Ref.TH.3556-CERN

SUPERSYMMETRIC GAUGE FIXING AND THE EFFECTIVE POTENTIAL

Robert D.C. Miller
CERN -- Geneva

A B S T R A C T

Supersymmetric (SUSY) covariant gauge fixing is analyzed at a component field level using the methods of Fukuda and Kugo ¹⁾. I find that for all models with D type breaking and almost all models with F type breaking supersymmetric covariant gauge fixing is a "bad gauge" choice (characterized by the unwelcome situation where the gauge multiplet picks up an x dependent VEV). SUSY R_{ξ} gauges avoid this problem.

Ref.TH.3556-CERN

March 1983

INTRODUCTION

For practical calculations with complicated SUSY GUTs ²⁾ the Wess-Zumino gauge supplemented with a conventional gauge choice (such as covariant gauge fixing) is often taken. However, such a gauge choice is manifestly non-SUSY. At a supergraph level one would prefer to work with a manifestly SUSY gauge fixing procedure. One standard procedure ³⁾ is to choose a SUSY version of covariant gauge fixing, which I denote by SCGF.

SUSY gauge fixing usually proceeds via the defining of a chiral constraining function F . We may recall that Fukuda and Kugo ¹⁾ observed that in conventional field theory a constraining function must vanish at all stationary points of the effective action (also observed independently by de Witt ⁴⁾). We will see that the proof in Ref. 1) readily extends to the present case of a chiral constraining function in a SUSY gauge theory.

The effective potential provides a means of searching for translationary invariant stationary points of the effective action. The vanishing of the chiral constraining function thus leads to a set of component field constraints which must be satisfied by any stationary point of the effective potential if it is to be considered a translationary invariant stationary point of the effective action.

For SCGF one such constraint requires a vanishing VEV for the D auxiliary fields. As a result the physical minima for all D type models and almost all F type models will not be translationary invariant (an auxiliary field, c of the vector superfield will be forced to pick up co-ordinate space dependence). In the language of Ref. 1), the gauge choice responsible is "bad" for the physical minima. In terms of the effective potential one invariably finds that instead of the physical minimum an apparently spurious stationary point appears. It is spurious in the sense that both it and the stationary value (the effective potential evaluated at the stationary point) are manifestly gauge dependent, yet we know that the latter situation cannot arise for a translationary invariant stationary point of the effective action as proved in Ref. 1). Finally, for models with no spontaneous breaking of SUSY, SCGF is found to be a good gauge choice.

In this letter both theory and illustrative examples of the above results are presented. The main conclusion to be drawn is that SCGF ought to be avoided in analyzing the effective potential of SUSY gauge theories. A parallel analysis for SUSY R_ξ gauges is made revealing that these can avoid the problems encountered with SCGF.

NOTATION AND SUSY GAUGE FIXING

The most general renormalizable SUSY gauge theory may be written as ⁵⁾

$$\mathcal{L} = \left\{ \left[\sum_R \bar{\phi}_R e^{g V_R} \phi_R \right] + \xi^a V^a \right\} \Big|_{\theta^2 \bar{\theta}^2} + \left\{ \left[\frac{1}{4k} \text{Tr}(W^\alpha W_\alpha) + P(\phi_R) \right] \right\} \Big|_{\theta^2} + \text{H.C.} \quad (1)$$

Here ϕ_R is a chiral superfield in the R representation of the gauge group G (assumed semi-simple for simplicity). g is the gauge coupling. V_R may be written as

$$V_R = V^a T_R^a \quad (2)$$

where V^a is the vector superfield in the adjoint representation of G . T_R^a are the group generators in representation R of G . T^a , with no subscripts, defines the usual fundamental representation which is normalized so that $\text{Tr}(T^a T^b) = \frac{1}{2} \delta^{ab}$. $\xi^a V^a$ denotes the Fayet-Iliopoulos D term present for Abelian G . W^α denotes the SUSY field strength, $W_\alpha = -\frac{1}{4} \bar{D}^2 e^{-V} D_\alpha e^V$, and finally $P(\phi_R)$ is the superpotential. Throughout this letter the conventions (e.g., spinor indexing) of Wess and Bagger ⁵⁾ are adhered to. The component field content of chiral and vector superfields is specified by the expansions

