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FINE-TUNING PROBLEM AND THE RENORMALIZATION GROUP

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A B S T R A C T

There is no fine-tuning problem in renormalization group improved perturbation theory for scalar fields if an appropriate set of short distance parameters is used. The gauge hierarchy problem has two possible issues: it may either be solved by the renormalization group approach itself with a strong scale dependence of the scalar mass, or it is reduced to the question "why is the mass of the scalar doublet normalized at the Planck scale much smaller than the Planck mass?" In the second case, a possible solution of the gauge hierarchy problem refers to a unification with gravity.

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Unified theories of weak, electromagnetic and strong interactions exhibit two largely separated mass scales: the scale M_W characterizing the spontaneous symmetry breakdown of the weak interaction gauge group $SU(2)$ and the unification scale M_X . In the context of these theories, three independent phenomena indicate that M_X is many orders of magnitude larger than M_W : the observed value of the Weinberg angle $\sin^2\theta_W$ ¹⁾, the stability of the proton²⁾ and the smallness of neutrino masses³⁾. Why is M_W so much smaller than M_X ? This problem, the gauge hierarchy problem⁴⁾, is one of the most crucial problems of unified gauge theories and has so far remained unsolved.

Many theories (for example, based on technicolour or on supersymmetry) have been constructed as attempts to solve this problem. Some of them try first to solve a technical problem, the so-called fine-tuning problem, which is related to the gauge hierarchy problem. It is sometimes believed that a solution of the fine-tuning problem should give a better starting point for the solution of the gauge hierarchy problem. The philosophy behind this approach assumes that the existence of a fine-tuning problem indicates that there can be no natural solution of the gauge hierarchy problem in the underlying model. In this article, we try to separate the different aspects of the fine tuning and naturalness problems, to show what is solved and therefore to clarify where the real problem persists.

Unified theories attempt to calculate the ratio M_W/M_X from some set of parameters α_i of the underlying model. (These parameters include scalar mass terms and self-couplings, gauge couplings, etc.). The phase transition from unbroken to broken $SU(2)$ symmetry is essentially^{*)} second order. This implies that depending on the α_i , the ratio M_W/M_X can take all values between zero and about one - or, equivalently, that the quadratic term in the effective potential for the weak scalar doublet can vary between about $+M_X^2$ and $-M_X^2$ ^{**)}. In particular, there are always values $\alpha_i^{(GH)}$ so that M_W/M_X is about 10^{-13} .

The naturalness problem is related to the question of what happens if we make small variations of the parameters α_i around the values $\alpha_i^{(GH)}$. A natural gauge hierarchy requires that small variations should not change M_W/M_X by many orders of magnitude.

*) Small radiative corrections which finally turn the second order phase transition into a slightly first order phase transition are completely negligible for our discussion.

**) This statement applies to the full effective potential including all radiative corrections. For a perturbation expansion, it is also valid in every order in the expansion, and in particular for the tree approximation.

The fine-tuning problem may arise if we consider some expansion method to calculate M_W/M_X from the α_i (for example, perturbation theory). In every order of this expansion, M_W/M_X is a given function of the α_i , and we may choose the α_i so that $M_W/M_X \approx 10^{-13}$. We then calculate the next order in the expansion for the same values α_i . If the resulting value for M_W/M_X differs by many orders of magnitude from the former one, the convergence of the expansion series is bad and we have to select new values α_i for every order in the expansion^{*)}. (In principle, it is possible that a fine-tuning problem disappears for high enough orders in the expansion, but in practice only the first few terms can actually be calculated.) Clearly, a fine-tuning problem is related to a particular expansion method. Furthermore, its existence may depend on the parameters α_i used to describe the model.

There are different aspects of fine-tuning problems which might arise in unified theories. The first fine-tuning problem concerns the relation between physical quantities (like the W boson mass) and the bare parameters appearing in the action used to define the functional integral describing the model. Renormalized perturbation theory formally relates the physical quantities and the bare parameters. This relation is renormalization scheme dependent.

