Ref.TH.3543-CERN

RECENT DEVELOPMENTS IN $N = 8$ SUPERGRAVITY *)

H. Nicolai
CERN - Geneva

A B S T R A C T

Some recent developments in (gauged) $N = 8$ supergravity are reviewed. Particular emphasis is placed on the connection between this theory and $N = 1$ supergravity in eleven dimensions and on the mechanism of spontaneous compactification by which spontaneously broken versions of supergravity are generated.

*) -----
Invited Lecture at the Europhysics Study Conference on "Electroweak Effects at High Energies", Erice (Sicily), February 1983.

1. - INTRODUCTION

During the past two years, we have witnessed a dramatic increase in "public interest" in supersymmetry and supergravity¹⁾. This interest cannot as yet be based on any solid experimental fact; there is no evidence that would force us to abandon the presently very successful standard model of strong and electro-weak interactions in favour of some supersymmetric theory. Nevertheless, it has become clear that supersymmetry has many attractive features to offer and that it will perhaps play a crucial role in the ultimate unification of fundamental particle interactions. This unification constitutes one of the most ambitious endeavors in theoretical physics because it necessarily involves the extrapolation in energy over many orders of magnitude where no experimental information is expected to become available. The main reason why supersymmetry and supergravity are so attractive is that, at the present time, no other candidate theories exist which may enable us to simultaneously solve the three outstanding problems of modern elementary particle physics. These are:

- i) the unification of gravity with the other fundamental interactions and the construction of a consistent, i.e., finite or renormalizable, theory of quantum gravity;
- ii) the explanation of the current proliferation of fundamental fields and coupling constants ("who ordered the muon?"), and
- iii) the hierarchy problem in its most fundamental form: why are the relevant mass scales of the standard model so tiny in comparison with the Planck mass of 10^{19} GeV, or why is the gravitational force so much weaker than the other fundamental forces ?

Supersymmetry describes bosons and fermions in a unified way as partners in a supermultiplet which thus contains particles of different spin. These supermultiplets form irreducible representations of the basic superalgebra

$$\{Q_{\alpha}^i, \bar{Q}_{\dot{\beta}j}\} = 2\delta^i_j \sigma^{\mu}_{\alpha\dot{\beta}} P_{\mu} \quad (1.1)$$

If there are N independent supersymmetries, the indices $i, j, \dots$ assume the values $1, \dots, N$ and the irreducible multiplets cover a range of spins of at least $\hbar N/4$. For $N > 4$, it turns out that there are no "matter supermultiplets" any more and that all relevant multiplets contain spin-2 particles. Hence a unification with gravity is forced upon us. From the algebra (1.1), it is also easy to see that local supersymmetry implies gravity: the commutator of two space-time dependent supersymmetry transformations yields a space-time dependent translation which is nothing but a general co-ordinate transformation.

The maximally extended theory which does not contain spins higher than $s = 2$ is $N = 8$ supergravity²⁾⁻⁴⁾. This theory, which is invariant under $N = 8$ independent supersymmetries, describes the interactions of one graviton, eight gravitinos, 28 spin-1, 56 spin-1/2 and 70 spin-0 fields. There is an alternative way to characterize this theory. Namely, the number of supersymmetry generators also increases as one goes to higher space-time dimensions because spinors in higher dimensions have more components. One can show that supergravities can be constructed in only up to eleven dimensions⁵⁾ and that $N = 1$ supergravity in eleven space-time dimensions⁶⁾ corresponds to $N = 8$ supergravity in four dimensions. This can be easily understood because the 32 components of a spinor in eleven dimensions decompose into eight four-components spinors in four dimensions. One may regard the eleven-dimensional theory as more fundamental since it gives rise not only to $N = 8$ supergravity in four dimensions but also to various $N \leq 8$ supergravity theories with and without spontaneous breaking of supersymmetry. In the general framework of Kaluza-Klein theories⁷⁾, the $N = 8$ theory is presumably obtained by "spontaneously compactifying"^{8),9)} the eleven dimensional theory on the seven-sphere S^7 ^{10),11)}. Only the low-energy modes are retained in this scheme, but one should realize that the full theory also contains an infinite tower of higher excited modes with masses of the order of the Planck mass and multiples thereof.

