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NON-LINEAR REPRESENTATIONS OF EXTENDED SUPERSYMMETRY
WITH CENTRAL CHARGES

S. Ferrara

CERN -- Geneva

L. Maiani

Istituto di Fisica dell'Università, Roma
and INFN, Sezione di Roma

and

P. C. West

Mathematics Department
King's College, London

ABSTRACT

A new class of non-linear representations of N extended supersymmetry is found. These non-linear realizations extend the Volkov-Akulov non-linear realizations to include central charges which can also be non-linearly realized. The general construction of the corresponding non-linear Lagrangians is presented. The coupling to N extended supergravity provides the simultaneous occurrence of the Higgs and super-Higgs effects. The corresponding formulae for spin- $\frac{3}{2}$ gravitinos and spin-1 vector boson masses are derived.

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1. INTRODUCTION

Non-linear realizations of global symmetries have been proven to be a useful tool in the past for exploring general properties of spontaneously broken theories, such as chiral Lagrangians with their related low-energy theorems [1]. In comparison, non-linear realizations of supersymmetry is a much less explored subject.

A class of non-linear representations of the graded extension of the $N = 1$ Poincaré algebra (N denotes the number of spinorial charges) has been constructed by Volkov and Akulov [2]. These representations have an obvious extension to the case of larger N . However, for $N > 1$ the supersymmetry algebra can admit central charges, as shown a long time ago by Haag, Lopuszanski and Sohnius [3]. These central charges naturally appear in model field theories with a spontaneously broken gauge internal symmetry, supersymmetry remaining unbroken [4]. Central charges also appear if massive multiplets are explicitly introduced into $N = 2$ supersymmetric Yang-Mills [5] theories and in $N = 8$ supergravity, with a supersymmetry breaking obtained through dimensional reduction from five space-time dimensions [6].

In this paper we extend the Volkov-Akulov Lagrangian to include non-linear realizations of N -extended supersymmetry with central charges. The N supersymmetries as well as the central charge symmetries are non-linearly realized. The resulting theory describes a system of N interacting Goldstone spinors (goldstinos) and $N(N-1)$ Goldstone bosons.

The interactions are determined by the supersymmetry-breaking parameter d [dimension of a $(\text{mass})^2$] and by $N(N-1)$ dimensionless constants g_{ij} , g'_{ij} (the couplings are symmetric in i, j).

In the present paper we also discuss non-linear realizations of local extended supersymmetry. Unlike the global case, we restrict our considerations to Lagrangians that are only invariant under linearized local supersymmetry transformations. As in the $N = 1$ case [7], this is sufficient to get all the information about the mass spectrum of the theory under the assumption that the full non-linear coupling exists.

The minimal coupling of non-linear representations to N -extended supergravity gives model-independent relations amongst the masses of the gravitinos and vector bosons of the supergravity multiplet. These formulae are obtained after requiring the cancellation of the cosmological constant. The resulting theory exhibits the simultaneous occurrence of the Higgs and super-Higgs effect: the $N(N-1)/2$ vector bosons combine with an equal number of central charge Goldstone bosons, whilst the N -goldstinos are eaten up by the corresponding N -gravitinos.

The coupling of our non-linear realizations to the supergravity Lagrangians with a cosmological constant has the further effect of distorting the central charges, so that they no longer commute with the spinorial generators or amongst themselves. The result is entirely equivalent to N -extended supersymmetry in de Sitter [8] space. As a consequence, this class of theories of spontaneously broken supergravity and vanishing cosmological constant leads to a broken $SO(N)$ gauge group. By letting some of the couplings g_{ij} vanish, one can also obtain intermediate situations where the $SO(N)$ group is partially or completely unbroken. The locally supersymmetric couplings can be computed in a simple way for $N = 2$ [9] and also for $N = 3$ [10]. For $N \geq 4$, scalar fields start to appear in the supergravity multiplet, and this complicates the analysis because the scalar fields could participate in the Higgs and super-Higgs effect through their potential. Owing to these complications, we confine our analysis to $N = 2$ and $N = 3$ extended supergravity.