It is the beauty of renormalizable quantum field theories that the model can be completely parametrized by an appropriate set of quantities which can, in principle, be measured. We will call these the physical parameters. Predictions on physical quantities can then be given in terms of measured physical parameters. For our case of interest - spontaneous symmetry breaking in a scalar field theory - the relevant physical parameters are scalar mass terms $\mu^2(M^2)$ and self-couplings $\lambda(M^2)$ normalized at some given energy scale M . They can be defined from scalar two-point and four-point functions, where M is the scale of external momenta or of some non-zero vacuum expectation value. [Knowledge of the functions $\mu^2(M^2)$ and $\lambda(M^2)$ determines the effective scalar potential and therefore the magnitude of spontaneous symmetry breaking. Strictly speaking, for negative values of $\mu^2(M^2)$, this parameter is not measurable. Nevertheless, in a phase with spontaneous symmetry breaking, it is directly related to the (positive) physical scalar mass.]

^{*)} The name "fine-tuning problem" is suggestive, but also somewhat misleading. The real problem is the problem of convergence of an expansion series and not a problem of tuning of parameters. Calculating the fine structure constant (which is known with high precision) from some short-distance couplings would also require fine tuning in every order of the perturbation expansion. Nobody would consider this feature as a problem.

The relation between the physical parameters and the bare couplings is usually defined by renormalized perturbation theory. Using dimensional renormalization, a small value of the mass of the scalar doublet, $\mu_D^2(M_W^2) \approx -(100 \text{ GeV})^2$, requires a fine tuning in the bare parameters if a large mass scale M_X is present in the theory. (Exceptions exist for supersymmetric theories.) On the other hand, it has been shown⁵⁾ that a different renormalization scheme (BPHZ) needs no fine tuning of the bare parameters. In any case, fine tuning of bare parameters is not really a relevant problem: we do not need to know the exact formal relation between physical and bare parameters (which furthermore depends on the renormalization scheme), and it is not important if some particular expansion method needs fine-tuning in the bare parameters or not. The relevant parameters are the physical parameters, since any predictions of a model must finally be expressed in terms of these.

A second, more serious possibility of a fine-tuning problem involves the relation between physical parameters at short and long distances. Unified theories relate physics at very short and very large distances. A unified theory is naturally parametrized with physical parameters at short distances, where the symmetry breaking effects are small. Testable predictions of these theories involve energy scales of the order M_W or lower. We therefore need a method to relate physical parameters at short and long distances. Typically, such a method is some kind of expansion involving the ratio of mass scales M_X/M_W :

$$\alpha_i(M_W^2) = \alpha_i(\alpha_j(M_X^2), M_X^2/M_W^2) = \alpha_i^{(0)}(M_W^2) + \alpha_i^{(1)}(M_W^2) + \dots \quad (1)$$

Since, in general, only a few orders in the expansion are calculable in practice, the series should converge quickly if predictions are to be made.

Imagine a perturbation series for $\mu_D^2(M_W^2)$ of the form

$$\frac{\mu_D^2(M_W^2)}{M_X^2} = \beta^{(0)}(\alpha_j(M_X^2), M_X^2/M_W^2) + \beta^{(1)}(\alpha_j(M_X^2), M_X^2/M_W^2) + \dots \quad (2)$$

where the $\beta^{(n)}$ are functions of the short-distance parameters and proportional to g^{2n} , with g the gauge coupling. We know it is possible to choose parameters $\alpha_i(M_X^2)$ so that $\beta^{(0)}$ is of the order 10^{-26} . However, if for the same parameters $\beta^{(1)}$ should turn out to have a non-zero value of the order $O(g^2)$, the

perturbation series converges badly and fine tuning is needed for many orders in the expansion. Such a fine tuning problem for the short-distance parameters would be much more serious than the one for the bare parameters: it would destroy the possibility of calculating all the long-distance parameters from the short-distance ones. Furthermore, it could indicate that a small ratio M_W/M_X is purely accidental (though possible) in the underlying model, implying that no natural explanation for this small mass ratio can be found.

