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GLUON CONDENSATE POTENTIALS

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A B S T R A C T

Attempts to represent the "gluon condensate", of Shifman, Vainshtein, and Zakharov, by a quark-antiquark potential, are critically reviewed. In particular, it is noted that the Leutwyler-Voloshin level shifts, in hydrogen-atom-like heavy quarkonia, are well reproduced by a static potential. But it is mass dependent. No adequate bridge is found between the field theory of Shifman, Vainshtein, and Zakharov, on the one hand, and popular potential models on the other.

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1. - INTRODUCTION

In accounting for heavy-quark-antiquark bound states great success has been reported for two apparently unrelated approaches. One involves a static potential in a non-relativistic Schrödinger equation. We refer to some recent reviews ¹⁾ of this approach. The other involves a relativistic quantum field theory (QCD) and certain quasiperturbative approximations ²⁾. It is natural to attempt to construct a bridge between these two approaches. Here we review some such attempts, and find them rather unsuccessful.

In their celebrated QCD approach, Shifman, Vainshtein, and Zakharov ²⁾ relied essentially on two novel ideas. The first might be called "pole dominance of quasiperturbative moments" - PQDM - the idea that certain dispersion relations are well approximated both by quasiperturbative calculations and by dominance of the pole representing the lowest bound state in the given channel. The second idea is that of the "gluon condensate" - effectively a modification of the gluon propagator at intermediate distances; this modification contributes the "quasi" in "quasiperturbative". If both of these two ideas are respected then one is led, for heavy quarks, and S states, to the "equivalent" potential of Bell and Bertlmann ³⁾:

$$V = -\frac{4}{3} \frac{\alpha_s}{r} + \frac{\pi^2}{144} \left\langle \frac{\alpha_s}{\pi} G G \right\rangle m r^4 \quad (1)$$

where α_s is the QCD coupling, where m is quark mass, and

$$\left\langle \frac{\alpha_s}{\pi} G G \right\rangle$$

is the "gluon condensate" parameter. This potential, used as a first order perturbation in the calculation of exponential moments, reproduces in non-relativistic approximation the quasiperturbative field theory moments of SVZ. Moreover, the non-relativistic approximation is numerically quite good in the case of charmonium ³⁾ and bottomonium ⁴⁾.

Unfortunately this potential is rather unlike those favoured by potential modellers. In particular the mass dependence of the non-Coulombic part is unwelcome to them, as also is the state dependence which appears when states other than S are considered ⁵⁾. Moreover when used non-perturbatively in the Schrödinger equation ³⁾ this potential gives rather different results than when used perturbatively, à la SVZ, in the moments. As a bridge between the two approaches, it is therefore rather inadequate. Nevertheless we insist that if both of the two main ideas of SVZ are respected, it is to this potential that we are led.

All the other potentials that we will consider ignore one of the two main ideas of SVZ, namely PDQM. They therefore set aside the SVZ work on charmonium, which depends on this approximation. It seems to us that this setting aside of PDQM, and the related SVZ results, deserves more emphasis than it is usually given. However, we are happy to see attempts to improve on PDQM, in view of our experience with it recalled above. This is especially so for very heavy quarkonia. These are expected to be very compact, and dominated by the Coulomb interaction. We found that for a pure Coulomb interaction the PDQM approach gives binding energies too small by a factor of 2 [Ref. 6), Table 1; note that some errors in the third and fourth column of this Table are corrected in Ref. 7), Table 1].

The Coulomb-dominated case, with the gluon condensate effect as a relatively small perturbation, has been worked out by Voloshin ⁸⁾ and Leutwyler ⁹⁾. Their work was not directed towards finding a potential. But Flory ¹⁰⁾ produced a potential from similar ideas after adding the approximation of static quarks. Comparing his results with those of Leutwyler and Voloshin we find this approximation quite inadequate for the lowest levels. However, all levels are well fitted when the cubic potential of Flory is supplemented by terms of lower degree in r - provided they are mass dependent.

