The answer is negative. Although such systems are large they are not dense enough. The nuclear mean free path $l$ in ordinary matter is of the order of metres and is larger than the drop size $L$.

Now, let us consider non-relativistic collisions ($E/N \lesssim 1$ GeV) of heavy nuclei. The nucleon mean free path is about 1 fm now, so it is several times smaller than the diameter of the heaviest nuclei. This implies that the macroscopic approach is reasonable, although corrections for finite-size effects can be significant.

Looking at data that were already available for such an energy region, people really found them reasonably described by various thermodynamical models. Moreover, even the hydrodynamical "blast wave" of the collective motion at the expansion stage seems to have been observed\textsuperscript{37}.

However, at energies around several GeV/N this picture fails. The structure of the nucleons comes into play. As we have argued above, they are "grained" or made of relatively small-size "constituent" quarks. The corresponding mean free path of such an object $l_q$ in nuclei is about 5 fm, being comparable with the dimensions of the heaviest nuclei. As a result, the probability that nucleons pass through each other without great energy loss is significant; this is known experimentally as "nuclear transparency". It makes the applications complicated; one can either select some "statistical subsystem" or select a particular sample of events.

The next level of consideration is the collision of two "constituent" quarks. This is essentially what we in fact observe in high-energy pp collisions, for usually the other two quarks in each proton just go through, and are therefore called "spectators". A rather interesting phenomenology of the fragmentation region, based on such a picture, was developed, but these questions are outside the scope of the present lectures.

Here the structure of the "constituent" quarks comes into play, but unfortunately we know very little about it. Presumably it contains a "current" (= point-like) valence quark surrounded by a gluonic field. From deep-inelastic data we know that these two components divide the total momentum about equally. So it is not quite clear what happens in the collision of "constituent" quarks. The "valence" quark seems to "go through" and the gluon subsystem interacts. This is essentially the Van Hove–Pokorsky picture\textsuperscript{38} supported, in particular, by observations that secondaries seem to originate mainly from some neutral flavour (gluonic?) subsystem. Can it be that such a gluonic subsystem is "thermalized"?

If one recollects our discussion about the non-perturbative forces in gluonic channels, which already violate the asymptotic freedom at $Q^2 \sim 10$ GeV\textsuperscript{2}, it is easy to imagine the typical mean free path in gluonic plasma to be as small as 0.1–0.3 fm. The perturbative estimates of the gluon mean free path given in Ref. 39 also point in the same direction, because $\sigma_{gg}$ contains large numerical factors. As a result, it was concluded that such "thermalization" of gluons might be possible up to ISR energies. Here it should be emphasized that in any case the asymptotic freedom does not allow the use of such an approach for asymptotically large energies. This means that temperatures available in experiments are limited from above, $T \lesssim 300–400$ MeV.

At still higher energies, such as those of the CERN Super Proton Synchrotron (SPS) collider, even "constituent" quarks become penetrable. The macroscopic consideration of matter excitation is no longer possible, and entropy is produced in a complicated non-equilibrium process.
However, macroscopic considerations are still applicable to the later stages of expansion, where they become even more reliable than in low-energy collisions, because now the particle density is larger. To illustrate this point, let us consider a "constituent" quark in the central region, produced at the SPS collider. If it interacts strongly with quarks in the ±1 rapidity interval, it has 10–20 companions in this kinematical region. This example evidently shows that a direct dynamical approach is in this case hopeless and methods of statistical mechanics are unavoidable.

Summarizing this discussion we could say that at least four possibilities may exist for the applications of the macroscopic approach, as follows:

1) AA collisions at non-relativistic energies \( E/N \lesssim 1 \) GeV, where the excited nuclear matter containing \( N \) and \( \pi \) (and, in a sense, \( \Delta \), etc.) is produced. The energy density \( \varepsilon \) is smaller than \( B_{\text{bag}} \).

2) AA collisions at energies \( E/N \sim \) several GeV, producing some plasma of "constituent" quarks; \( \varepsilon \) is in between \( B_{\text{bag}} \) and \( B_{\text{vac}} \).

3) Collisions at energies of several GeV/quark, even perhaps "qq" subcollisions, with \( \varepsilon \gtrsim B_{\text{vac}} \). Production of quark–gluon plasma.