The appropriate tool for studying relations between short- and long-distance parameters is renormalization group improved perturbation theory. Does a fine-tuning problem exist for this expansion? Let us look at the renormalization group equation (RGE) for the doublet scalar mass term $\mu_D^2(M^2)$. For M^2 between M_W^2 and M_X^2 , we can consider an effective theory where contributions to the RGE involve only the "light" particles with masses of the order M_W or smaller. All contributions from "heavy" particles with masses of the order M_X are suppressed by factors M^2/M_X^2 . One finds

$$M^2 \frac{\partial}{\partial M^2} \mu_D^2(M^2) = \frac{1}{2} A \mu_D^2(M^2) \quad (3)$$

The anomalous dimension A is a function of the dimensionless parameters of the effective theory below M_X . In a perturbation expansion, the n -th order contribution is a polynomial of degree n in the squared gauge and Yukawa couplings and in the quartic scalar couplings ($\alpha_i \equiv g^2, U_i^2, \dots, \lambda$).

$$A = C_i \alpha_i(M^2) + C_{ij} \alpha_i(M^2) \alpha_j(M^2) + \dots \quad (4)$$

For large enough Yukawa or scalar couplings, A is positive and there is an infrared fixpoint at $\mu_D^2 = 0$ [for details, see Ref. 6)].

The solution of Eqs. (3) and (4) is of the form

$$\begin{aligned} \mu_D^2(M_W^2) &= K(\alpha_i(M_S^2), M_S^2/M_W^2) \mu_D^2(M_S^2) \\ &= \left(\frac{M_W^2}{M_S^2} \right)^{\frac{\bar{A}}{2}} \mu_D^2(M_S^2) \end{aligned} \quad (5)$$

with

$$\bar{A} = \int_1^{\ln(M_S/M_W)} dt A(t) / \ln(M_S/M_W) \quad (6)$$

$$t = \ln(M/M_W)$$

$\bar{A}$ is the mean value of the anomalous dimension over the range $M_W \leq M \leq M_S$, and can be calculated once the evolution of the $\alpha_i(M^2)$ is known. It depends on the $\alpha_i(M_S^2)$ and the ratio M_S^2/M_W^2 . The perturbation series for κ converges quickly for small α_i . For $M_W^2 \ll M_S^2 \ll M_X^2$, there is no fine-tuning problem in the relation between the short-distance parameter $\mu_D^2(M_S^2)$ and the long-distance parameter $\mu_D^2(M_W^2)$.

The good behaviour of the evolution of $\mu_D^2(M^2)$ critically depends on the specific form of the RGE (3). For $M_S^2 \ll M_X^2$, this form is already dictated by the fact that $\mu^2(M^2)$ is the only physical parameter with dimension mass squared in the effective theory below M_X . What happens to this equation for energy scales around or above M_X ? (We assume that M is still below the Planck mass M_P .) If the right-hand side of Eq. (4) were to get additional contributions proportional to M_X^2 , we would clearly be confronted with a fine-tuning problem: $\mu_D^2(M^2)$ would be driven through zero and the scale M_0 , where $\mu_D^2(M_0^2)$ is equal to zero, would be different order by order in perturbation theory, thus requiring a fine tuning to obtain $\mu_D^2(M_W^2)/M_X^2 \approx 10^{-26}$. Fortunately, such a catastrophe does not happen.

For the whole range $M^2 < M_P^2$, the evolution of $\mu_D^2(M^2)$ is of the form of Eq. (3), governed by an anomalous dimension. A second order phase transition between phases with broken and unbroken $SU(2)$ symmetry implies a fixpoint at $\mu_D^2 = 0$. At the transition, the correlation length becomes infinite, and this is, of course, true for every renormalization scale M . (This is an aspect of scale invariance at a second order phase transition. It is in the nature of the renormalization group approach itself that the evolution of couplings can never lead to a crossing of a second order phase transition.) The parameter $\mu_D^2(M^2)$ measures the deviation from the phase transition. The right-hand side of its evolution equation (3) has dimension mass squared. For sufficiently small deviations, it must be proportional to $\mu_D^2(M^2)$, since it vanishes for $\mu_D^2 = 0$. Up to corrections of the order $\mu_D^2(M^2)/v^2(M^2)$, where $v^2(M^2)$ is some other (large) mass scale of the theory (for example, M_X^2), the RGE for $\mu_D^2(M^2)$ is completely determined by its anomalous dimension A .