$$\begin{aligned} \phi = & A(x) + i \theta \sigma^m \bar{\theta} \partial_m A(x) + \frac{1}{4} \theta^2 \bar{\theta}^2 \square A(x) \\ & + \sqrt{2} \theta \psi(x) - \frac{i}{\sqrt{2}} \theta^2 \partial_m \psi(x) \sigma^m \bar{\theta} + \theta^2 F(x) \end{aligned} \quad (3)$$

and

$$\begin{aligned} V = & C(x) + i \theta \chi(x) - i \bar{\theta} \bar{\chi}(x) + \frac{i}{2} \theta^2 (M(x) + i N(x)) - \frac{i}{2} \bar{\theta}^2 (M(x) - i N(x)) \\ & - \theta \sigma^m \bar{\theta} V_m(x) + i \theta^2 \bar{\theta} \left[\bar{\lambda}(x) + \frac{1}{2} \bar{\sigma}^m \partial_m \chi(x) \right] - i \bar{\theta}^2 \theta \left[\lambda(x) + \frac{1}{2} \sigma^m \partial_m \bar{\chi}(x) \right] \\ & + \frac{1}{2} \theta^2 \bar{\theta}^2 \left[D(x) + \frac{1}{2} \square C(x) \right] \end{aligned} \quad (4)$$

the physical fields being A , ψ , V_m and λ and the rest are auxiliary fields [see 5) for further details on notation if necessary].

To (1) one must add a gauge fixing term. In this letter I consider SUSY gauge fixing generated by a chiral constraining function denoted by F^a . The gauge fixing term is then written as ($F = F^a T^a$)

$$\mathcal{L}_{\text{G.F.}} = -\frac{1}{8\alpha k} \text{Tr}(\bar{F} F) \Big|_{\theta^2 \bar{\theta}^2} \quad (5)$$

Equation (5) will generate a ghost sector, $\mathcal{L}_F$ by the usual Fadeev-Popov procedure, however, it plays no part in the present letter and need not be specified ^{*)}.

In this letter I will primarily be concerned with supersymmetric covariant gauge fixing (SCGF) defined by

$$\mathbb{F} = \bar{D}^2 V \quad (6)$$

Remarks will also be made on SUSY R_ξ gauges which we may define by

$$\mathbb{F} = \bar{D}^2 V + \omega_R \phi_R \quad (7)$$

with a sum on R understood (ω_R are unspecified complex constants).

It will be important later to have component field expansions for the constraining function $\mathbb{F}$. Expressed in terms of the $y^m, \theta, \bar{\theta}$ basis ($y^m = x^m + i\theta\sigma^m\bar{\theta}$) one obtains for (7) [(6) is of course a specific case of (7)]

$$\begin{aligned} \mathbb{F} = & [2i (M(y) - iN(y)) + \omega_R A_R(y)] \\ & + \theta^2 [-2 (D(y) + \square C(y) - i\partial^m V_m(y)) + \omega_R F_R(y)] \\ & + \text{Fermionic terms.} \end{aligned} \quad (8)$$

we will also need $D^2 \mathbb{F}$, for which one finds (in $\bar{y}^m, \theta, \bar{\theta}$ basis)

$$\begin{aligned} D^2 \mathbb{F} = & 4 (2 (D(\bar{y}) + \square C(\bar{y}) - i\partial^m V_m(\bar{y})) - \omega_R F_R(\bar{y})) \\ & + 4 \bar{\theta}^2 (-2i \square (M(\bar{y}) - iN(\bar{y})) - \omega_R \square A_R(\bar{y})) \\ & + \text{Fermionic terms.} \end{aligned} \quad (9)$$

Finally for SCGF ($\omega_R = 0$) the gauge fixing term becomes (in $x^m, \theta, \bar{\theta}$ basis)

^{*)} The anticommutativity of the chiral ghost multiplet ensures none of its component fields can attain VEV's, my thanks to T. Kugo for this simplified explanation.