4) Collisions of arbitrary high energies are connected with arbitrary large energy densities. However, mixing takes place only at later stages of the collision. During expansion of the system it goes through all phases.
3.1 INTRODUCTION

In the following we will not discuss points (1) and (2) from the list of possibilities presented at the end of Lecture 2. The non-relativistic region [point (1)] has been considered many times elsewhere, the author is not an expert on such a topic, and, also important, such an energy region seems not to be very interesting for CERN physicists.

Possibility (2), although very interesting, has not been studied so far. One possible explanation can be the fact noted above, that in this energy region the selection of a subsystem or subset of events seems to be unavoidable, which makes data analysis more complicated.

So, our main topics are the most questionable possibilities (3) and (4), connected with the highest available excitation energies. It should be kept in mind that the “mixing” and “equilibrium” under discussion are, of necessity, very approximate. The real parton distribution may be very complicated and, for example, anisotropical even for a “local” group of partons in some accompanying reference frame. Still, the introduction of typical energy (related to temperature) can be a meaningful approximation. After all, it is the qualitative changes in the system that we look for.

3.2 SPACE-TIME PICTURE OF THE COLLISIONS

All the following considerations apply to some restricted space–time regions in the collisions, and therefore are connected with some restricted kinematical regions in the observed spectra. These limitations should be kept in mind when evaluating the relevance of the whole approach.

Figure 3.1 shows the projection of four-dimensional space–time on the two-dimensional plane: the time-longitudinal coordinate. At negative times, two relativistic nucleons approach each other. Two “constituent” quarks collide at the origin, and others (being at different positions in the transverse plane!) just go ahead as “spectators”.

The collision time \( t_{\text{coll}} \approx 1/E_{\text{cm}} \) is very short; therefore only rare hard collisions of constituents can take place during \( t_{\text{coll}} \). Evidently, their cross-section is very small and they show up only at the “tails” of spectra or at largest \( p_T, M \), etc.

The important moment of time (which is different in the lab. system for elements of matter with different rapidities) is the relaxation time \( t_{\text{rel}} \). From this moment onwards, the notion of local temperatures \( T \) makes sense.

As the system expands and becomes dilute, the interaction becomes “frozen” at some other moment of time \( t_0 \), after which secondary hadrons propagate freely. In other words: there is the lowest “final” temperature \( T_F \).

*) We do not consider events in which more than one pair of quarks participate. Such event rates and their experimental consequences are discussed in Ref. 40.
So in its individual history any element of matter passes through the stage of colliding jets of partons, local mixing at $T \sim T_p$, and expansion up to $T \sim T_f$. Of course, it is highly non-trivial that $T_f$ is essentially higher than $T_p$, and so far it has not been proved that $T_f$ exists at all. However, we do find empirical evidence for it.

3.3 TRANSVERSE MOMENTUM DISTRIBUTION

It is quite clear that in a truly macroscopic case we would mainly see phenomena connected with the final stage (or with $T_f$), because other phenomena are veiled by the interaction. It is like looking at the Sun and seeing only its outer thin region with $T \sim 6000^\circ$, but not its much hotter interior.

So, as was noted by Pomeranchuk long ago, the distribution of all particles should be

$$dN/dp_t^2 \approx \exp\left(-E_f/T_f\right), \quad E_f^2 = p_t^2 + m^2$$  \hspace{1cm} (3.1)

with the same $T_f$. Experimentally this is really true (see some examples in Fig. 3.2). The thermodynamical interpretation of this experimental fact was emphasized by many people, in particular by Hagedorn at CERN.

In Ref. 41 it was pointed out that the thermodynamical expression (3.1) works "too well" in the following sense. In a truly macroscopic system there should also be collective transverse expansion which, by means of Doppler effect, should create certain deviations from purely thermal spectra, in particular for heavy particles. Such phenomena were indeed observed at lower energies (see, for example, Ref. 37).

It was suggested in Ref. 41 that bag-type effects (similar to those displayed in Fig. 2.1) essentially reduce the pressure in the temperature region of interest, reducing also the collective velocity.

It was recently discovered at the SPS collider that the average $p_t$ of pions is higher than that at the CERN Intersecting Storage Rings (ISR). If this increase is due to the appearance of the transverse "blast wave", then for heavier particles such as $\bar{p}$ such an increase should be stronger. This question will probably be answered soon enough by experiments.