In a phase where the unifying group is spontaneously broken, the doublet mass term $\mu_D^2(M^2)$ has several contributions. There is the mass term $\mu_\phi^2(M^2)$ of the scalar multiplet to which the doublet belongs (for simplicity, we assume that the model contains only one doublet), and there are contributions from the cubic and quartic couplings involving the multiplet and the vacuum expectation value (VEV) ϕ_x characterizing the spontaneous breakdown of the unifying group. [There may even be several such VEV's with different mass scales. In minimal SU(5), the field ϕ is a 5-plet and ϕ_x is the VEV of a 24-plet.] However, the mass term $\mu_\phi^2(M^2)$ does not appear explicitly in the renormalization group equations. These equations only involve the eigenvalues of the scalar mass matrix evaluated at a non-symmetric renormalization point characterized by $\phi_x \neq 0$. The doublet mass term $\mu_D^2(M^2)$ is such an eigenvalue and it is the evolution of $\mu_D^2(M^2)$ which is characterized by an anomalous dimension rather than the evolution of $\mu_\phi^2(M^2)$. Since $\mu_\phi^2(M^2)$ appears only implicitly in the RGE through the relation between $\mu_D^2(M^2)$ and $\mu_\phi^2(M^2)$, its evolution is completely determined by the evolution of $\mu_D^2(M^2)$ and the evolution of the various cubic and quartic scalar couplings. As long as $\mu_\phi^2(M^2)$ and M^2 are much larger than ϕ_x^2 , the symmetry breaking effects are small and the evolution of $\mu_\phi^2(M^2)$ is given by the same anomalous dimension as for $\mu_D^2(M^2)$. However, this changes completely when the scales are comparable to ϕ_x^2 . As a consequence, the relation between $\mu_D^2(M_W^2)$ and $\mu_D^2(M_S^2)$ has the form of Eq. (5) up to a scale M_S^2 of the order M_p^2 . No fine-tuning problem exists for this relation between long-distance and short-distance parameters!

Finally, we have to discuss a third possible class of fine-tuning problems involving relations between different sets of short-distance parameters used to parametrize the model. In particular, this concerns the relation between $\mu_D^2(M_S^2)$ and $\mu_\phi^2(M_S^2)$ which can be used alternatively as short-distance parameters. The relation between both quantities can be calculated in perturbation theory. It depends on the exact definition of these parameters. Assuming for simplicity that there is only one quartic coupling γ between ϕ and ϕ_x , one has

$$\mu_D^2(M_S^2) = \mu_\phi^2(M_S^2) + \gamma(M_S^2) \phi_x^2 + \text{higher order terms} \quad (7)$$

Let us suppose that $\mu_D^2(M_S^2)$ is very small compared to $\mu_\phi^2(M_S^2)$. If the higher order terms are of the order $\gamma^2 \phi_x^2$ (or $g^4 \phi_x^2$ etc.), fine tuning would be needed if we want to express $\mu_D^2(M_S^2)$ as a function of $\mu_\phi^2(M_S^2), \gamma(M_S^2)$ etc. On the contrary, no fine tuning is needed if we take $\mu_D^2(M_S^2)$ as the physical parameter and we calculate $\mu_\phi^2(M_S^2)$ from it. There are two arguments why this kind of fine-tuning problem should not be taken very seriously. First, there is no reason to start with $\mu_\phi^2(M_S^2)$ as the physical parameter rather than with $\mu_D^2(M_S^2)$. It is even more likely that $\mu_D^2(M_S^2)$ is the relevant physical parameter: this is the parameter appearing explicitly in the RGE, and, in view of a unification with gravity in Kaluza-Klein theories, where spontaneous symmetry breaking is related to properties of the ground state of an "internal space", the deviation from a phase transition $\mu_D^2(M_S^2)$ is a much more basic quantity than the mass term of some scalar multiplet. Secondly, the higher order terms in Eq. (7) depend on the precise definition of $\mu_D^2(M_S^2)$ and $\mu_\phi^2(M_S^2)$. It is not difficult to find definitions where

$$\mu_D^2(M_S^2) = \mu_\phi^2(M_S^2) + \gamma(M_S^2) \phi_x^2 \quad (8)$$

to all orders in perturbation theory. [After a definition of $\mu_D^2(M_S^2)$, the relation (8) may even be used to define $\mu_\phi^2(M_S^2)$.]