After cancellation of the cosmological constant, the simplest invariant Lagrangian leads to gravitino and vector boson masses which are uniquely determined in terms of d , k , and g_{ij} , k being the gravitational constant. For $N = 2$ the two gravitinos have the same mass, and this result persists to all orders. For $N = 3$ we get an apparently more interesting situation. It is possible to break the $O(3)$ symmetry by giving different masses to the triplet of vector bosons, because of the three different coupling constants g_{ij} in the non-linear global Lagrangian. Even restricting to the simplest case, we would expect gravitino mass splitting to appear in higher-order corrections as a reflection of the

$O(3)$ breaking in the vector boson sector. In fact we find that it is possible to add further terms to the minimal Lagrangian which give arbitrary mass splitting to the gravitinos already at the tree level. It should be possible to have one or more gravitinos with vanishing mass, although this does not correspond in general to the recovery of one or more supersymmetries. In this paper our attention has been focused on the invariance properties of the theory. A physically meaningful Lagrangian must also satisfy further conditions, e.g. the absence of Velo-Zwanziger anomalies [7,11], at the classical level, or the absence of unwanted ultraviolet divergences, at the quantum level. The discussion of different gravitino masses of Section 4 could also be affected by these problems. Investigation of these further questions is beyond our present scope.

The plan of the paper is the following: In Section 2 we discuss non-linear realizations of N-extended global supersymmetry with central charges. In Section 3 we derive the locally supersymmetric Lagrangians in the linearized approximation and discuss the Higgs and super-Higgs effects. The possibility of gravitino mass splitting is discussed in Section 4, where we also comment on the possibility of constructing, in the case of local supersymmetry, manifestly invariant non-linear Lagrangians.

2. NON-LINEAR REALIZATIONS OF GLOBAL POINCARÉ SUPERSYMMETRY

We consider the global N-extended supersymmetry algebra in its most general form, as described by Haag, Lopuszanski and Sohnius [3] *):

$$\{Q_{\alpha}^i, \bar{Q}_{\beta}^j\} = -2(\gamma^{\mu})_{\alpha\beta} P_{\mu} \delta^{ij} - 2Z^{ij} \mathbb{1}_{\alpha\beta} - 2Z'^{ij} (\gamma_5)_{\alpha\beta} \quad (2.1)$$

where Q_{α}^i are a set of N-translationally invariant spin 1/2 Majorana charges; Z^{ij} and Z'^{ij} are a set of $N(N-1)$ operators belonging to the centre of the algebra, antisymmetric in ij . The group composition in the infinitesimal implies the commutation relations:

*) In this paper we use a Majorana representation, with real γ -matrices and the metric (1, 1, 1, -1).

$$[\bar{\epsilon}^i Q^i, \bar{\epsilon}^j Q^j] = -2 \bar{\epsilon}^i \gamma^\mu \epsilon^i P_\mu - 2 \bar{\epsilon}^i \epsilon^j Z_{ij} - 2 \bar{\epsilon}^i \gamma_5 \epsilon^j Z'_{ij} \quad (2.2)$$

We consider a group element of the form

$$\exp \left[\frac{1}{d} (\bar{S} Q - i C_{ij} Z_{ij} - i C'_{ij} Z'_{ij}) \right] \exp(-ix \cdot P) \quad (2.3)$$

where d is a dimensionful constant, $\dim d = 2$. The elements (2.3) form a manifold parametrized by coordinates: ζ_i , C_{ij} , and C'_{ij} .

In the spirit of non-linear realizations, ζ_i , C_{ij} , and C'_{ij} are to be considered as fields, functions of space-time coordinates, which provide a basis for a non-linear representation of the commutation rules (2.1).