$$\begin{aligned}
 \mathcal{L}_{GF} &= -\frac{1}{8\alpha k} \text{Tr} (D^2 V \bar{D}^2 V) \Big|_{\theta^2 \bar{\theta}^2} \\
 &= -\frac{1}{2\alpha k} \text{Tr} \left[(D_\mu U + \square C U)^2 + (\partial^\mu V_m(x))^2 + M \square M(U) \right. \\
 &\quad \left. + N \square N(U) + \text{Fermionic terms} \right]
 \end{aligned} \tag{10}$$

one observes the presence of $-(1/2\alpha k)\text{Tr}(\partial^\mu V_m)^2$, displaying that (6) is the SUSY version of covariant gauge fixing.

THE FUKUDA-KUGO THEOREM AND STATIONARY POINT CONSTRAINTS

Fukuda and Kugo¹⁾ made a close study of gauge dependence in the effective potential of conventional gauge field theories. They stated and proved the following theorem :

The effective potential V at any stationary point gives average energy densities of the (quasi-) stationary state of "vacuum" and is a physical quantity. So the value of V at any stationary point should not depend on the gauge.

In addition to this they also proved that at any stationary point of the effective action the constraining function must vanish, $F = 0$ (an observation also made independently by de Witt⁴⁾).

In this Section I generalize the latter result to the SUSY case with a chiral constraining function.

We may write

$$\mathcal{Z}[J] \sim \int \mathcal{D}[V] \Delta_F e^{s[V]} e^{-\frac{1}{8\alpha} \int \bar{F}^a F^a dz} \tag{11}$$

where $dz = d^4x d^4\theta$, s denotes the action and Δ_F is the Fadeev-Popov determinant. If we replace $F^a(z)$ by $F^a(z) + C^a(z)$ with arbitrary chiral parameter C , then Δ_F remains unchanged^{*)} and $\mathcal{Z}[J]$ is thus completely independent of C . It follows that

*) One recalls the Fadeev-Popov trick to see this is so.

$$\frac{\delta Z[J]}{\delta \bar{c}^b(z_1)} \Big|_{J=c=\bar{c}=0} = 0 \quad (12)$$

whence observing that ³⁾

$$\frac{\delta \bar{c}^a(z_1)}{\delta \bar{c}^b(z_2)} = -\frac{1}{4} D^2 \delta(z_{12}) \delta^{ab} \quad (13)$$

we conclude (employing integration by parts) that

$$\langle D^2 F^b \rangle = 0 \quad (14)$$

which is the sought after generalization. Equation (14) is a result valid at all stationary points of the effective action.

ANALYSIS OF VARIOUS MODEL TYPES

Before we continue it is useful to stress what (14) conveys. Equation (14) is a condition that will be satisfied by all stationary points of the effective action. The effective potential is employed to search for translationaly invariant stationary points of the effective action. A stationary point of the effective potential is then required to satisfy (12) if it is to be a candidate translationaly invariant stationary point of the effective action. Thus not all stationary points of the effective potential are a priori guaranteed to be translationaly invariant stationary points of the effective action.

From (14) and (9), using lower case letters to denote VEV's ($\langle A \rangle = a, \dots$) all stationary points of the effective action must satisfy (for SUSY R_ξ gauge fixing)

$$\begin{aligned} 2(d + \Box c) - \text{Re}(\omega_R f_R) &= 0 \\ -2\partial^m \psi_m - \mathcal{F}_m(\omega_R f_R) &= 0 \\ -2\Box m - \mathcal{F}_m(\omega_R \Box a_R) &= 0 \\ -2\Box n - \text{Re}(\omega_R \Box a_R) &= 0 \end{aligned} \quad (15)$$

Translational invariant stationary points of the effective action must satisfy

$$\begin{aligned} 2d - \text{Re}(\omega_R f_R) &= 0 \\ f_m(\omega_R f_R) &= 0 \end{aligned} \tag{16}$$

A stationary point of the effective potential not satisfying (16) will be termed spurious, as we know it cannot be a translational invariant stationary point of the effective action. For SCGF (16) reduces to the requirement that

$$d = 0 \tag{17}$$

for a non-spurious stationary point of the effective potential. [Observe that in the case of the effective action [see Eq. (15)] $d \neq 0$ implies that C must pick up a non-trivial x dependence. For such solutions, SCGF is defined as a bad gauge choice ¹⁾ as they cannot be determined via the effective potential.] Let us classify SUSY gauge theories into three categories : i) those with D type breaking, ii) those with F type breaking, and iii) those with no spontaneous SUSY breaking. We can then draw some general conclusions as regards SCGF.