Resuming the different aspects of our discussion, we conclude that fine tuning is not an intrinsic problem of scalar field theories with large ratios of scales of spontaneous symmetry breaking. We can always choose a short-distance parameter set containing $\mu_D^2(M_S^2)$. Then the predictions are stable order by order in a renormalization group improved perturbation expansion. From this point of view, fine-tuning problems are artifacts of badly-chosen expansion methods or badly-chosen short-distance parameters.

Let us now turn to the problem of naturalness. Are the long-distance parameters stable with respect to variations of the short-distance ones? We first introduce the concept of a natural relation between long-distance and short-distance parameters; we call a relation of the type of Eq. (1) natural, if a small relative change of any one of the short-distance parameters $\alpha_i(M_S^2) \rightarrow \alpha_i(M_S^2) + \delta\alpha_i(M_S^2)$ (with $|\delta\alpha_i/\alpha_i| \ll 1$) induces only a small relative change for the long-distance parameter. It is often sufficient to linearize the problem and we define

$$K_{ik}(\alpha_j(M_S^2), M_S^2/M_W^2) = \frac{\partial \alpha_i(M_W^2)}{\partial \alpha_k(M_S^2)} \Big|_{M_S^2/M_W^2 \text{ fixed}} \quad (9)$$

[The quantity κ in Eq. (5) is a special case of this.] The relation between $\alpha_i(M_W^2)$ and the short-distance parameters is natural if κ_{ik} is (roughly) of the order $\alpha_i(M_W^2)/\alpha_k(M_S^2)$ or smaller for all $\alpha_k(M_S^2)$. Some comments are in order:

- the question whether a relation $\alpha_i(M_W^2) = \alpha_i(\alpha_j(M_S^2), M_S^2/M_W^2)$ is natural or not does not relate to a particular expansion method (as is the case for the fine-tuning problem). The relations between long-distance and short-distance parameters are properties of the "full" theory (even if it is only known approximately).
- Natural relations between long- and short-distance parameters reduce the question of whether some long-distance parameter can be regarded as natural to the problem of naturalness of the short-distance parameters. If short-distance parameters having typical values within a range $\alpha_i^{(S)} - \delta\alpha_i^{(S)} \leq \alpha_i(M_S^2) \leq \alpha_i^{(S)} + \delta\alpha_i^{(S)}$ are regarded as natural (whatever this notion means exactly), the long-distance parameters are natural if they have typical values within a range $\alpha_i^{(L)} - \delta\alpha_i^{(L)} \leq \alpha_i(M_W^2) \leq \alpha_i^{(L)} + \delta\alpha_i^{(L)}$ with $\delta\alpha_i^{(L)} = \kappa_{ik} \delta\alpha_k^{(S)}$. [$\mu_D^2(M_W^2) \approx \approx -(100\text{GeV})^2$ would not be a typical value in a range $-M_X^2 \leq \mu_D^2(M_W^2) \leq M_X^2$, but it would be natural if the range is of the order M_W^2 .]
- Very small values of long-distance parameters like $\mu_D^2(M_W^2)/M_X^2 \approx 10^{-26}$ can only be regarded as natural if either κ_{ik} or $\alpha_k(M_S^2)$ are very small (for all k).
- The question whether parameter relations are natural or not may depend on the set of short-distance parameters $\alpha_i(M_S^2)$ chosen to parametrize the model.

Let us now look at Eq. (5) relating $\mu_D^2(M_W^2)$ and $\mu_D^2(M_S^2)$. The dependence of $\mu_D^2(M_W^2)$ on all short-distance parameters except $\mu_D^2(M_S^2)$ is only implicit via the dependence of $\bar{A}$ on these parameters. The derivative with respect to these parameters is given by

$$\kappa_{ok} = \frac{\partial \mu_D^2(M_W^2)}{\partial \alpha_k(M_S^2)} = - \frac{\partial \bar{A}}{\partial \alpha_k(M_S^2)} \ln\left(\frac{M_S}{M_W}\right) \cdot \mu_D^2(M_W^2) \quad (10)$$

Since $\bar{A}$ depends smoothly on the $\alpha_k(M_S^2)$, the κ_{ok} are all of the order $\mu_D^2(M_W^2)$. For the derivative with respect to $\mu_D^2(M_S^2)$, one has

$$\kappa_{oo} = \frac{\partial \mu_D^2(M_W^2)}{\partial \mu_D^2(M_S^2)} = \frac{\mu_D^2(M_W^2)}{\mu_D^2(M_S^2)} = \kappa \quad (11)$$