The action of a group element characterized by parameters ϵ_i , ω_{ij} , ω'_{ij} induces a motion on the manifold (2.3) which is easily seen to be of the form

$$\delta x^\mu = -\frac{i}{d} (\bar{\epsilon}^i \gamma^\mu S^i) = -\xi^\mu \quad (2.4)$$

$$\delta S_i = d \epsilon_i + \xi^\nu \partial_\nu S_i \quad (2.5)$$

$$\delta C_{ij} = d \omega_{ij} + \frac{i}{4} (\bar{\epsilon}_i S_j - \bar{\epsilon}_j S_i) + \xi^\nu \partial_\nu C_{ij} \quad (2.6)$$

$$\delta C'_{ij} = d \omega'_{ij} + \frac{i}{4} (\bar{\epsilon}_i \gamma_5 S_j - \bar{\epsilon}_j \gamma_5 S_i) + \xi^\nu \partial_\nu C'_{ij} \quad (2.7)$$

The starting point for the construction of invariant Lagrangians is the determination of the "covariant derivatives" of the fields introduced above. To this end, one considers an element of the graded Lie algebra of the form

$$g^{-1} \delta g = dx^\mu \left\{ -i f_\mu^a P_a + \frac{i}{d} C_{\mu ij} Z_{ij} + \frac{i}{d} C'_{\mu ij} Z'_{ij} + \frac{1}{d} \bar{g}_\mu^i Q^i \right\} \quad (2.8)$$

where g is the element (2.3). An explicit computation gives

$$f_{\mu}^a = f_{\mu}^a + \frac{i}{d^2} (\bar{S}^i \gamma^a \partial_{\mu} S^i) \quad (2.9)$$

$$C_{\mu ij} = \partial_{\mu} C_{ij} - \frac{i}{4d} [\bar{S}_i \partial_{\mu} S_j - (i \rightarrow j)] \quad (2.10)$$

$$C'_{\mu ij} = \partial_{\mu} C'_{ij} - \frac{i}{4d} [\bar{S}_i \gamma_5 \partial_{\mu} S_j - (i \rightarrow j)] \quad (2.11)$$

$$g_{\mu i} = \partial_{\mu} S_i \quad (2.12)$$

Under a supersymmetry transformation, the above quantities behave as covariant vectors, e.g.:

$$\delta f_{\mu}^a = \xi^{\nu} \partial_{\nu} (f_{\mu}^a) + (\partial_{\mu} \xi^{\nu}) f_{\nu}^a \quad (2.13)$$

with ξ^{ν} given by Eq. (2.4). The matrix f_{μ}^a can be viewed as a covariant vierbein, and its inverse, f_a^{μ} , can be used to transform world-vector indices into "flat" indices, e.g.:

$$C_{a ij} = f_a^{\mu} C_{\mu ij} \quad (2.14)$$

$$\delta (C_{a ij}) = \xi^{\nu} \partial_{\nu} (C_{a ij}) \quad (2.15)$$

A general invariant action for the geometrical fields ξ , C , C' , has the form

$$A = \int d^4x (\det f) \mathcal{L}(C_a, C'_a, g_a) \quad (2.16)$$

where $\mathcal{L}$ is a Lorentz-invariant function. The action (2.16) describes the interaction of the N -Goldstone fermions of the N -extended supersymmetry, with the $N(N-1)$ Goldstone bosons, each one associated with the spontaneous breaking of the corresponding central charge.

The minimal Lagrangian, generalizing the Volkov-Akulov action [2], is

$$\begin{aligned} \mathcal{L}_{MIN} = & -\frac{d^2}{2} (\det f) - \frac{1}{4} (\det f) \eta^{ab} \sum_{ij} [g_{ij}^2 C_{a ij} C_{b ij} \\ & + g_{ij}^{\prime 2} C'_{a ij} C'_{b ij}] \end{aligned} \quad (2.17)$$

The dimensional constant d is introduced in order for ζ to be a well-normalized Majorana spinor. In fact,

$$-\frac{d^2}{2}(\det f) = -\frac{d^2}{2} - \frac{i}{2} \bar{S}_i \not{\partial} S_i + \dots \quad (2.18)$$

Correspondingly, we have introduced $N(N-1)$ dimensionless constants g_{ij} , g'_{ij} such that, for example,

$$g_{ij} C_{ij} = \partial_\mu Z_{ij} - \frac{i}{4d} g_{ij} (\bar{S}_i \partial_\mu S_j - (i \rightarrow j)) \quad (2.19)$$

where z_{ij} are normalized scalar fields [no sum over repeated indices is implied in Eq. (2.19) and the constants g_{ij} are symmetric in ij].