2. - BETHE-SALPETER KERNEL

Let us recall the essential difficulty ⁸⁾ of representing the gluon condensate insertion, in the gluon propagator, by a potential. The first approximation to the Bethe-Salpeter kernel contains the zero order gluon propagator :

$$(X^2)^{-1} \quad (2)$$

with

$$X^2 = \vec{X}^2 + X_4^2$$

where X_4 is Euclidean time. In the case of weak coupling, and so of slowly moving fermions in Coulomb bound states, a good approximation is to replace this by

$$\delta(X_4) \int dX_4 (X^2)^{-1} = \pi \delta(X_4) |\vec{X}|^{-1} \quad (3)$$

The right-hand side is instantaneous in (Euclidean) time, and the coefficient of the δ function gives the familiar Coulomb potential. This approximation, together with non-relativistic approximations for fermion propagators, reduces the two-time relativistic Bethe-Salpeter equation to the one-time non-relativistic Schrödinger equation.

Consider now the gluon condensate effect. Disregarding complications associated with tensor indices, we have to cope with the propagator modification

$$(X^2)^{-1} \rightarrow (X^2)^{-1} + C (X^2)^{-1} \quad (4)$$

where C is a constant. An attempt to repeat the above argument confronts us with the expression

$$\delta(X_4) \int dX_4 (\vec{X}^2 + X_4^2)^{-1} \quad (5)$$

- which diverges and makes no sense.

So an effective potential cannot be found in the familiar way. One must look at how other relative-time dependent factors, in any particular application, suppress the divergence.

For example, when the quasiperturbative moments are considered, the relevant integrals converge - and can be reproduced in non-relativistic approximation by the "equivalent" (for this purpose) potential (1). As we discuss in a moment, the level shifts in heavy hydrogenic quarkonia also converge - and can be well reproduced by a different "equivalent" potential. Since for these physical quantities we get finite expressions, we are not tempted to "regulate" somehow the Bethe-Salpeter kernel to force out somehow a potential in the more usual way ¹¹⁾.

Of course the expression (5) becomes convergent if (4) is modified to fall off sufficiently at large x^2 . This possibility has been explored by Gromes ¹²⁾. Here we continue to explore rather the possibility originally envisaged by SVZ, that the objects in question are small enough for the short-distance expansion (4) to be decisive, and any fall off at larger distances unimportant.

We note briefly that Soni and Tran ¹³⁾ do not encounter divergences like (5). But this is because while allowing the gluon condensate effect in propagators in closed loops in diagrams, they omit it from the basic one-gluon exchange diagram. They omit therefore the only effect which SVZ include, and so have some quite different philosophy.

3. - LEUTWYLER AND VOLOSHIN

These authors ^{8),9)} are not concerned to derive a potential, but we find it convenient to approach the potential of Flory ¹⁰⁾ by way of their work. They argue that for very heavy quarks and antiquarks the low lying bound states, because of their small size, will be dominated by the Coulomb interaction $-(4/3)\alpha_s/r$. The condensate effect can then be added as a perturbation. The direct approach then, from our point of view, would be to add the condensate term to the propagator as a

perturbation in the bound state Bethe-Salpeter equation. However, Leutwyler and Voloshin find a different approach more convenient. They begin with a non-relativistic Schrödinger equation for the Coulomb bound states. This system they suppose to interact with a constant (in both space and time) colour octet field

$$\vec{F}^a$$

This is supposed to have vacuum expectation values, to simulate the condensate term in the gluon propagator,

$$\langle \Omega | g^a F_i^a F_j^a | \Omega \rangle = \frac{1}{3} \delta_{ij} \frac{1}{8} g^{ab} \langle \Omega | g^2 \vec{F}^c \cdot \vec{F}^c | \Omega \rangle \quad (6)$$

with

$$\langle \Omega | g^2 \vec{F}^c \cdot \vec{F}^c | \Omega \rangle = -\frac{1}{4} \langle g^2 G_{\mu\nu}^c G_{\mu\nu}^c \rangle = -\pi^2 \langle \frac{\alpha_s}{\pi} G G \rangle \quad (7)$$

in terms of the conventional gluon condensate parameter.

The Hamiltonian of the combined system is

$$H = H^0 + H_1 \quad (8)$$

The unperturbed Hamiltonian is

$$H^0 = H_1^0 = V_1 + p^2/m \quad (9)$$

and

$$H^0 = H_2^0 = V_2 + p^2/m \quad (10)$$

for quark-antiquark colour-singlet and colour-octet states respectively, with

$$V_1 = -\frac{4}{3} \frac{\alpha_s}{r} \quad V_2 = +\frac{1}{6} \frac{\alpha_s}{r} \quad (11)$$

where with quark-antiquark separation $\vec{x}$, $r = |\vec{x}|$. The interaction is of dipole form :

$$H_I = (-g/2)(\lambda_q^a - \lambda_{\bar{q}}^a) \vec{x} \cdot \vec{F}^a \quad (12)$$

where the λ are SU(3) generators for quark and antiquark.

