Ref.TH.3552-CERN

A LIMIT ON THE PIONIC COMPONENT OF THE NUCLEON
THROUGH SU(3) FLAVOUR BREAKING IN THE SEA

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A B S T R A C T

It is shown that deep inelastic scattering data provide a very strong bound ($5.0 \pm 1.5\%$) on the fraction of the momentum of the nucleon which is carried by pions. In the CBM this translates into a bag radius greater than 0.87 ± 0.10 fm.

Ref.TH.3552-CERN
March 1983

There is currently a great deal of interest in phenomenological models of hadron structure of relevance to nuclear physics ¹⁾⁻⁴⁾. In particular, much attention has been paid to those extensions of the MIT Bag Model which incorporate chiral symmetry by coupling the pion field to the bag surface ^{1)-3),5)}. Considerable indirect evidence has been accumulated, through calculations of static baryon properties ^{6),7)}, pion-nucleon scattering ⁸⁾, and photoproduction ⁹⁾, which indicates that this is a sensible approach. However, it is rather disturbing that no-one has yet provided direct experimental evidence of a pionic component in the nucleon.

This is even more worrying in view of the enormous differences between the Little Bag Model (LBM) and the Cloudy Bag Model (CBM). In the former ³⁾ it is proposed that the pion coupling to the bag surface should be highly non-perturbative, compressing the bag to a radius of order 0.4 fm. In the latter ^{1),2)} the hadron size is supposed to be given by non-perturbative QCD effects (perhaps related to, but not dominated by pionic corrections) to be of the order (0.8, 1.0) fm. Given that the average internucleon separation in nuclear matter is of order 1.8 fm, it is a priori obvious that the consequences of the two models in a microscopic theory of nuclear structure could be quite different ^{2),10)}. The purpose of this note is to show that existing deep-inelastic scattering data provide quite a stringent limit on the pionic content of the nucleon. To be specific, pions do not carry more than about 5% of the momentum of a nucleon in an infinite momentum frame. Within the CBM this indeed corresponds to a bag radius greater than 0.87 ± 0.1 fm.

In order to obtain this result, let us begin with the observation by Sullivan ¹¹⁾ that there is a contribution to the sea component of the nucleon structure function associated with the process shown in Fig. 1. (This has been discussed recently in connection with the EMC effect ¹²⁾⁻¹⁴⁾, because a relatively small enhancement in the number of pions per nucleon in Fe can explain the observed increase in the structure function for $x < 0.3$.) This contribution has the form ¹¹⁾

$$\delta F_{2N}^{\mu}(x) = \int_x^1 dy f(y) F_{2\pi}^{\mu}(x/y), \quad (1)$$

where $F_{2\pi}^{\mu}$ is the structure function of the pion, and $f(y)$ is the momentum distribution of the pion in an infinite momentum frame. [In writing Eq. (1) we have omitted any explicit mention of the Q^2 dependence of δF_{2N}^{μ} and $F_{2\pi}^{\mu}$ which is required by QCD ^{15),16)}. Its inclusion would not change our discussion in any significant way.] Very simply, Eq. (1) says that we should integrate over

the probability, $f(y)$, of finding a pion with a fraction y of the momentum of the nucleon and finding a quark in that pion with momentum fraction x (that is x/y of y).

A simple calculation gives $f(y)$ in terms of the coupling constant, $g = 13.5$, and the form factor $F(t)$, at the πNN vertex. That is ¹¹⁾

$$f(y) = \frac{3g^2}{16\pi^2} y \int_{\frac{M^2 y^2}{1-y}}^{\infty} \frac{dt}{(t + m_\pi^2)^2} \frac{t |F(t)|^2}{}, \quad (2)$$

with $t = (\vec{q}^2 - q^0^2)$ equal to minus the four-momentum of the exchanged pion. In order to illustrate the behaviour of $f(y)$ we have assumed a simple form factor

$$F(t) = \exp \left[-\lambda (t + m_\pi^2) / m_\pi^2 \right]. \quad (3)$$

[It has been shown ⁷⁾ that Eq. (3) is an excellent approximation to the form factor in the CBM, with $\lambda = -0.106 m_\pi^2 R^2$.] Figure 2 shows the pion momentum distribution for a range of values of λ . It has two essential features. Firstly, $f(y)$ peaks very near $y = 0.25$ over the entire range of λ considered. Secondly, the maximum value of $f(y)$ increases rather rapidly as λ decreases.