The introduction of matter fields is straightforward [12]. For simplicity we restrict ourselves to the case of one, complex scalar field, with central charges determined by:

$$[Z_{ij}, \phi] = i\sqrt{d} q_{ij} \phi \quad (2.20)$$

q_{ij} being a set of real constants. To simplify the notation, we set $q'_{ij} = 0$ and ignore the axial central charges Z'_{ij} . A supersymmetry transformation acts on ϕ according to

$$\delta \phi = \xi^\mu \partial_\mu \phi + i q_{ij} \xi_{ij} \phi \quad (2.21)$$

$$\xi_{ij} = \frac{i}{2\sqrt{d}} (\bar{\epsilon}_i S_j - \bar{\epsilon}_j S_i)$$

One easily checks the relation

$$(\delta_1 \delta_2 - \delta_2 \delta_1) \phi = 2(\bar{\epsilon}_2 \not{\gamma} \epsilon_1) i \partial_\mu \phi + 2\sqrt{d} q_{ij} \bar{\epsilon}_1^i \epsilon_2^j \phi \quad (2.22)$$

in agreement with Eqs. (2.2) and (2.20).

We may also introduce the covariant derivative of ϕ ,

$$D_\mu \phi = \partial_\mu \phi - \frac{i}{d\sqrt{d}} q_{ij} (\bar{S}_i \partial_\mu S_j) \phi \quad (2.23)$$

One checks directly that

$$D_a \phi = f_a^\mu D_\mu \phi \quad (2.24)$$

transforms like ϕ . As a consequence, the action

$$A_{\text{MATTER}} = \int d^4x (\det f) \mathcal{L}_{\text{MATTER}}(\phi, \phi^*, D_a \phi, D_a \phi^*) \quad (2.25)$$

is invariant if $\mathcal{L}_{\text{matter}}$ is Lorentz-invariant as well as invariant under the global U(1) transformations associated with the central charges, according to Eq. (2.20). The generalization to an arbitrary number of matter fields is straightforward.

The actions (2.17) and (2.25) summarize the low-energy relations which follow from current and supercurrent algebra. In agreement with more general considerations [13], all supersymmetries are spontaneously broken, with the same order parameter d . The situation is different in the case of the symmetries associated with the central charges. For $g_{ij} \neq 0$, the corresponding central charge is spontaneously broken. In this case, we may introduce a new field:

$$\psi = \exp \left[\frac{z_i}{\sqrt{a}} g_{ij} C_{ij} \right] \phi \quad (2.26)$$

and ψ is inert under the action of Z_{ij} . Thus the Lagrangian for ψ is no longer restricted by U(1) symmetries (e.g. arbitrary mass-splittings are allowed). For $g_{ij} = 0$, central charges are conserved, and the corresponding fields z_{ij} in Eq. (2.19) reduce to ordinary, neutral, matter fields, which can be dropped altogether. In this situation, Eqs. (2.17) and (2.25) provide an extension of the Volkov-Akulov action to the case of the N-extended supersymmetries, with conserved and linearly realized central charges.

3. HIGGS AND SUPER-HIGGS EFFECT IN EXTENDED SUPERGRAVITY

In this section we discuss the coupling of the non-linear realizations introduced in Section 2 to the N-extended supergravity. In the $N = 1$ case, the coupling of the Volkov-Akulov Lagrangian to the supergravity action with a cosmological term has been used by Deser and Zumino [7] to obtain a model-independent relation between the gravitino mass and the supersymmetry-breaking parameter d . In our case, we find that a Higgs effect takes place, in parallel with the super-Higgs effect, whereby the Goldstone bosons of the broken central charges combine with the massless vector fields of the supergravity multiplet to produce massive vector

bosons. As anticipated in Section 1, we discuss in detail the $N = 2$ case, and indicate in the end the results for $N = 3$.