The perturbation of a given colour-singlet Coulomb level $|1\rangle$ is given by

$$\delta E = \langle 1 | \langle \Omega | H_I (E_1^0 - H_0^0)^{-1} H_I | \Omega \rangle | 1 \rangle \quad (13)$$

$$= \frac{\pi^2}{18} \langle \frac{\alpha_s}{\pi} G G \rangle \langle 1 | X_i (H_0^0 - E_1^0)^{-1} X_i | 1 \rangle \quad (14)$$

The unperturbed Coulomb levels are

$$E_{n\ell} = 2m - m\beta^2 / (4n^2) \quad (15)$$

where

$$\beta = (4/3)\alpha_s \quad (16)$$

From (13) these levels are shifted by ⁹⁾

$$\delta E_{n\ell} = A n^6 \mathcal{E}_{n\ell} \quad (17)$$

where

$$A = m\pi^2 \langle \frac{\alpha_s}{\pi} G G \rangle (m\beta)^{-4} \quad (18)$$

$$\mathcal{E}_{n\ell} = \frac{2}{9n^3} \left\{ \frac{\ell+1}{2\ell+1} [F(n,\ell) - F(-n,\ell)] + \frac{\ell}{2\ell+1} [F(n,\ell-1) - F(-n,\ell-1)] \right\} \quad (19)$$

$$F(n,\ell) = 2n [n^2 - (\ell+1)^2] + \frac{(n+\ell+2)(n+\ell+1)}{qn+16} \left[\frac{(n-\ell)(n+\ell+1)}{qn+16} + \frac{4(2n-\ell)^2}{4n+8} \right] \quad (20)$$

For large n , ϵ has the expansion

$$\begin{aligned} \mathcal{E}_{n\ell} = \frac{4}{81} \left\{ 35 - \frac{1015}{81n^2} + \frac{12704}{6561n^4} + \dots \right. \\ \left. + \frac{\ell(\ell+1)}{n^2} \left(-30 + \frac{446}{27n^2} + \frac{34304}{2187n^4} + \dots \right) \right. \\ \left. + \frac{\ell^2(\ell+1)^2}{n^4} \left(9 - \frac{16384}{2187n^4} + \dots \right) \right\} \quad (21) \end{aligned}$$

where the missing terms, ..., are higher powers of n^{-2} .

The quantities ϵ are of order unity, so the ratio of the shift (17) to the Coulomb part of the unperturbed energy (15) is of order

$$\begin{aligned} 4\pi^2 \langle \frac{\alpha_s}{\pi} G G \rangle (m\beta)^{-4} n^6 \beta^{-2} \\ \approx (2.5 n^2 \beta^{-1/2} m/m)^4 \quad (22) \end{aligned}$$

where

$$M^4 = \langle \frac{\alpha_s}{\pi} G G \rangle \quad (23)$$

Consider the conventional value ²⁾

$$M \approx 330 \text{ MeV} \quad (24)$$

(probably an underestimate ^{3)-7),14)-19)}, and

$$\beta \approx .3$$

Then for (22) to be less than unity even for $n=1$ requires

$$m \gtrsim 5 \text{ GeV}$$

So only for rather massive quarks is the result (17) plausible even for the lowest states. For a given quark mass it becomes rapidly implausible with increasing n . Both these points are emphasized by Leutwyler⁹⁾ and Voloshin⁸⁾.