Thus, the relation (5) between $\mu_D^2(M_W^2)$ and $\mu_D^2(M_S^2)$ is natural. [In contrast, if we had chosen a set of short-distance parameters containing $\mu_\phi^2(M_S^2)$ instead of $\mu_D^2(M_S^2)$, the relation between $\mu_D^2(M_W^2)$ and $\mu_\phi^2(M_S^2)$ would not be natural if $\mu_D^2(M_S^2)$ is much smaller than $\mu_\phi^2(M_S^2)$.]

The gauge hierarchy problem is now reduced to the question: can the value of $\mu_D^2(M_S^2)$ related to $\mu_D^2(M_W^2) \approx -(100 \text{ GeV})^2$ by Eq. (5) be considered as natural? As we have shown, a positive answer implies that there is neither a naturalness nor a fine-tuning problem for the long-distance parameters, including a very small value $\mu_D^2(M_W^2)/M_X^2 \approx 10^{-26}$.

The answer to the gauge hierarchy problem depends critically on the value of $\bar{A}$. For $\bar{A}$ positive, $\mu_D^2(M_P^2)/\mu_D^2(M_W^2)$ is larger than one (we take $M_S \approx M_P$ for definiteness). For $|\bar{A}| \ll 1$ we find $\mu_D^2(M_P^2)$ of the same order as $\mu_D^2(M_W^2)$. In the context of unified theories of weak, electromagnetic and strong interactions it seems to be difficult to give a natural explanation why $\mu_D^2(M_P^2)$ should be so small. However, we have the big advantage that the question of naturalness is now reported to the Planck scale. It is not excluded that unification with gravity will provide us with a mechanism explaining why the theory is so near to a phase transition.

A second possibility exists if $\bar{A} \approx 2$. For this case $\mu_D^2(M_P^2)$ is of the order of M_P^2 and this can certainly be regarded as a natural value. Then the gauge hierarchy problem could be solved in the context of unified gauge theories (without gravity). A mechanism for a natural gauge hierarchy based on $\bar{A} \approx 2$ has been described in Ref. 6). Here we would like to add that $\bar{A}$ must not necessarily become greater than two below the scale M_X , which would require strong Yukawa or quartic scalar couplings. We may have large values of $\bar{A}$ between M_X and M_P where many fields contribute to the RGE.

Before summarizing our conclusions let us add two comments on issues which have been widely discussed and sometimes obscured the real content of the gauge hierarchy problem: the problem of quadratic divergences and the use of supersymmetry to "improve" the gauge hierarchy problem. The quadratic divergences only affect the formal relation between physical parameters and bare couplings. (We recall that this relation depends on the renormalization scheme.) They do not contribute at all to the RGE. They play no role in the relations between physical parameters at short and long distances or between different sets of short-distance parameters. In brief, the quadratic divergences are completely irrelevant for the naturalness and fine-tuning problems involving the physical parameters.

The situation is analogous to statistical mechanics near a second order phase transition. As an example, we may take a lattice of a ferromagnetic substance. The critical temperature depends on the cut-off, which is given in this case by the lattice spacing a . If we make a perturbation expansion, we typically get contributions of the order $1/a$ to the critical temperature in every order of the expansion - this corresponds to the quadratic divergences in a scalar field theory. However, the critical temperature is not a universal quantity. (It also depends on the precise form of the short-distance interaction and on the lattice type.) Only (small) deviations from the critical temperature can be treated as universal quantities characterized by the renormalization group behaviour. Our physical parameters correspond to universal quantities. Especially $\mu_D^2(M^2)$ corresponds to a deviation from the critical temperature.