In this contribution, we will review some of the progress that has been recently made in understanding the basic mechanism of spontaneous compactification and its implications for supergravity. Although various aspects are not yet fully understood, it is now clear that the rich structure of supergravity theories may also lead to a better understanding of spontaneous compactification.

2. - SPONTANEOUS COMPACTIFICATION OF ELEVEN-DIMENSIONAL SUPERGRAVITY

Eleven-dimensional supergravity was constructed in Ref. 6). It is based on the following multiplet of fields: an elfbein E_M^A which describes ordinary gravity in eleven dimensions, a 32-component Majorana spinor Ψ_M which is the analog of the gravitino in four dimensions and an antisymmetric three-index tensor A_{MNP} . The latter is subject to the Maxwell-like Abelian gauge transformations

$$\delta A_{MNP} = \partial_{[M} \Lambda_{NP]} \quad (2.1)$$

which are necessary to balance the bosonic and fermionic physical degrees of freedom. Under local supersymmetry transformations of parameter ϵ , these fields transform into each other according to⁶⁾

$$\begin{aligned}
 \delta E_M^A &= -\frac{i}{2} \bar{\epsilon} \tilde{F}^A \psi_M \\
 \delta A_{MNP} &= \frac{\sqrt{2}}{8} \bar{\epsilon} \tilde{\Gamma}_{[MN} \psi_{P]} \\
 \delta \psi_M &= D_M \epsilon + \\
 &+ \frac{\sqrt{2}}{288} i (\tilde{\Gamma}_M^{NPQR} - 8 \delta_M^N \tilde{\Gamma}^{PQR}) \epsilon F_{NPQR}^*
 \end{aligned}
 \tag{2.2}$$

The eleven-dimensional $\tilde{\Gamma}$ matrices which appear in (2.2) are 32×32 matrices and may be represented as direct products of ordinary 4×4 γ matrices and 8×8 matrices Γ_m which generate the Clifford algebra in seven dimensions

$$\Gamma_\mu = \gamma_\mu \otimes \mathbb{1} \quad \tilde{\Gamma}_m = \gamma^5 \otimes \Gamma_m
 \tag{2.3}$$

Here and in what follows, we will always split eleven-dimensional indices $M = 1, \dots, 11$ into four-dimensional ones $\mu, \dots = 1, \dots, 4$ and seven-dimensional ones $m, \dots = 5, \dots, 11$.

There is a unique Lagrangian which is left invariant under (2.2). However, we will need here only the bosonic field equations which follow from this Lagrangian. They are

$$\begin{aligned}
 R_{MN} - \frac{1}{2} g_{MN} &= -\frac{1}{48} \{ 8 F_{MPQR} F_N^{PQR} - g_{MN} F^2 \} \\
 F_{MNPQ}{}_{;M} &= \\
 &= -\frac{\sqrt{2}}{1152} \epsilon^{M_1 \dots M_8 NPQ} F_{M_1 \dots M_4} F_{M_5 \dots M_8}
 \end{aligned}
 \tag{2.4}$$

Spontaneous compactification means that these equations admit solutions which "spontaneously" split eleven-dimensional space-time into a product of four-dimensional space-time and some seven-dimensional "internal" manifold and thereby "explain" the four-dimensionality of space-time. That this is indeed the case, was first noted in Ref. 9). The second of Eqs. (2.4) is solved by

$$F^{\mu\nu\varrho\sigma} = 3\sqrt{2} m e^{-1} \epsilon^{\mu\nu\varrho\sigma}
 \tag{2.5}$$

all other F 's = 0

*) -----
We adhere to the notation and conventions of Ref. 11).