Returning to Eq. (1) we see that the pion structure function is evaluated at x/y . As usual we expect that the valence component of the pion should dominate for $x/y > 0.1$. Since y is typically 0.25, this implies that the pionic contribution to the nucleon structure function for $x > 0.03$ involves only non-strange quarks. Thus, if the pion is an important component of nucleon structure it should contribute to breaking the $SU(3)$ flavour symmetry $[SU(3)_F]$ of the sea. Of course, it is generally expected that $SU(3)_F$ will be broken because of the larger strange quark mass, and it would be unreasonable to attribute the entire excess of non-strange sea quarks to the pion. Nevertheless it seems quite reasonable to use any evidence for $SU(3)_F$ breaking to impose a limit on the pionic contribution to the nucleon structure function.

Integrating Eq. (1) over x , we find that

$$\int_0^1 \delta F_{2N}^\mu(x) dx = \left[\int_0^1 F_{2\pi}^\mu(\xi) d\xi \right] \left[\int_0^1 dy y f(y) \right]. \quad (4)$$

There is now quite good data on the pion structure function from Drell-Yan ¹⁷⁾⁻¹⁹⁾, which gives $\int_0^1 F_{2\pi}^\mu(\xi) d\xi = 0.15 \pm 0.04$. Further, from the definition of $f(y)$ we recognize the second integral on the right of Eq. (4) as the average fraction of

the nucleon momentum carried by the pion - $\langle y \rangle_\pi$. Finally, if as we argued above the SU(2) excess in the sea makes a contribution, $F_{2N, \text{excess}}$, greater than that from the pion alone, we obtain the bound

$$\langle y \rangle_\pi \leq F_{2N, \text{excess}} / (0.15 \pm 0.04). \quad (5)$$

As a simple estimate of the SU(3) breaking in the sea we take the estimate of Feynman and Field (FF) that for a free proton²⁰⁾

$$\bar{U} = \int_0^1 dx \times \bar{u}(x) = 0.015, \quad (6a)$$

$$\bar{D} = \int_0^1 dx \times \bar{d}(x) = 0.021, \quad (6b)$$

$$\bar{S} = \int_0^1 dx \times \bar{s}(x) = 0.011. \quad (6c)$$

In fact, their estimate of the excess of $\bar{D}$ over $\bar{U}$ was based on the Gottfried sum rule^{15),20)}

$$\int_0^1 \frac{dx}{x} \left(F_2^{ep}(x) - F_2^{en}(x) \right) = \frac{1}{3} + \frac{2}{3} \int_0^1 [\bar{u}(x) - \bar{d}(x)] dx, \quad (7)$$

where the left-hand side was taken as 0.27. The CFS group at Fermilab has found supporting evidence for $\bar{D} > \bar{U}$ by about 30% on the basis of the Drell-Yan process in proton-nucleus collisions²¹⁾.

In contrast, there is essentially no direct evidence to support the claim that $\bar{S} < \bar{U}$. Of course, the CDHS group has reported that²²⁾

$$\frac{2\bar{S}}{(\bar{U} + \bar{D})} \Big|_{Fe} = 0.52 \pm 0.09, \quad (8)$$

but this is in Fe, where the EMC results have shown that the sea is modified¹²⁾. It is nevertheless striking that taken at face value FF would predict $2\bar{S}/(\bar{U} + \bar{D}) = 0.61$. If one attributes all of the SU(3)_F breaking in the nucleon to pionic effects, which are doubled in Fe, this ratio becomes 0.44. Clearly the FF estimate of SU(3)_F breaking is near to being correct and we shall use it to estimate $F_{2N, \text{excess}}$. However, high statistics di-muon data on H₂ and D₂ to really pin down S and $\bar{S}$ would be extremely valuable, and should be given a high priority.