In the context of renormalized field theory the bare parameters have no physical meaning. However, one could also look at these parameters from a different viewpoint: the field theory describing unified theories of weak, electromagnetic and strong interactions may be consistent in itself up to infinitely small distances (this is not sure !) Nevertheless, at distances small compared with the Planck length it loses sense as an isolated theory - gravity has to be included. This may lead one to consider the Planck scale as a physical cut-off, and then the bare couplings become some new kind of physical short-distance parameters at the Planck scale. The problems of naturalness and fine tuning do not change radically in this picture. Especially, we have the supplementary parameter $(\mu_\phi^0)^2$, the bare mass of the scalar multiplet calculated with a cut-off at the Planck scale. The relation between $\mu_\phi^2(M_P^2)$ and $(\mu_\phi^0)^2$ involves the quadratic divergences

$$\mu_\phi^2(M_P^2) = (\mu_\phi^0)^2 + f(\alpha_i(M_P^2)) M_P^2 \quad (12)$$

The questions of naturalness and fine tuning now depend on the choice of $(\mu_\phi^0)^2$, $\mu_\phi^2(M_P^2)$ or $\mu_D^2(M_P^2)$ as short-distance parameter. For $\bar{A} \approx 2$, all of them are of the order M_P^2 and there is no gauge hierarchy problem. For $|\bar{A}| \ll 1$, $(\mu_\phi^0)^2$ and $\mu_\phi^2(M_P^2)$ are typically of the order M_P^2 whereas $\mu_D^2(M_P^2)$ is of the order of M_W^2 . The above discussion applies unchanged: the relevant parameter is likely to be $\mu_D^2(M_P^2)$ and the gauge hierarchy problem asks for an explanation of its small value from a unification with gravity.

There is therefore no need for the quadratic divergences to vanish in the context of a unified theory which is not supposed to be valid up to infinitely small distances. There is only one thing we can learn from non-vanishing quadratic divergences: the phase transition is not characterized by a special symmetry of the model in this case. If some symmetry would enforce $\mu_D^2(M^2)$ to vanish, this would also be true for $(\mu_\phi^0)^2$ since it is also a coefficient of the operator $\phi_D^\dagger \phi_D$ quadratic in the scalar doublet. Thus, such a symmetry would force the quadratic divergences to vanish and imply that all the parameters $\mu_D^2(M_P^2)$, $\mu_\phi^2(M_P^2)$ and $(\mu_\phi^0)^2$ are zero.

The first attempts to use supersymmetry for a solution of the gauge hierarchy problem were based on such a symmetry. The vanishing of the quadratic divergences opened this possibility and the supersymmetric relations between scalars and fermions suggested to use a chiral symmetry. Unfortunately, no satisfactory model could be constructed along these lines. Later on, many models, using different mechanisms, have been constructed to obtain a massless scalar doublet. However, no satisfactory explanation has been given so far why $\mu_D^2(M_W^2)$ is finally not zero but of the order of $-(100 \text{ GeV})^2$. It was argued that supersymmetry offered at least one advantage: if some very small parameter is introduced from the beginning (as an appropriate short-distance parameter) in order to obtain the relation $M_W^2/M_X^2 \approx 10^{-26}$, this gauge hierarchy would be stable against radiative corrections due to non-renormalization theorems in supersymmetric field theories. Our discussion has shown that this is not a problem any more: stability against radiative correction is already a property of renormalization group improved perturbation theory (using appropriate short-distance parameters) and no need for supersymmetry arises from this side. The remaining gauge hierarchy problem is on the same level for supersymmetric and non-supersymmetric theories: to give a natural explanation of the value $\mu_D^2(M_P^2)$.

In conclusion, we resume some aspects of the gauge hierarchy problem in scalar theories of spontaneous symmetry breakdown.

- a) Fine-tuning problems are always related to a particular expansion method and to a particular choice of the set of parameters used to describe the model.
- b) There is no fine-tuning problem for renormalization group improved perturbation theory if an appropriate set of short-distance parameters is used. All fine-tuning problems are artifacts either of badly chosen expansion methods or of a badly chosen parameter set.

- c) The relation between long-distance parameters and short-distance parameters is natural if we use an appropriate set of short-distance parameters. The problem of naturalness can therefore be postponed to the Planck scale and the gauge hierarchy problem is reduced to the search of a natural explanation of the scalar doublet mass normalized at the Planck scale.
- d) There is a possibility to solve the gauge hierarchy problem in the context of a unified theory for weak electromagnetic and strong interactions alone. This requires strong renormalization effects for the mass term of the scalar doublet. If this possibility is not realized, a satisfactory solution of the gauge hierarchy problem probably requires a theory of unification with gravity.

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