The $N = 2$ supergravity multiplet contains one graviton, associated to the vierbein e_μ^a , two gravitinos $\psi_{\mu i}$, and one vector field which we denote by

$$A_{\mu ij} = -A_{\mu ji} = \frac{\epsilon_{ij}}{2\sqrt{2}} A_\mu \quad (3.1)$$

where A_μ is canonically normalized. The supergravity Lagrangian [9] is

$$\mathcal{L}_{SG} = \frac{eR}{2\kappa^2} - \frac{i}{2} \bar{\psi}_\mu R_i^\mu - e (F_{\mu\nu ij})^2 + ik \bar{\psi}_\mu (\hat{F}_{\rho\sigma ij} + \frac{1}{2} k_5 \hat{F}_{\rho\sigma ij}) \psi_\sigma + \dots \quad (3.2)$$

+ (four-fermion terms)

where

$$e = \det(e_\mu^a), \quad R_i^\mu = \epsilon^{\mu\rho\sigma} \gamma_5 \gamma_\nu D_\rho \psi_\sigma \quad (3.3)$$

and k is the gravitational coupling; R is the Einstein curvature scalar, $R_{\mu i}$ the Rarita-Schwinger operator, and $\hat{F}_{\mu\nu ij}$ the supercovariant field strength, associated with the vector field in (3.1). To Eq. (3.2) we add the supersymmetric cosmological term [14]

$$\mathcal{L}_c = \frac{3}{4} \frac{\mu^2}{\kappa^2} e + \frac{i}{2} \mu e \bar{\psi}_\mu \sigma^{\rho\sigma} \psi_\mu + \frac{ik\mu}{4} \bar{\psi}_\mu \gamma_5 \gamma_\nu \psi_\sigma A_{\rho ij} \epsilon^{\mu\rho\sigma} \quad (3.4)$$

The full action obtained by adding Eqs. (3.2) and (3.4) is invariant under the following local supersymmetry transformations:

$$\delta e_\mu^a = ik \bar{\epsilon}_i \gamma^a \psi_\mu \quad (3.5)$$

$$\delta \psi_{\mu i} = \frac{2}{\kappa} (D_\mu \epsilon_i + \frac{\kappa\mu}{2} A_{\mu ij} \epsilon_j) + \frac{1}{2\kappa} \gamma_\mu \epsilon_i - 2\sigma^{\rho\sigma} \hat{F}_{\rho\sigma ij} \gamma_\mu \epsilon_j \quad (3.6)$$

$$\delta A_{\mu ij} = \frac{i}{2} (\bar{\epsilon}_i \psi_{\mu j} - (i \rightarrow j)) \quad (3.7)$$

where D_μ stands for the gravitational covariant derivative with spin 3/2-torsion.

We now wish to add the Lagrangian Eq. (2.17) to $\mathcal{L}_{SG} + \mathcal{L}_c$. Since, in general,

there are only $N(N-1)/2$ vector fields in the supergravity multiplet, we may drop out the Goldstone bosons associated with the axial central charges, which correspond to set $g'_{ij} = 0$, keeping $g_{ij} \neq 0$. As usual, to maintain local supersymmetry, one has to add new couplings as well as new terms to the transformation laws. We do not attempt to determine the full Lagrangian and the complete transformation laws. Following the strategy of Deser and Zumino [7], we shall restrict the complete Lagrangian to terms bilinear and trilinear in the various fields. The resulting Lagrangian is locally supersymmetric if we include variations at most linear in the fields, for bilinear terms, and field independent for the trilinear part. At this level the variations of the fields are those given in Eqs. (2.5), (2.6), and (3.5) to (3.7). Under the assumption that the iteration to higher order of the same procedure produces a fully invariant Lagrangian, the approximation we are using is sufficient for studying the mass spectrum of the theory, in tree-level. We write

$$\mathcal{L} = \mathcal{L}_{SG} + \mathcal{L}_C + \mathcal{L}_H + \delta\mathcal{L} \quad (3.8)$$

where

$$\mathcal{L}_H = -\frac{d^2}{2}e - \frac{i}{2}e \bar{\psi}_i \not{\partial} \psi_i - \frac{1}{4} \sum_{ij} g_{ij}^2 C_{\mu ij} C_{\mu ij} \quad (3.9)$$