Clearly the pion exchange process of Fig. 1 does predict that the excess of $\bar{D}$ to $\bar{U}$ should be in the ratio 5 to 1 in the proton. To first order this is consistent with Eqs. (6a) and (6b). If one were less circumspect this might suggest that some other mechanism must provide an equal excess of $\bar{U}$ and $\bar{D}$ over $\bar{S}$ of about 20%. While this would sharpen our bound considerably we shall take the more conservative approach, and use all of the excess of non-strange over strange quarks $\{(\bar{U}+\bar{D})/2-\bar{S}\} = 0.007$ to estimate $F_{2N,\text{excess}}$. A trivial calculation then yields $F_{2N,\text{excess}} = (10/9) \times 0.007 = 0.008$. Together with Eq. (5) this gives the required bound on the average fraction of the momentum of the nucleon carried by the pion

$$\langle y \rangle_{\pi} \leq 5 \pm 1.5 \% \quad (9)$$

In Fig. 3 we show the average fraction of the momentum of the nucleon carried by pions, $\langle y \rangle_{\pi}$, as a function of the cut-off parameter, λ , at the $NN\pi$ vertex [see Eq. (3)]. Clearly Eq. (9) is a very strong constraint on that parameter. It is not possible to accept a value of λ smaller than $0.039^{+0.012}_{-0.006}$.

We also show in Fig. 3 the CBM radius corresponding to each value of λ . The lower bound on the bag radius in the CBM implied by Eq. (9) is $R = 0.87 \pm 0.10$ fm. Of course, there are many defects in the static bag model, and one cannot insist too strongly on an absolute value of R . One expects the bag to have some surface thickness ²³⁾⁻²⁵⁾, and this together with centre-of-mass and recoil corrections could change the simple relationship between R and λ . Nevertheless, we expect this upper bound to be a good indication of the size of the region within which quarks are confined in the nucleon. The concept of a little bag with a size of order (0.3-0.5) fm is definitely excluded.

Let us briefly consider some objections which might be raised to the present calculation. For example, we have considered only that graph involving a direct interaction between the γ^* and the pion. There are a number of other graphs which should be included in principle. While those graphs could have important effects on the valence distribution, the small- x or sea contribution should be adequately described by Fig. 1. Secondly, we have neglected the off-shell behaviour of the pion structure function ($t \neq -m_{\pi}^2$). However, the typical momentum of the virtual pion is $\sqrt{t} \sim 300$ MeV/c, which is not far off-shell.

One might also ask whether we have a rigorous upper bound. It is logically possible that the strange sea could be bigger than the non-strange sea apart from pionic effects. However, with m_s some 200 MeV heavier than $m_{u,d}$, no

one has ever suggested a physical mechanism for it. Finally, all our arguments relied on integrated quantities, such as $\bar{D}$, $\bar{U}$ and $\bar{S}$. This was unavoidable in view of the lack of experimental information about the shapes of $s(x)$, $\bar{s}(x)$ and $[\bar{d}(x) - \bar{u}(x)]$. It is therefore appropriate to close with a plea for better measurements of these three quantities in the free proton. Such data would not only allow us to sharpen the bound on $\langle y \rangle_\pi$ considerably, but would also deepen our understanding of how chiral symmetry is realized in Nature.

ACKNOWLEDGEMENTS

I would like to thank Chris Llewellyn Smith for informative discussions related to this work.

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FIGURE CAPTIONS

- Figure 1 The contribution of the pion to the structure function of the nucleon.
- Figure 2 The probability $f(y)$ of finding a pion carrying a fraction y of the momentum of the nucleon, for several values of the cut-off parameter λ (or bag radius R) - see Eq. (3).
- Figure 3 The average fraction of the nucleon's momentum carried by the pion as a function of λ (or bag radius R). The shaded area represents the bound obtained in this work.

- Figure 1 -

- Figure 2 -

- Figure 3 -