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WHAT DO THE DATA FOR THE T FAMILY SAY ABOUT

$$\underline{\langle 0 | (\alpha_s / \pi) G_{\mu\nu}^a G_{\mu\nu}^a | 0 \rangle ?}$$

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A B S T R A C T

We show that the data on $^3S_1$ upsilononium are consistent with the standard value for the gluon condensate parameter

$$\langle 0 | (\alpha_s / \pi) G_{\mu\nu}^a G_{\mu\nu}^a | 0 \rangle$$

contrary to a claim recently made by Zalewska and Zalewski.

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In a recent paper ¹⁾, Zalewska and Zalewski present a QCD sum rule analysis of new more accurate data on the T family to estimate the gluon condensate parameter $\langle 0 | (\alpha_s/\pi) G_{\mu\nu}^a G_{\mu\nu}^a | 0 \rangle$. They conclude from their analysis that the data require a value for $\langle 0 | (\alpha_s/\pi) G_{\mu\nu}^a G_{\mu\nu}^a | 0 \rangle$ which exceeds the standard value obtained from charmonium ^{2),3)} by almost two orders of magnitude. In this note we will show that their analysis is not correct, and that the ϵ data are perfectly consistent with the canonical value for the gluon condensate :

$$\langle 0 | \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{\mu\nu}^a | 0 \rangle \simeq (330 \text{ MeV})^4. \quad (1)$$

In the QCD sum rule analysis one calculates moments of the polarization operator of some current. In our case we are interested in the vector current of heavy quarks $j_\mu(x) = \bar{b}(x) \gamma_\mu b(x)$,

$$M_n = \frac{1}{n!} \left(-\frac{d}{dQ^2} \right)^n \Pi(Q^2) \Big|_{Q^2=0} = \frac{1}{\pi} \int \frac{ds}{s^{n+1}} \text{Im} \Pi(s), \quad (2)$$

where $Q^2 = -q^2$ and

$$(q_\mu q_\nu - g_{\mu\nu} q^2) \Pi(q^2) = \int d^4x e^{iqx} T \langle 0 | j_\mu(x) j_\nu(0) | 0 \rangle. \quad (3)$$

Calculating the first order α_s and gluon condensate contributions to $\Pi(Q^2)$ one obtains for M_n the expression ²⁾

$$M_n^{\text{theor}} = A_n [1 + a_n \alpha_s - b_n \phi_b] \quad (4)$$

where the values of a_n and b_n have been tabulated in the Table,

$$\phi_b = \frac{4\pi^2}{9} \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle / (4m_b^2)^2$$

and

$$A_n = \frac{3 \cdot 2^n (n+1) (n-1)!}{4\pi^2 (2n+3)!!} \cdot \frac{1}{(4m_b^2)^n}.$$

With $m_b \simeq 4.2 \text{ GeV}$ it can be seen that the gluon condensate contribution to M_n with the value (1) for the matrix element $\langle 0 | (\alpha_s/\pi) G^2 | 0 \rangle$ is completely negligible for the T system. It is also clear that first-order perturbation theory is only justified when $a_n \alpha_s \ll 1$, i.e., for the moments M_n up to $n=8$ or 9 .

Experimentally, four resonances have been identified in the $b\bar{b}$ vector channel. Their mass values and electronic widths are

$$\begin{aligned}
 Y : m &= 9.4597 \text{ GeV}, & \Gamma(e^+e^-) &= 1.07 \text{ keV} \\
 Y' : &= 10.0187 & &= 0.486 \\
 Y'' : &= 10.3507 & &= 0.425 \\
 Y''' : &= 10.5727 & &= 0.266
 \end{aligned} \tag{5}$$

For the continuum states above threshold one normally assumes a θ function behaviour

$$\text{Im } \Pi^{\text{cont}}(s) = \frac{1}{4\pi^2} \left(1 + \frac{\alpha_s}{n}\right) \theta(s-s_0). \tag{6}$$

It should be emphasized, however, that this is only a rough model and in applications to spectroscopy only those moments (or ratios of moments) have been used in which the continuum contribution is less than 30% ^{2),3)}.

With the data of Eq. (5) and the continuum of Eq. (6) we can calculate the experimental moments M_n^{exp} , which contain the unknown parameter s_0 .

Let me explain what the authors of Ref. 1) did. Assuming that the gluon condensate contribution is negligible for the lowest moments they chose the quantity M_1^2/M_2 (which is independent of m_b) to solve for s_0 at a fixed value for α_s , and used the resulting s_0 as input in $r_2 = M_2/M_1$ to solve for m_b . With these values for α_s , m_b , and s_0 they solved each $r_n = M_n/M_{n-1}$ for the gluon condensate and found that this had to be about fifty times as large as the value of Eq. (1), changing only by a factor two from $n = 2$ to $n = 10$.