Models with D type breaking will always have $d \neq 0$ at the physical stationary point. Thus SCGF is a bad gauge choice for determining the physical stationary point in all models with D type breaking. The analysis of the effective potential for such models will reveal stationary points at which $d \neq 0$, these are spurious by definition and their spurious nature is reflected in the fact that invariably both the stationary point and stationary value will be manifestly gauge dependent.

Generally models with F type breaking (which possess $f \neq 0$ at the physical stationary point) will also have $d \neq 0$ at that point (counter examples are not the norm, although they can be constructed). Thus generally speaking SCGF of F type models faces the same difficulties discussed above for D type models.

Finally, consider models which realize a SUSY vacuum. The physical stationary point satisfies $f = d = 0$, it will thus show up correctly in an effective potential analysis. SCGF is a good gauge choice for determining the physical stationary point in such models. Some of these features are best appreciated when illustrated by explicit examples.

ILLUSTRATIONS VIA THE TREE-LEVEL EFFECTIVE POTENTIAL

For an Abelian gauge group the tree-level effective potential, V_0 , may be derived straightforwardly. The result is (SCGF)

$$V_0 = \frac{1}{2} \left(\frac{\alpha-1}{\alpha} \right) D^2 + \sum_R e^{g Q_R c} (\bar{F}_R F_R - \bar{F}_R F_R |_{P=0}) \quad (18)$$

with F and D equations of motion given by

$$F_R = - \frac{\partial P(\bar{A})}{\partial \bar{A}_R} - \frac{ig}{2} Q_R (M + iN) A_R \quad (19)$$

$$\left(\frac{\alpha-1}{\alpha} \right) D = - \frac{1}{2} - \frac{g}{2} \sum_R e^{g Q_R c} Q_R \bar{A}_R A_R \quad (20)$$

where we recall that P is the superpotential [see (1)]. Here Q_R are the $U(1)$ charges of the various representations (R) . (Note that V_0 is not the often quoted $\frac{1}{2}D^2 + \Sigma \bar{F}F$ result of the Wess-Zumino gauge.)

One observes three curious features of (18). V_0 is not defined for the SUSY Feynman ($\alpha = 1$) gauge, the $\frac{1}{2}D^2$ term vanishes in the SUSY Landau gauge ($\alpha = 0$) and finally (18) apparently violates the positive definiteness of the SUSY effective potential. [However, (18) clearly shows the latter to be a pure gauge artefact.] Before giving any specific examples note that the stationary value for any non-spurious stationary point is given by [see (17)]

$$V_0 = \sum_R e^{g Q_R c} \bar{f}_R f_R \quad (21)$$

displaying "positive definiteness".

Let us first consider a model in which SUSY is not spontaneously broken. The model is a $U(1)$ SUSY gauge theory with a Fayet-Iliopoulos D term with one minimally coupled [see (1)] chiral superfield, S , of charge $+1$ and vanishing superpotential, $P = 0$. For this model (18) yields

$$V_0 = \frac{1}{2} \left(\frac{\alpha}{\alpha-1} \right) \left(\frac{1}{2} + \frac{g}{2} e^{g c} \bar{A}_S A_S \right)^2 \quad (22)$$

There are two stationary points of V_0

- i) $a_S = \bar{a}_S = 0$ (m, n and c undetermined) for which

$$V_0 = \frac{1}{8} \left(\frac{\alpha}{\alpha-1} \right) \quad ; \quad d = - \frac{1}{2} \left(\frac{\alpha}{\alpha-1} \right) \quad (23)$$

and

ii) $\bar{a}_s a_s = -\xi e^{-gC}/g$ (m and n undetermined), for which

$$V_0 = 0 ; d = 0 \quad (24)$$

The potential does yield the absolute minimum as predicted earlier. The stationary point in (23) is spurious as $d \neq 0$ [see (17)]. Observe that as mentioned the stationary value belonging to such a spurious stationary point is invariably gauge dependent and can be forced negative. Finally observe that with $\xi = 0$ we obtain a model with no spurious stationary points.