4. - CUBIC FLORY POTENTIAL

In the Leutwyler-Voloshin formula (14), the factor

$$(H_8^0 - E_1^0)^{-1} = \left(V_8 + \frac{p^2}{m} - E_1^0 \right)^{-1} \quad (25)$$

is troublesome because the term (p^2/m) does not commute with the other quantities involved. If we could somehow regard m as very large, and commutators containing m^{-1} as very small, we could write

$$X_i (H_8^0 - E_1^0)^{-1} X_i = \frac{1}{2} \{ \bar{X}^i, (H_8^0 - E_1^0)^{-1} \} \quad (26)$$

In the same "large m " spirit we could write

$$(H_8^0 - E_1^0)^{-1} |1\rangle = (H_8^0 - H_1^0)^{-1} |1\rangle = (V_8 - V_1)^{-1} |1\rangle \quad (27)$$

$$\langle 1 | (H_8^0 - E_1^0)^{-1} = \langle 1 | (H_8^0 - H_1^0)^{-1} = \langle 1 | (V_8 - V_1)^{-1} \quad (28)$$

using the fact that $|1\rangle$ is an eigenstate of H_1^0 .

Then we would have

$$\delta E = \langle 1 | \delta V_a | 1 \rangle \quad (29)$$

where

$$\delta E_{a\ell} = A n^6 \varepsilon_{a\ell} \quad (36)$$

$$\delta V_a = \frac{\pi^2}{18} \langle \frac{\alpha_s}{\pi} G G \rangle 8 r^3 / (q\beta) \quad (30)$$

$$= A \frac{4}{81} (m\beta r)^3 \quad (31)$$

where A is given by (18).

Flory¹⁰⁾ considers essentially the same theoretical framework as Voloshin and Leutwyler, but neglects a priori the motion of the quarks. Effectively he takes m to be infinite from the beginning. It is not surprising then that (30) is closely related to his result. Actually, he arrives at a more elaborate addition to the Coulomb potential

$$\delta V_F = \frac{8}{5} \left\{ \left[\left(\frac{3\alpha_s}{2r} \right)^2 + \frac{5\pi^2}{72} \langle \frac{\alpha_s}{\pi} G G \rangle r^2 \right]^{1/2} - \frac{3\alpha_s}{2r} \right\} \quad (32)$$

This contains terms of all orders in $\langle \alpha_s G G / \pi \rangle$. We will comment later on the higher order terms. For the moment we are interested only in the term of first order, for the others are relatively unimportant at small r . Expanding out, we find

$$\delta V_F = \delta V_a \left(1 + o \left(\langle \frac{\alpha_s}{\pi} G G \rangle r^4 \right) \right) \quad (33)$$

as expected.

For Coulomb wave functions²⁰⁾

$$\frac{4}{81} \langle m^3 \beta^3 r^3 \rangle_{n\ell} = n^6 \varepsilon_{a\ell} \quad (34)$$

$$\varepsilon_{a\ell} = \frac{4}{81} \left(35 + \frac{25}{n^2} + \frac{\ell(\ell+1)}{n^2} \left(-30 - \frac{6}{n^2} \right) + \frac{\ell^2(\ell+1)^2}{n^4} 3 \right) \quad (35)$$

The corresponding level shifts are

For large n the leading terms in (21) are reproduced correctly. But for low n this "infinite mass" approximation is rather poor, as displayed in the Table. For the ground state the disagreement is by a factor of about 2.

This disagreement of course is quite independent of the mass m . This is the only parameter defining a scale of length for the Coulomb wave functions. The explicit relative mass dependence of the terms neglected in (30) is simply cancelled by the mass dependence implicit elsewhere, for given n and l . In these conditions the "infinite mass" approximation is hard to motivate.

5. - CORRECTIONS

That the Flory cubic potential gives correctly the leading terms for large n in (21) suggests that we might improve the fit by adding - quite arbitrarily, potentials of lower degree. Thus a quadratic potential

$$\delta V_6 = \frac{4}{81} B (m\beta r)^2 \quad (37)$$

gives additional shifts

$$\delta E_{6nl} = B n^6 \epsilon_{6nl} \quad (38)$$

with

$$\epsilon_{6nl} = \frac{4}{81} \langle m^2 \beta^2 r^2 \rangle_{nl} n^{-6} \quad (39)$$

and 20)

$$n^{-6} \langle m^2 \beta^2 r^2 \rangle_{nl} = \frac{10}{n^2} + \frac{2}{n^4} - \frac{l(l+1)}{n^2} \frac{6}{n^2} \quad (40)$$

The choice

$$B = -\frac{304}{81} A \quad (41)$$

produces accord with the next-to-leading terms in (21) (all three of them, remarkably!).