is the linearized, goldstino-Goldstone boson action, and $\delta\mathcal{L}$ contains all corrections needed for maintaining local supersymmetry. We find

$$\begin{aligned} \delta\mathcal{L} = & -\frac{ikd}{2} (\bar{\psi}_{\mu i} \gamma^{\mu} \psi_i) - \frac{i}{2} (\bar{\psi}_i \psi_i) \\ & - \frac{ik}{4} \bar{\psi}_i \gamma_{\mu} \psi_j A_{\mu ij} - \frac{1}{4} \sum_{ij} g_{ij}^2 \left(\frac{k^2 d^2}{4} A_{\mu ij} A^{\mu}_{ij} \right. \\ & \left. - kd D_{\mu} C_{ij} A^{\mu}_{ij} \right) - \frac{i}{4} \sum_{ij} g_{ij}^2 \left(k \bar{\psi}_i \psi_j - \right. \\ & \left. - \frac{1}{4} d \bar{\psi}_i \gamma_{\mu} \psi_j \right) \left(C^{\mu}_{ij} - \frac{kd}{2} A^{\mu}_{ij} \right) \end{aligned} \quad (3.10)$$

The vanishing of the cosmological constant requires

$$\mu^2 = \frac{2}{3} d^2 k^2 \quad (3.11)$$

Furthermore, choosing the gauge $\zeta = 0$ and $C_{ij} = 0$ in Eq. (3.8) we obtain

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{SG} + i m_{3/2} \bar{\Psi}_{\mu i} \sigma^{\mu\nu} \Psi_{\nu i} - 2 \sum_{ij} m_{ij}^2 A_{\mu ij} A^{\mu ij} \\ & + \frac{i}{2} \kappa m_{3/2} \bar{\Psi}_{\mu i} \gamma_5 \gamma_\nu \Psi_{\sigma j} A_{\rho ij} \varepsilon^{\mu\nu\sigma\rho} \end{aligned} \quad (3.12)$$

with

$$m_{3/2} = \mu/2 \quad (3.13)$$

$$m_{ij}^2 = \frac{3}{16} g_{ij}^2 m_{3/2}^2 \quad (3.14)$$

The final theory describes, besides gravitation, massive gravitinos interacting with a massive vector boson, the charges associated to the latter interaction being

$$g = \kappa m_{3/2} \quad (3.15)$$

We now briefly consider the $N = 3$ extended supergravity theory coupled to the $N = 3$ non-linear realization with central charges.

In this case the supergravity multiplet consists of the following fields:

$$e_\mu^a, \Psi_\mu^i, A_{\mu ij}, \lambda \quad (3.16)$$

where $i, j = 1, \dots, 3$ and λ is an $O(3)$ singlet Majorana spin-1/2 fermion.

The $N = 3$ supergravity Lagrangian with cosmological constant was given in Ref. [13], and we do not repeat its form here.

For our purpose, the only interesting new term is the extra term in the local supersymmetry variation of $\delta A_{\mu ij}$ compared to Eq. (3.7):

$$\delta' A_{\mu ij} = \frac{i}{2\sqrt{2}} \varepsilon_{ij\kappa} \bar{\epsilon}_\kappa \gamma_\mu \lambda \quad (3.17)$$

This extra term in turn requires the additional terms $\delta\mathcal{L}'$ to $\delta\mathcal{L}$ given by (3.10). They are

$$\delta \mathcal{L}' = -\frac{i}{4\sqrt{2}} \sum_{ij} g_{ij}^2 k \bar{S}_k \gamma_r \lambda \varepsilon_{ijk} (C r_{ij} - \frac{kd}{2} A^{\mu}_{ij}) \quad (3.18)$$

The full invariant-action up to trilinear terms becomes

$$\mathcal{L}_{SG} + \mathcal{L}_C + \mathcal{L}_N + \delta \mathcal{L} + \delta \mathcal{L}' \quad (3.19)$$

The triplet of vector bosons $A_{\mu ij}$ can acquire three different masses depending on the arbitrary couplings g_{ij} , whilst the spin-1/2 field λ remains massless. In fact, to this order in the fields, no restriction on the coupling constants g_{ij} emerges from the coupling of $\mathcal{L}_M$ to supergravity. The extension to N-extended supergravity with $N > 3$ implies the further discussion of the scalar potential of the supergravity multiplets. Our results can be applied only for those extrema of the potential not giving a v.e.v. to the scalars.