Inserting (2.5) into the right-hand side of the first of Eqs. (2.4), one easily sees that these become two independent equations, one for ordinary space-time

$$R_{\mu\nu} = 12 m^2 g_{\mu\nu} \quad (2.6)$$

and another one for the "internal" seven-dimensional manifold

$$R_{mn} = -6 m^2 g_{mn} \quad (2.7)$$

Thus, the mass parameter m in (2.5) serves as an order parameter for spontaneous compactification, and the four-dimensionality of space-time is a consequence of the fact that the field strength F_{MNPQ} has four indices⁹⁾:

At this point, the choice of four- and seven-dimensional manifolds is still quite arbitrary. Equations (2.6) and (2.7) only require them to be Einstein spaces: a four-dimensional one with negative cosmological constant and a seven-dimensional one with positive cosmological constant. One can now further restrict the set of solutions by demanding that the resulting theory in four dimensions have eight supersymmetries¹⁰⁾. To do so, one must analyze the transformation law of Ψ_m [see Eq. (2.2)] in the background (2.5), (2.6) and (2.7). Inserting (2.5) into $\delta\Psi_m$, one finds, using the representation (2.3),

$$\delta\Psi_m = (D_m - \frac{1}{2} m \Gamma_m) \epsilon \quad (2.8)$$

where Γ_m acts trivially on four-dimensional Dirac indices. For supersymmetry transformation parameters ϵ which correspond to unbroken supersymmetries in four dimensions, the right-hand side of (2.8) must vanish. A necessary condition for this to happen is the integrability constraint

$$\begin{aligned} [D_m - \frac{1}{2} m \Gamma_m, D_n - \frac{1}{2} m \Gamma_n] \epsilon &= \\ &= \left(+ \frac{1}{4} R_{mn}{}^{pq}(E) \Gamma_{pq} + \frac{m^2}{2} \Gamma_{mn} \right) \epsilon \stackrel{!}{=} \\ &\stackrel{!}{=} 0 \end{aligned} \quad (2.9)$$

But (2.9) immediately implies

$$R_{mnpq}(E) = -m^2 (g_{mp} g_{nq} - g_{mq} g_{np}) \quad (2.10)$$

and therefore the "internal" manifold is the maximally symmetric space in seven dimensions: the seven-sphere S^7 . In a similar way, one can show that the evaluation of the integrability constraint on the four-dimensional manifold leads to

$$R_{\mu\nu\sigma\rho}(E) = 4m^2(g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma}) \quad (2.11)$$

The curvature tensor (2.11) describes a four-dimensional anti de Sitter space. The integrability constraint thus singles out the unique choice of $\text{AdS} \times S^7$ as the compactification that preserves eight supersymmetries. Conversely, one can also prove that this is indeed the only such compactification¹¹⁾. In order to exhibit the eight remaining supersymmetries explicitly, it is necessary to distinguish between four-dimensional co-ordinates x^μ and seven-dimensional co-ordinates y^m . The supersymmetry transformation parameter ϵ which is a function of both x^μ and y^m may then be expanded according to

$$\epsilon(x, y) = \epsilon^I(x) \otimes \eta^I(y) + \dots \quad (2.12)$$

where ϵ^I are four-dimensional and η^I seven-dimensional spinors, and the index I labels the independent supersymmetries. Inserting (2.12) into (2.8) and observing that the differential operator in (2.8) only acts on seven-dimensional spinor indices, one obtains

$$(D_m - \frac{1}{2} m \Gamma_m) \eta^I(y) = 0 \quad (2.13)$$

For S^7 , there are precisely eight such covariantly constant spinors¹⁰⁾. Furthermore, the natural metric on S^7 is $\text{SO}(8)$ invariant, and therefore the label $I = 1, \dots, 8$ is an $\text{SO}(8)$ index. The four-dimensional parameters $\epsilon^I(x)$ in (2.12) must therefore be identified with the eight local supersymmetry transformation parameters of $N = 8$ supergravity in four dimensions. The eight gravitino fields of the four-dimensional theory are obtained in an analogous manner by expanding the gravitino field Ψ_μ of the eleven-dimensional theory

$$\Psi_\mu(x, y) = \psi_\mu^I(x) \otimes \eta^I(y) + \dots \quad (2.14)$$

Equations (2.12) and (2.14) in conjunction with (2.2) guarantee that

$$\delta \psi_\mu^I(x) = D_\mu \epsilon^I(x) + \dots \quad (2.15)$$

which is the expected transformation law²⁾⁻⁴⁾.