Linear and constant potentials

$$\delta V_2 = \frac{4}{81} C m\beta r \quad (42)$$

$$\delta V_4 = \frac{4}{81} D \quad (43)$$

produce additional shifts

$$\delta E_{cnl} = C n^6 \epsilon_{cnl} \quad (44)$$

$$\delta E_{dnl} = D n^6 \epsilon_{dnl} \quad (45)$$

where

$$\epsilon_{cnl} = \frac{4}{81} n^{-6} \langle m\beta r \rangle_{nl} \quad (46)$$

$$\epsilon_{dnl} = \frac{4}{81} n^{-6} \langle 1 \rangle_{nl} \quad (47)$$

and 20)

$$n^{-6} \langle m\beta r \rangle_{nl} = \frac{3}{n^4} - \frac{l(l+1)}{n^2} \frac{1}{n^4} \quad (48)$$

$$n^{-6} \langle 1 \rangle_{nl} = n^{-6} \quad (49)$$

Taking

$$C = (58/10) A \quad (50)$$

and

$$D = (-113/100)A \quad (51)$$

we find (without any very systematic search for a best fit) that the potential

$$S V_{41b} = \frac{4\pi^2}{81\beta} \left\langle \frac{\alpha_s}{\pi} G G \right\rangle \left\{ r^3 - \frac{304}{81} \frac{r^2}{m\beta} + \frac{53}{10} \frac{r}{m^2\beta^2} - \frac{113}{100} \frac{1}{m^3\beta^3} \right\} \quad (52)$$

Gives first order level shifts in agreement with those of Leutwyler and Voloshin (17) to within one per cent. The Table shows the effect of subtracting from ϵ_{nl} successively ϵ_{anl} , $(B/A)\epsilon_{bnl}$, $(C/A)\epsilon_{cnl}$ and $(D/A)\epsilon_{dnl}$.

6. - HIGHER ORDERS IN $\frac{\alpha_s GG}{\pi}$

The potential (33) of Flory was obtained from ideas like those of Leutwyler and Voloshin, with the simplification that the quarks are not allowed to move. The problem is then just to calculate the ground state energy of the fixed-in-position quark-antiquark pair. Second order perturbation theory readily gives the cubic potential δV_a of (30). Flory, however, retains terms of all orders in $\langle \alpha_s GG/\pi \rangle$. His treatment is quite terse, and we have not been able to follow it completely.

To illustrate our difficulties in a trivial context, consider a very simple model. Let a system with just two states, called "1" and "g", interact with a constant field F which causes transitions. The Hamiltonian is

$$H = H^0 + H_I \quad (53)$$

$$H^0 = \alpha \sigma_3 \quad (54)$$

$$H_I = F \sigma_1 \quad (55)$$

where the σ are Pauli matrices. The perturbed eigenvalue E, corresponding to the unperturbed $E^0 = -\alpha$, has the Brillouin-Wigner expansion

$$E = E^0 + \langle 1 | H_I \frac{1-P}{E-H^0} H_I | 1 \rangle + \langle 1 | H_I \frac{1-P}{E-H^0} H_I \frac{1-P}{E-H^0} H_I | 1 \rangle + \dots \quad (56)$$

where P is the projection operator

$$P = |1\rangle \langle 1| \quad (57)$$

It seems to us that in the corresponding expression of Flory, his (3) or (6), E has been replaced in the energy denominators by E^0 - but the compensating extra terms of the Rayleigh-Schrödinger expansion not added. This is our first difficulty.

In the trivial model, because H_I has no 8-8 matrix elements, the Brillouin-Wigner series terminates

$$E = -\alpha + F^2 / (E - \alpha) \quad (58)$$

whence

$$E = -\sqrt{\alpha^2 + F^2} \quad (59)$$

If we had replaced E by E^0 in the denominator we would have obtained only the approximation

$$E \approx -\alpha - F^2 / (2\alpha) \quad (60)$$

So far we supposed F fixed. Suppose now that it fluctuates. Then we have to specify how it fluctuates. Flory seems to use a pairing algorithm, for mean values of powers of F, corresponding to Gaussian fluctuations. This is a new assumption, not made by SVZ, whose motivation is not clear to us. That is our second difficulty.