4. MANIFESTLY INVARIANT FORMULATION AND GRAVITINO MASSES

The results of the previous section can be made more transparent with the observation that all the terms in Eqs. (3.8) to (3.10) which contain the coupling constants g_{ij} can be absorbed, to the order of our approximation, in the single expression^{*)}

$$-\frac{k^2 d^2}{16} \sum_{ij} g_{ij}^2 (B_{r ij})^2 \quad (4.1)$$

with

$$B_{r ij} = A_{r ij} - \frac{2}{kd} C_{r ij} - \frac{i}{2d} (\bar{S}_i \psi_{rj} - (i \rightarrow j)) + \frac{i}{4kd^2} \bar{S}_i \gamma_r S_j \quad (4.2)$$

It is easy to check that the variation of $B_{\mu ij}$ under a local supersymmetry transformation is at least quadratic in the fields, i.e. $B_{\mu ij}$ is invariant within the approximation considered.

Under the central charge transformations given in Section 1, the variations of $A_{\mu ij}$ and $\psi_{\mu i}$ are

^{*)} For simplicity, we restrict ourselves to the $N = 2$ case; for $N > 2$, extra terms must be added to $B_{\mu ij}$, which can be easily determined from the results of Section 3.

$$\delta A_{\mu ij} = \frac{2}{\kappa} \partial_{\mu} \omega_{ij} \quad (4.3)$$

$$\delta \psi_{\mu i} = 0 \quad (4.4)$$

so that $B_{\mu ij}$ is invariant.

Thus each term in the sum given by Eq. (4.1) is separately invariant, and this explains why arbitrary mass terms for the vector fields are possible.

It is possible to improve the previous consideration by recalling that in the presence of a cosmological term, i.e. $\mu \neq 0$, the Abelian central charge algebra is promoted to the non-Abelian $SO(N)$ algebra. As a consequence, Eqs. (4.3) and (4.4), as well as (2.5) and (2.6), acquire extra terms of order μ which can be determined as follows.

The commutation rules of the Z_{ij} generator for $\mu \neq 0$ are

$$[Z_{ij}, Z_{kl}] = -\frac{i\mu}{2} (\delta_{il} Z_{jk} - \delta_{ik} Z_{jl} - (i \rightarrow j)) \quad (4.5)$$

Correspondingly, the transformation laws of the fields become

$$\begin{aligned} \delta A_{\mu ij} &= \frac{2}{\kappa} \partial_{\mu} \omega_{ij} - \mu (\omega_{ik} A_{\mu kj} - (i \rightarrow j)) \\ \delta \psi_{\mu i} &= -\mu \omega_{ij} \psi_{\mu j} \\ \delta S_i &= -\mu \omega_{ij} S_j \end{aligned} \quad (4.6)$$

$$\delta C_{\mu ij} = \partial_{\mu} \omega_{ij} - \mu [\omega_{ik} C_{\mu kj} - (i \rightarrow j)]$$

where $C_{\mu ij}$ is still defined by Eq. (2.8) but now Z_{ij} obey the commutation relations given by Eq. (4.5); $C_{\mu ij}$ provides the non-Abelian generalization of the covariant derivatives $C_{\mu ij}$ introduced in Section 2. From Eq. (4.6) it follows that

$$\delta B_{\mu ij} = -\mu (\omega_{ik} B_{\mu kj} - (i \rightarrow j)) \quad (4.7)$$

It is obvious that the term (4.1), although invariant under local supersymmetry, is not invariant under (4.7). However, with the well-known procedure

of non-linear realizations, we can replace $B_{\mu ij}$ with the $SO(N)$ -invariant fields

$$\tilde{B}_{\mu ij} = B_{\mu ij} + \frac{1}{d} (C_{ik} B_{\mu kj} - (i \rightarrow j)) + \dots \quad (4.8)$$

where dots indicate terms of higher order in C_{ij} . Within our approximation, the term (4.1) is equivalent to

$$-\frac{\kappa^2 d^2}{16} \sum_{ij} g_{ij}^2 \tilde{B}_{\mu ij}^2 \quad (4.9)$$

A similar construction can be applied to the gravitino fields.