It requires some more work to prove that the resulting four-dimensional theory is gauged $N = 8$ supergravity⁴⁾; this has been demonstrated in Refs. 10) and 11)^{*)}. For the proof, one must construct the analogous ansätze to (2.14) for the other fields as well and show that the spectrum of $N = 8$ supergravity emerges. This involves the identification of zero eigenmodes of certain differential operators on S^7 which is a non-trivial task. That the construction is actually possible may be traced to the unique mathematical properties of S^7 .

3. - SPONTANEOUS SYMMETRY BREAKING

One of the advantages of spontaneous compactification is that it allows us not only to derive gauged $N = 8$ supergravity from eleven-dimensional supergravity but also to construct spontaneously broken versions of supergravity. While (2.5), (2.10) and (2.11) characterize the only solution of the field equations (2.4) with eight local supersymmetries, there are other solutions of (2.4) which exhibit spontaneous breaking of supersymmetry. However, most of these spontaneously broken solutions cannot be related to gauged $N = 8$ supergravity in four dimensions. In this section, three such solutions and the status of their interpretation with regards to $N = 8$ supergravity will be briefly discussed.

The first solution of this type was exhibited in Ref. 12). It is characterized by the relations [cf. Eqs. (2.5), (2.10) and (2.11)]

$$\begin{aligned} F^{\mu\nu\varrho\sigma} &= 2\sqrt{2} m e^{-1} \varepsilon^{\mu\nu\varrho\sigma} \\ R_{\mu\nu\varrho\sigma}(E) &= \frac{10}{3} m^2 (g_{\mu\varrho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\varrho}) \\ R_{mnpq}(E) &= -m^2 (g_{mp} g_{nq} - g_{mq} g_{np}) \end{aligned} \quad (3.1)$$

In addition, the field strength F_{mnpq} now has a non-vanishing expectation value given by

$$F_{mnpq} = \frac{\sqrt{2}}{6} m \varepsilon_{mnpqrst} \bar{\psi} \Gamma^{rst} \psi \quad (3.2)$$

where the covariantly constant spinor ψ satisfies the equation

$$(D_m + \frac{1}{2} m \Gamma_m) \psi(y) = 0 \quad (3.3)$$

Note that the sign in (3.3) is opposite to the one occurring in (2.13); therefore the parity of ψ is opposite to that of $\eta^I(y)$. One can now verify that

^{*)}-----
To be precise, the correspondence has only been established for the linearized theory so far but is expected to extend to all orders.

(3.1) and (3.2) also solve the field equations (2.5)¹²⁾. Since the quantity $\bar{\psi} \Gamma_{mnp} \psi$ is the torsion which "parallelizes" the seven-sphere, this solution is sometimes called the "parallelized solution".

An especially nice feature of this solution is that it may be interpreted as a spontaneously broken version of $N = 8$ supergravity in four dimensions [such an interpretation was first suggested in Ref. 13)]. One can show that it corresponds to a solution of $N = 8$ supergravity with a vacuum expectation value of the pseudoscalar fields B^{IJKL} . For suitably chosen ψ , this vacuum expectation value is explicitly given by¹¹⁾

$$\langle B^{IJKL} \rangle \sim a_{mnp} \Gamma_{[IJ}^{\mu\nu} \Gamma_{KL]}^{\rho\sigma} \quad (3.4)$$

where a_{mnp} are the octonionic structure constants. All supersymmetries are broken by this solution and the surviving gauge symmetry is the exceptional group G_2 ^{11),14)}. One can furthermore explicitly verify that the condition for a stationary point of the four-dimensional scalar field potential⁴⁾ is satisfied by the solution (3.4)¹⁵⁾. Thus, the connection between the "parallelized solution" and the symmetry breaking of gauged $N = 8$ supergravity is rather well understood.