![Graph showing distribution of various secondaries over $E_f = \sqrt{p_t^2 + m^2}$ (Ref. 41). The solid lines all have the same slope of 130 MeV, the dashed one for $\bar{p}$ corresponds to "blast wave" as predicted by hydrodynamical expansion into "empty" vacuum.](image-url)

Fig. 3.2 Distribution of various secondaries over $E_f = \sqrt{p_t^2 + m^2}$ (Ref. 41). The solid lines all have the same slope of 130 MeV, the dashed one for $\bar{p}$ corresponds to "blast wave" as predicted by hydrodynamical expansion into "empty" vacuum.
3.4 MULTIPLICITY AND INCLUSIVE RAPIDITY DISTRIBUTION

Although most of the secondaries observed are "cooled" to the low final temperature $T_p$, their total number contains important information about earlier stages of the collision as well. This can be best expressed by means of the famous Boltzmann statement: entropy never decreases. If the final temperature $T_f$ is fixed, the total entropy and the particle number are just proportional, so the total produced entropy can be directly measured.

If the entropy is produced instantaneously by the shock wave, as assumed by Landau\cite{L}, the following relation holds:

$$\langle n \rangle \sim s^{1/4}$$  \hspace{1cm} (3.2)

In Fig. 3.3 this prediction is compared with the data. It is seen that up to highest ISR\cite{ISR} energies the agreement is nice, but recent SPS data\cite{SPS} definitely deviate from formula (3.2).

The same is true for the predictions of the hydrodynamical calculations for inclusive rapidity distribution (see Fig. 3.4). Here also, some deviations at ISR energies in the central region are seen.

The reason for such deviations is not difficult to explain: they appear because "instantaneous" shock waves do not exist, and in reality entropy production (in some complicated non-equilibrium process) is smaller. However, it is very interesting that predictions based on such macroscopic considerations are at least reasonable up to the energies of the ISR!

As the application of the macroscopic theory to the latest stages is being considered, let us mention the following interesting fact. The hydrodynamical equations (2.2) have a (one-dimensional) scaling solution, corresponding to a "plateau" in the rapidity distribution. This solution is rather simple and bears a remarkable resemblance to the Universe expansion. For example, each observer finds that all others move outwards with a velocity proportional to the distance (as in Hubble’s law).

So, let us summarize our scenario for the "qq" collisions. For $t < \tau_{\text{rel}}$ the system is in the first approximation transparent ($l \ll L$), and in the second one some amount of entropy is produced by the scattering of the partons. At $t \sim \tau_{\text{rel}}$ the local thermalization is reached, so macroscopic considerations based on thermodynamics and hydrodynamics become applicable.

The remaining problem is to formulate some meaningful procedure for estimating the produced entropy and for providing the initial conditions for the hydrodynamical equations.

![Fig. 3.3 Charged-particle multiplicity versus energy, including ISR (Ref. 42) and SPS (Ref. 43) data.](image)

![Fig. 3.4 Pseudorapidity distribution as compared to predictions of the Landau hydrodynamical model, including the same data as in Fig. 3.3.](image)
3.5 NON-EQUILIBRIUM PROCESSES AT EARLIER STAGES OF THE COLLISIONS

It was mentioned in Section 3.3 that for truly macroscopic systems all we see is the cool final stage. However, there always exist non-equilibrium processes which can be used for the studies of the earlier stages.

Even for the Sun, one can pick up neutrinos from the interior and thus measure the interior temperature of the Sun. A similar idea, first proposed by Feinberg⁴⁴, was to look for penetrating particles such as lepton pairs or direct photons, which come freely out of the system created in high-energy hadron collisions.

Another type of phenomenon is the surface effect: quarks with high \( p_z \) can be “evaporated” from the surface at an earlier, hotter stage. Of course, this effect is suppressed by the (surface/volume) \( \sim (l/L) \) ratio, but it can still be seen in some part of the \( p_z \) spectrum.

The last idea of this type explores the finite time of the expansion. Let us consider the production of new flavours (say charmed quarks) by the reactions \( \bar{q}q \rightarrow \bar{c}c \), \( gg \rightarrow \bar{c}c \), etc. In equilibrium the rates of such reactions should be balanced by the inverse ones \( \bar{c}c \rightarrow gg \), \( \bar{q}q \), etc., but in the real condition with a small enough number of charmed quarks the latter rate is negligible. There is not enough time for c quarks to reach equilibrium, so they are produced in the initial stages, and then simply remain there till the final break-up. Similar phenomena are well known in cosmology, for instance the \(^4\text{He} \) fraction in the Universe was “frozen” in exactly the same way.