The analogue of $B_{\mu ij}$ is

$$\begin{aligned} \phi_{\mu i} = & \psi_{\mu i} - \frac{1}{2\kappa d} \gamma_{\mu} S_i - \frac{2}{\kappa d} (\partial_{\mu} S_i + \frac{\kappa}{2} A_{\mu j} S_j) \\ & + \frac{2}{d} \sigma_{\rho\sigma} \gamma_{\mu} \hat{F}_{\rho\sigma ij} S_j \end{aligned} \quad (4.10)$$

$\phi_{\mu i}$ is invariant under local supersymmetry and transforms as a vector under local $SO(N)$ variation. The $SO(N)$ -invariant gravitino fields are

$$\tilde{\Phi}_{\mu i} = \phi_{\mu i} + \frac{1}{d} C_{ik} \phi_{\mu k} + \dots \quad (4.11)$$

The Lagrangian (3.8) can now be simply rewritten as follows*):

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{SG}(\tilde{e}, \tilde{\Phi}_{\mu}, \tilde{B}_{\mu}) + i m_{3/2} \tilde{e} \tilde{\Phi}_{\mu}^i \sigma^{\mu\nu} \tilde{\Phi}_{\nu}^i \\ & - \tilde{e}^2 \sum_{ij} m_{ij}^2 (\tilde{B}_{\mu ij})^2 + \frac{i}{2} \kappa m_{3/2} \tilde{\Phi}_{\mu i} \gamma_5 \gamma_{\nu} \tilde{\Phi}_{\nu j} \tilde{B}_{\mu ij} \varepsilon^{\mu\nu\rho\sigma} \end{aligned} \quad (4.12)$$

Each term in (4.12) is now separately invariant under local supersymmetry and $SO(N)$ transformations up to variations at most quadratic in the fields. We are now ready to discuss the question of gravitino masses.

Equation (4.12) implies degenerate gravitinos. However, we are free to add to (4.12) any gravitationally invariant term constructed out of $\tilde{e}_{a\mu}$, $\tilde{B}_{\mu ij}$, $\tilde{\Phi}_{\mu i}$. In particular, an arbitrary mass term,

*) To be complete, one should also redefine a new invariant vierbein $\tilde{e}_{a\mu}$. Since we demanded vanishing cosmological terms, it is not necessary to replace $e_{a\mu}$ with $\tilde{e}_{a\mu}$ within our approximation.

$$i \delta m_{ij} \bar{\tilde{\Phi}}_{\mu i} \sigma^{\mu\nu} \tilde{\Phi}_{\nu j} \quad , \quad \delta m_{ij} = \delta m_{ji} \quad (4.13)$$

is allowed, and it may give arbitrary mass splittings to the N-gravitinos. Of course, this is possible only if $SO(N)$ is spontaneously broken ($g_{ij} \neq 0$), otherwise the construction of $\tilde{\Phi}_{\mu i}$ would not be possible*).

To conclude, we would like to stress that the validity of our results relies upon the assumption that the construction of an invariant Lagrangian can be carried out to all orders. The considerations of the present section indicate that this task may be accomplished if one succeeds in constructing fields which are invariant under local supersymmetry and $SO(N)$, up to general coordinate and local Lorentz transformations.

The construction of the invariant fields requires the determination of a finite field-dependent supersymmetry transformation. Its actual determination is not straightforward since the supersymmetry algebra is field-dependent. However, the fact that in the case of internal symmetries [1], as well as for $N = 1$ supergravity [15], this procedure can be made to work, indicates that these difficulties may be only of a technical nature.

*) The term (4.13) is the counterpart of the spin-1 mass terms (4.9). The possibility of gravitino mass splitting is similar to the non-vanishing of nucleon mass terms in non-linear chiral dynamics.

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