Next we consider a model with D type breaking. This is essentially massive SUSY QED which requires two chiral superfields S and T for which

$$P(S, T) = \xi S T \quad (25)$$

where ρ denotes the "electron" mass. A Fayet-Iliopoulos D term is then added to induced spontaneous SUSY breaking. V_0 is given by (18) with the D and F equations of motion

$$\begin{aligned} F_S &= -\xi \bar{A}_t - \frac{i g}{2} (M + iN) A_S \\ F_t &= -\xi \bar{A}_s + \frac{i g}{2} (M + iN) A_t \\ \left(\frac{\alpha-1}{\alpha}\right) D &= -\frac{\xi}{2} - \frac{g}{2} (e^{gC} \bar{A}_s A_s - e^{-gC} \bar{A}_t A_t) \end{aligned} \quad (26)$$

By considering all the stationary conditions ($\partial V_0 / \partial \text{field} = 0$) I find only one consistent solution and this is given by

$$\begin{aligned} i)' \quad a_s &= a_t = 0 ; m, n \text{ \& } C \text{ undetermined.} \\ \text{with} \quad V_0 &= \left(\frac{\alpha}{\alpha-1}\right) \frac{\xi^2}{8} ; d = -\frac{\xi}{2} \left(\frac{\alpha}{\alpha-1}\right) \end{aligned} \quad (27)$$

This model contains no non-spurious stationary points! Just as predicted in the last section, a physical solution does not appear, however, this is no longer perplexing as we know from the same section that SCGF is a bad gauge choice for this type of model.

F type models unfold in much the same way and an example is not necessary.

COMMENT ON SUSY R_ξ GAUGES

The problems of SCGF stem from the requirement that d must vanish at all translationary invariant stationary points of the effective action (17). However, for SUSY R_ξ gauges we no longer have such a demanding constraint as seen in (16). In other words SUSY R_ξ gauges can accommodate models with $d \neq 0$ at physical stationary points since $d \neq 0$ no longer implies [see (15)] that the auxiliary field C has to pick up an x dependent VEV. Thus for appropriate choice of ω_R the physical stationary point will be translationary invariant (indicating good gauge choice) and can be determined via the effective potential.

CONCLUSIONS

We may safely conclude that SCGF is not a useful gauge choice when discussing or analyzing a SUSY effective potential. SUSY R_ξ gauges on the other hand offer a viable alternative if one wishes to pursue SUSY gauge fixing in SUSY effective potential analyses.

Maybe one lesson learnt is that blindly choosing an admissible gauge is not always sufficient.

ACKNOWLEDGEMENTS

I am indebted to Dr. T. Kugo for many useful discussions, comments and suggestions.

REFERENCES

- 1) R. Fukuda and T. Kugo - Phys.Rev. D13 (1976) 3469.
- 2) B.A. Ovrut - "Supersymmetry at Ordinary Energy", IAS Preprint (May 1982) ;
M. Dine - "Realistic Supersymmetric Models", in Proceedings of the Johns
Hopkins Workshop on Current Problems in Particle Theory, Florence (June 1982) ;
J. Ellis, L.E. Ibáñez and G.G. Ross - "SUSY Grand Unification", CERN Preprint
TH.3382 (August 1982).
- 3) M. Grisaru, W. Siegel and M. Roček - Nuclear Phys. B159 (1979) 451.
- 4) B. de Witt - Phys.Rev. D12 (1975) 1843.
- 5) J. Wess and J. Bagger - "Supersymmetry and Supergravity", Princeton Lecture
Notes (spring 1981).
- 6) S. Coleman and E. Weinberg - Phys.Rev. D7 (1973) 1888.