The theoretical model for such phenomena⁴⁵ consists of two separate parts. The first can be called “local”; it deals with the probability of a given reaction per unit time per unit volume at a given temperature. To give an example, the ideal \( \bar{q} \) and q gas with temperature \( T \) produces \( e^+e^- \) pairs at the following rate:

\[
dW_{e^+e^-}/dT\,d^4x(T) = (\pi a^2/108) \, T^4 . \tag{3.3}
\]

Other reactions can be considered in a similar way.

The second ingredient is more complicated: it is the temperature distribution over space–time:

\[
\Phi(T) = \int \, d^4x \, \delta (T - T(x)) . \tag{3.4}
\]

(thermal stage)

This quantity depends on the expansion properties.

As already explained, this function is meaningful only in the interval \( T_1 < T < T_7 \). Evidently, it is much smaller at \( T_1 \) than at \( T_7 \) because of the expansion. It turns out that both the Landau solution and the scaling one with the rapidity “plateau” give about the same results: \( \Phi(T) \sim T^{-7} \). What is most important, however, is that \( \Phi(T) \) should not depend on the particular process considered. The experimental probability is written as follows:

\[
W = \int \, dT \, \Phi(T) (dW/d^4x)(T) . \tag{3.5}
\]

In Fig. 3.5 we compare the calculation in this model⁴⁵ with data on lepton pairs. No free parameters were used in this calculation, and the continuum⁴⁶ part of the \( \mu^+\mu^- \) spectrum below \( M \lesssim 4 \) GeV agrees well with the data. Above the same point, the Drell–Yan scaling regime seems to show up, so it is natural to ascribe it to hard parton annihilation.

![Fig. 3.5 Dilepton mass spectrum in the plasma model (solid curve) (Ref. 45) and Drell–Yan parton approach (dashed curve) is compared with data. The existence of the non-resonance continuum at small masses is especially interesting.](image)

⁴⁴ In dense quark plasma hadronic resonances such as \( \rho, \psi \), etc. certainly do not exist. However, people were so very surprised to find an \( e^+e^- \) “continuum” below the \( \rho \) mass that they called them “the anomalous” ones.
Fig. 3.6 The slope of the $E_1$ inclusive spectrum calculated in Ref. 46 from various sets of data. The "plateau" seems to be present in all of them.

Fig. 3.7 Energy dependence of the "plateau" height $T$ and pre-exponent $A$ in the fit $d\sigma/dp^2_1 = A(E_1) \exp (-E_1/T(E_1))$. 
Charm production by this mechanism was estimated to take place in about one per thousand of inelastic events at ISR energies.

Also, a reasonable description of the inclusive hadronic spectrum was reached for the $p_T \approx 1$–3 GeV region. However this is more difficult since the “evaporated” quark should further fragment into hadrons etc., so these predictions are not so convincing by themselves. Anyway, decay of “evaporated” quarks results in significant background for jet-type events due to hard-parton scattering.

An interesting phenomenon was found by Zhirov. Figure 3.6 shows the slope of the $E_\perp$ distribution as a function of transverse energy $E_\perp$. It is seen that in any reaction considered, it seems to flatten between $E_\perp \approx 2$ and 3 GeV, and then to increase again. Corresponding values of $T$ at the plateau agree well with the initial temperature $T_i$ as found from dilepton analysis. Also, its energy dependence is about $T_i \sim S^{0.2}$, which is reasonable in the hydrodynamic model (see Fig. 3.7). The preexponent $A_1$ behaves as $s^{-1}$, demonstrating Lorentz contraction at initial stage.

To conclude this discussion, we may say that some empirical arguments seem to demonstrate that the thermodynamical approach is reasonable for the discussion of $E_\perp$ spectra up to $E_\perp$ about 4 GeV. So, the non-equilibrium processes naturally fill the gap between the “soft” exponent at $E_\perp \lesssim 1$ GeV and the “hard” power law at larger $E_\perp$, connected with hard scattering.

The success of such a model implies sufficiently strong mixing, providing meaningful temperatures of about 300–400 MeV at ISR energies, well above the transitions discussed in Lecture 2.

Further experimental tests of all these ideas are badly needed.

3.6 THE ROLE OF CONSTITUENT QUARKS IN COLLISIONS WITH NUCLEI AND VERY HIGH ENERGY HADRONIC COLLISIONS

I would also like to make several remarks concerning the role of constituent quarks in soft processes in general.