A second solution is obtained by "squashing" the seven-sphere¹⁶⁾. There are two Einstein metrics on S^7 , one corresponding to the "round" S^7 and another one corresponding to a distorted S^7 ¹⁷⁾. The curvature tensor of the "squashed solution" is no longer given by (2.10), but since the equations of motion (2.2) only contain the Ricci-tensor and not the full curvature tensor, it is easy to see that the squashed S^7 still solves the equations of motion. The most interesting feature of this solution is that precisely one supersymmetry survives the squashing¹⁶⁾. Accordingly, there is one covariantly constant spinor η with

$$\left(D_m - \frac{1}{2} m \Gamma_m \right) \eta(y) = 0 \quad (3.5)$$

where D_m is now constructed from the squashed metric on S^7 . It turns out that this spinor is not only covariantly constant, but actually constant^{16),18)}, i.e.,

$$\eta(y) = \text{constant} \quad (3.6)$$

In contrast to the "parallelized" solution, it is not clear whether the squashed solution is interpretable as a spontaneously broken version of $N = 8$ supergravity. Whereas it has been demonstrated that the former gives vacuum expectation values to the 35 zero-mass pseudoscalars only, the latter will give vacuum expectation values not only to the 35 zero-mass scalars but also to the higher excited modes¹⁹⁾. One reason for this difference is that both unbroken and the parallelized solution live on the "round" S^7 which may be represented as the coset-space $SO(8)/SO(7)$ whereas the squashed S^7 is the distance sphere in the eight-dimensional quaternionic projective space¹⁷⁾ which is metrically inequivalent to $SO(8)/SO(7)$.

Finally, there is a third solution which combines squashing and parallelization^{18),19)}. It is straightforwardly obtained by noting once more that the equations of motion only contain the Ricci tensor. All supersymmetries are broken for this solution [(in fact, whenever $F_{mnpq} \neq 0$, all supersymmetries are broken¹⁸⁾]. The most curious feature of this solution is that, owing to the constancy of η in (3.6), the parallelizing torsion is also constant¹⁸⁾. This is vaguely reminiscent of group manifolds although there is no seven-dimensional semi-simple compact Lie group.

The constancy of the parallelizing torsion implies that the pseudoscalars of gauged $N = 8$ supergravity do not get vacuum expectation values in striking contrast to the parallelized solution without squashing. This is further evidence that squashed solutions should be viewed as spontaneously broken versions of the eleven-dimensional theory itself rather than of $N = 8$ supergravity. This conclusion is reinforced by the observation that the constant spinor (3.6) cannot be represented as a linear combination of the spinors $\eta^I(y)$ of Eq. (2.13) with constant coefficients: the unbroken supersymmetry associated with η is a linear combination of supersymmetries originating from higher modes in the expansion (2.12).

4. - PHYSICS ?

The outstanding problem in the context of $N = 8$ supergravity is, of course, the question of whether this theory has anything to do with physics. On one hand, the theory should predict sufficiently many (almost) massless fermions, on the other hand the supersymmetries should be sufficiently broken in order not to be in blatant conflict with present day phenomenology. We have seen that the theory does admit spontaneous symmetry breaking, and that the different patterns of symmetry breaking are beautifully related to the unique mathematical properties of S^7 . However, we do not know yet how many massless fermions the various broken versions predict; in principle, it could be that none are left. In this

respect, only the squashed solution is in comparatively good shape. Since there is one supersymmetry left, the associated gravitino will pair up with the graviton to form a $N = 1$ supermultiplet. If the residual gauge symmetry coincides with the isometry group $SO(5) \times SO(3)$ of the squashed S^7 as was asserted in Ref. 16), there will be $13 = 10 + 3$ massless gauge bosons, and $N = 1$ supersymmetry then predicts 13 massless spin-1/2 partners. One can furthermore show that these are the only massless states of this solution¹⁹⁾: there are no massless pseudoscalars (essentially because the covariantly constant spinor of opposite parity is not available) and since $N = 1$ supersymmetry would require such pseudoscalars as partners of scalars and matter spin-1/2 fields the latter must also be absent.