It is obvious that this procedure is extremely dangerous. The continuum contribution to M_1 is about 70% and this moment should therefore not be used to fix any of the parameters. Also, since the aim is to fit the moment ratios r_n (which have very small errors, about 1%) one should not select the extreme point r_2 (see the Figure) to fix the parameters. This easily introduces systematic errors which can lead to large discrepancies when calculating higher moments. This is essentially what has happened. The quark mass m_b has been underestimated which means that without a gluon condensate all higher ratios (which go like $1/m_b^2$) lie above the data and a large gluon condensate is needed to bring them down. This result, however, is completely spurious.

A simple search in the parameter space (m_b, α_s, s_0) reveals that there is a large range of values for the parameters which reproduce each ratio (for $n = 2-9$) within the experimental errors without a gluon condensate. Following Zalewska and Zalewski we have taken these errors to be one per cent. The best χ^2 is obtained for

$$\alpha_s = 0.155, m_b = 4.23 \text{ GeV}, s_0 = (11.20 \text{ GeV})^2 \quad (7)$$

giving $\chi^2/\text{D.O.F.} \approx 1.0/5$. For comparison, the parameter set of Zalewska and Zalewski gives $\chi^2 \approx 30$. The value for m_b in Eq. (7) is very close to the value obtained before in Ref. 3). The resulting theoretical and experimental moments are tabulated in the Table, and the ratios are plotted in the Figure. From this it can be seen that not only the ratios but also the moments are reproduced very accurately. One can easily calculate from the Table that the two quantities M_1^2/M_2 and r_2 used by Zalewska and Zalewski are reproduced very accurately with the parameter set (7). In fact the two equations which were solved by Zalewska and Zalewski to obtain s_0 and m_b have more than one solution (taking account of the experimental errors). M_1 and r_2 are only weakly dependent on s_0 , and it is perfectly possible to find acceptable values for s_0 and α_s for any m_b between 4.17 and 4.25. In higher moments it makes, however, a great difference which m_b is chosen.

We conclude from this that the theoretical moments (4) with $\phi_b \equiv 0$ reproduce the data perfectly well and cannot be used to determine the gluon condensate.

The analysis which leads to the parameter set (7) has been performed with unexpanded ratios, i.e., the ratios $r_n = M_n/M_{n-1}$ have not been expanded to first-order in α_s as done by Zalewska and Zalewski. Theoretically there is no preference for either ratios since both have the same accuracy (up to terms of $O(\alpha_s)$). Our conclusion holds for unexpanded as well as expanded ratios although it is more difficult to find a set of parameters for which $\chi^2/\text{D.O.F.} < 1$ in the latter case. With expanded moments the parameter set of Zalewska and Zalewski gives $\chi^2 \approx 50$. Actually, since our analysis has reproduced all moments M_n (not only the ratios r_n) very accurately the question of expanded or unexpanded moments is irrelevant. The differences between the two, however, give an idea of the error in the theoretical expressions. With expanded moments the parameter set (7) gives $\chi^2/\text{D.O.F.} \approx 10/5$. About half of this comes from r_9 . It turns out

that for the highest ratios ($n = 8,9$) the error in the theoretical expressions is about twice the experimental error which means that even when more accurate data might become available, second order α_s corrections to the moments M_n have to be calculated before it may become possible to determine the gluon condensate parameter from the upsilon data.

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n	a_n	b_n	$M_n^{\text{theor}} \times 10^{2n+1}$	$M_n^{\text{exp}} \times 10^{2n+1}$
1	0.734	3.4	0.315	0.307
2	0.706	13.3	0.188	0.183
3	0.508	32.7	0.151	0.149
4	0.216	64.6	0.138	0.136
5	-0.140	112.0	0.135	0.133
6	-0.543	177.9	0.137	0.135
7	-0.981	265.2	0.143	0.141
8	-1.448	377.1	0.152	0.149
9	-1.938	516.5	0.162	0.159
10	-2.977	686.4	0.173	0.172

Table - The values for the coefficients a_n and b_n in Eq. (4) for $n = 1-10$; and the theoretical and experimental moments for the parameter set (7). The errors in the experimental moments are determined by the errors in $\Gamma(e^+e^-)$ and vary from 5 to 10%.

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FIGURE CAPTION

The ratios $r_n = M_n/M_{n-1}$ plotted versus n . The experimental points are indicated by $\times$ and the theoretical values by $-$. The error in the experimental ratios is about 1%.

FIG. 1.