There has been a long series of disputes between those people, on the one hand, who consider the nuclei as “transparent” and who make some kind of parton-like models, and, on the other hand, people who consider them as “black”, and suggest something like collective interaction with a nuclear “tube”. The proponents of the former approach mainly considered fragmentation properties with its weak $A$-dependence, whilst those advocating the latter one have stressed the noticeable $A$-dependence in the central region.

It now seems that both were right in some way. The key point which eliminates contradictions between them is the “grained” structure of hadrons, emphasized in Section 1.6.

The interaction is indeed strong on the “constituent” quark-size scale, but the existing nuclei are just too “dilute”. As a result, even for the largest nuclei there is a good chance that the “constituent” quarks will go through the nuclei without coming across another one to interact with. This explains many features of the fragmentation region; these are discussed in Refs. 47.

If the incoming “constituent” quarks interact, this happens at quite separate points in the transverse plane and therefore such interactions are relatively independent (= additive). For instance, the multiplicity on heavy target should approach the Anisovich ratio:

$$\langle n_{\pi^+} / \langle n_p \rangle \rangle \rightarrow 2/3,$$

(3.6)

and this is really what happens. Note that for $A = 1$ this ratio is close to one, for only one pair of quarks interact.

Such independence of subcollisions becomes most apparent in correlation data. A rather complete discussion is given in a recent paper, from which I have borrowed Fig. 3.8. It shows the dependence of the Wroblewski ratio $D/\langle n \rangle$ (where $D$ is the multiplicity dispersion) as a function of the number of recoil protons, $N_p$. It is seen that going from a typical one-quark collision at small $N_p$ to that of all-quark ones at large $N_p$, this quantity really falls by $\sqrt{2}$ for pions and $\sqrt{3}$ for protons, confirming the existence of two or three independent subcollisions.

Also interesting, that good description of data in Ref. 49 is reached with the so-called “effective target” model for constituent quark collision with a “nuclear tube”, being just a variant of the hydrodynamical approach. It also suggests that at constituent quark level the interaction is strong enough, whilst the nuclear “diluteness” makes the picture more complicated.

As far as the existence of constituent quarks as building blocks of hadrons is recognized, one should also reconsider our description of hadron-hadron collisions. The interesting question is about the probability of the processes in which more than one pair of quarks take part, and about its role in inclusive spectra, multiplicity distribution, etc.

As noted in Ref. 51, these questions become more relevant at highest energies because the quark-quark cross-section is known to increase with $s$. This should lead to smaller spectator contribution, additional multiplicity growth, etc., as $s \rightarrow \infty$.

In recent work these questions are considered at quantitative level up to the energies of the SPS collider. It is shown there that although shadowing in total cross-section is not changed dramatically from the ISR to the SPS, the probability for double interactions involving more than two quarks is nearly doubled; at the SPS, it reaches about the
same fraction as that for the interactions involving two quarks. In this paper diffractive dissociation and multiplicity distribution are also considered, and the importance of new experiments separating two- and multiquark collisions is emphasized.

Now, the main lesson from these studies is that strong fluctuation of the parameters of hadron-hadron and hadron-nuclei interactions from one event to another is due to the "grained" substructure of the hadrons and that they are just a mixture of completely different processes. In order to understand them one should consider separately more fundamental processes at constituent quark level.

4. CONCLUSIONS

Let us summarize our general statements, which are as follows:
1) The non-perturbative phenomena in QCD show up already at a scale of about 1 GeV⁻¹, being much smaller than the confinement length. Their relation to the confinement phenomenon is also not evident.
2) It is suggested that such phenomena are connected with strong short-range fluctuations in the QCD vacuum, presumably of an instanton nature.
3) Such fluctuations induce a substructure inside hadrons, identified with the "constituent" quark.
4) The existence of at least three phases of high-density and/or high-temperature matter is suggested.
5) High-energy collisions, especially by heavy ions, present the only possibility to create such matter in the laboratory.
6) Some data on pp and pA collisions can be explained by the production of small expanding clusters of quark–gluon plasma already in collisions of "constituent" quarks.

Acknowledgements

These lectures were kindly organized by the CERN Academic Training Programme Committee, and I am much indebted to its members for such an opportunity. I would also like to express my gratitude to the CERN Theory Division for its hospitality and for many fruitful discussions. I also wish to thank my friends and colleagues who shared with me their understanding of non-perturbative phenomena in QCD, in particular A.I. Vainshtein, M.A. Shifman and V.I. Zakharov.
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