Even if there are no massless spin-1/2 fermions left, this is not necessarily disastrous. It has been pointed out in Ref. 11) that, in the spontaneously broken versions, one cannot a priori rule out the possibility that some of the previously massive states become (almost) massless. If this were true, higher modes would not only affect the ultra-violet behaviour²⁰⁾ but also have important implications for low energy physics.

Another possibility is that the physical particles which we observe are bound states²¹⁾. This possibility also suggests itself in view of the fact that conventional Kaluza-Klein theories only predict vector-like fermions. In Ref. 3), it was shown that unbroken $N = 8$ supergravity possesses a "fake" local chiral $SU(8)$ invariance which could become dynamical at the quantum level. Since gauged $N = 8$ supergravity also has this local $SU(8)$ invariance⁴⁾, one may conjecture that some chiral subgroup thereof survives the symmetry breaking and that this residual group should be identified with one of the usual GUT groups.

It remains to be seen which of these contending scenarios eventually emerges as the correct one. Since supergravity confronts us with unprecedented complexities, we are barely beginning to understand its implications, and much work remains to be done.

REFERENCES

- 1) See, e.g.,
P. van Nieuwenhuizen, Phys. Rep. 68 (1981) 189.
- 2) B. de Wit and D.Z. Freedman, Nucl. Phys. B130 (1977) 105;
B. de Wit, Nucl. Phys. B158 (1979) 189.
- 3) E. Cremmer and B. Julia, Phys. Lett. 80B (1978) 48; Nucl. Phys. B159 (1979) 141.
- 4) B. de Wit and H. Nicolai, Phys. Lett. 108B (1981) 285; Nucl. Phys. B208 (1982) 323.
- 5) W. Nahm, Nucl. Phys. B135 (1978) 149.
- 6) E. Cremmer, B. Julia and J. Scherk, Phys. Lett. 76B (1978) 409.
- 7) T. Kaluza, Sitzungsber. Preuss. Akad. Wiss. Berlin KI (1921) 966;
O. Klein, Z. Phys. 37 (1926) 895.
- 8) E. Cremmer and J. Scherk, Nucl. Phys. B103 (1976) 399.
- 9) P. Freund and M. Rubin, Phys. Lett. 97B (1980) 233.
- 10) M.J. Duff and C. Pope, preprint ICTP/82/83-7, to appear in "Supergravity '82",
edited by S. Ferrara, J. Taylor and P. van Nieuwenhuizen.
- 11) B. Biran, B. de Wit, F. Englert and H. Nicolai, CERN preprint TH.3489 (1982).
- 12) F. Englert, Phys. Lett. 119B (1982) 339.
- 13) M.J. Duff, CERN preprint TH.3451 (1982).
- 14) R. D'Auria, P. Fré and P. van Nieuwenhuizen, CERN preprint TH.3453 (1982).
- 15) B. Biran, B. de Wit, F. Englert and H. Nicolai, in preparation.
- 16) M. Awada, M.J. Duff and C. Pope, preprint ICTP/82-82/4 (1982).
- 17) J. Bourguignon and A. Karcher, Ann. Sci. d'Ecole Normale Sup. 11 (1978) 71.
- 18) F. Englert, M. Rooman and P. Spindel, preprint Brussels university (1983).
- 19) H. Nicolai and P. van Nieuwenhuizen, to be published.
- 20) M.J. Duff, in "Supergravity '81", edited by S. Ferrara and J. Taylor,
Cambridge Univ. Press.
- 21) J. Ellis, M.K. Gaillard and B. Zumino, Phys. Lett. 94B (1980) 343.