To simulate the pairing algorithm, suppose iF to be a multiple of the co-ordinate of an harmonic oscillator

$$F = iK(\alpha + \alpha^\dagger) \quad (61)$$

where K is a real constant, and α and $\alpha^\dagger$ are destruction and creation operators with respect to a "soft-gluon vacuum" $|0\rangle$:

$$\alpha|0\rangle = \langle 0|\alpha^\dagger = 0 \quad (62)$$

The factor i is required in (61) to make

$$\langle 0|F^2|0\rangle = -K^2 \quad (63)$$

negative, in analogy with (7). The energy of the oscillator, following Leutwyler and Voloshin and Flory, is neglected in comparison with the energy separation 2α of the "1" and "8" states of the system to which it is coupled.

When this new problem is tackled by Brillouin-Wigner perturbation theory, the allowed intermediate states (all states other than the unperturbed state in question) include some differing from the initial state only by addition of "soft-gluons". Flory, however, seems to insist on excitation from "1" to "8" in intermediate states. That is our third difficulty.

Moreover, the energy denominators associated with these extra intermediate states are of the form

$$(E - E^0) \quad (64)$$

- because the energy of the "soft-gluons" is neglected. An iterative development beginning with $E = E^0$ encounters an immediate infinity. In fact the initial state, because of the possibility of adding zero-energy "soft-gluons", is highly degenerate, and the application of perturbation theory correspondingly delicate. We can attempt a non-

perturbative solution. Then because F commutes with everything else it can be diagonalized first and replaced by one of its eigenvalues - i.e., by any imaginary c number. The problem is then again that of a fixed field, and

$$E = -\sqrt{\alpha^2 + F^2} \quad (65)$$

But F^2 is here any negative number. We have a continuous spectrum - becoming imaginary for $(-F^2)$ large. This situation is rather different from that which Flory seems to envisage. For these various reasons we think that the higher order terms in (32), until a fuller derivation appears, should be treated with reserve.

7. - CONCLUSION

Our most constructive remark is that the Leutwyler-Voloshin level shifts can be remarkably well fitted by the static potential (52). This potential differs considerably from the quartic term in (1). While (1) is constructed to reproduce gluon condensate shifts in Shifman-Vainshtein-Zakharov moments, (52) is designed to reproduce gluon condensate level shifts in hydrogen-like heavy quark-antiquark systems. The two potentials differ because they are fitted to different aspects of the field theory - and field theory is not potential theory. However, (52) is like (1) in being mass dependent. In this it differs obstinately from the mass-independent potentials of current potential models ¹⁾.

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1	0	1.468235	-1.494727	0.729321	-0.055864	-0.000061
2	0	1.585217	-0.451819	0.034691	-0.014383	-0.013511
2	1	0.998390	-0.297906	0.049602	0.008707	0.009579
3	0	1.661034	-0.204535	0.005972	-0.003721	-0.003645
3	1	1.361433	-0.174917	0.008132	-0.000843	-0.000767
3	2	0.803842	-0.117969	0.010165	0.002626	0.002702
4	0	1.690125	-0.115430	0.001854	-0.001213	-0.001200
4	1	1.514012	-0.106358	0.002239	-0.000701	-0.000687
4	2	1.175499	-0.088389	0.002831	0.000148	0.000161
4	3	0.702006	-0.061883	0.003275	0.000975	0.000988
5	0	1.703803	-0.073975	0.000753	-0.000503	-0.000499
5	1	1.588943	-0.070314	0.000855	-0.000368	-0.000364
5	2	1.364886	-0.063024	0.001029	-0.000127	-0.000123
5	3	1.042954	-0.052155	0.001222	0.000166	0.000170
5	4	0.640138	-0.037788	0.001355	0.000434	0.000437
6	0	1.711282	-0.051406	0.000363	-0.000243	-0.000242
6	1	1.630726	-0.049658	0.000395	-0.000200	-0.000199
6	2	1.472354	-0.046164	0.000457	-0.000115	-0.000114
6	3	1.241639	-0.040938	0.000534	-0.000005	-0.000004
6	4	0.946796	-0.033999	0.000609	0.000115	0.000116

TABLE - Approximation of Voloshin-Leutwyler level shifts by a series of power law potentials. The first two columns list the quantum numbers n and l . The third column lists the quantity ϵ_n of Leutwyler. The subsequent columns list the residues ϵ_n^l after subtracting successively contributions of the cubic, quadratic, linear, and constant potentials of (52).