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HIERARCHY OF FERMION MASSES IN SUPERSYMMETRIC COMPOSITE MODELS

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A B S T R A C T

The combined effect of chirality and supersymmetry (through Goldstone partnership) can generate a variety of light mass scales for spin $\frac{1}{2}$ composites. The general ideas are illustrated in a one-family example with technicolour-type breaking of the standard low energy group.

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1. - There has been recent interest in the possibility that supersymmetry (SUSY) may play a rôle in the (hypercolour) dynamics that would bind preons inside composite quarks and leptons. It is indeed conceivable that SUSY is efficient at producing composite fermions that are light compared to their inverse sizes, $\approx \Lambda_H > \text{a few hundred GeV}$.

In ordinary gauge theories the only known mechanism requiring light spin $\frac{1}{2}$ fermions is an unbroken chiral symmetry ¹⁾: unfortunately, several arguments ²⁾, as well as the QCD example, make it likely that such symmetries are all spontaneously broken. For SUSY theories, however, examples ³⁾ and arguments ^{4),5)} suggest the occurrence of different chirality breaking patterns, with some chiral subgroup possibly remaining unbroken. Furthermore, spontaneously broken global symmetries do also imply light fermions, as supersymmetric partners of (pseudo) Goldstone bosons.

Models exploiting the chirality protection mechanism have already been discussed in the literature ⁶⁾. The idea that quarks and leptons could be Goldstone partners has also been suggested recently ⁷⁾ and several aspects of the scheme have been discussed ^{*)}.

In this note we want to argue that a combination of both mechanisms for fermionic mass protection offers the best hopes for generating a structure (hierarchy) of mass scales between zero and Λ_H and hence for producing phenomenologically interesting composite models. We stress that, in any case, the interplay between chirality ('t Hooft) consistency conditions ¹⁾ and Goldstone partnership constraints has to be understood before attempting the construction of realistic schemes.

We shall start our discussion by making several general remarks on the above-mentioned interplay stressing in particular how certain composite fermions may enjoy double mass protection from their dual rôle as Goldstone partners (kept light by SUSY) and as "'t Hooftons" (kept light by chirality). In the second part of the paper, we illustrate the general ideas of a specific model for one generation of quarks and leptons. The model has, besides pedagogical value, some attractive phenomenological features, e.g. :

- i) it allows the gauging of the standard low energy group $G_W = SU(3)_C \times SU(2)_L \times U(1)_Y$;

*) As will become apparent later, our conclusions differ from those of Ref. 7) in various respects.

- ii) the breaking of G_W to $SU(3)_C \times U(1)_Q$ occurs via a technicolour-type composite Higgs mechanism; there are 16 light composites forming a family of quarks and leptons;
- iii) the quark doublet has a mass much smaller than M_W ; the charged lepton is lighter than the quark doublet and the neutrino has zero mass (before SUSY breaking).

In spite of the above, we do not consider this model as the best one can achieve. It is likely that more realistic models can be constructed along the general lines discussed below.

2. - The (pseudo) Goldstone structure of a SUSY gauge theory is like that of an ordinary gauge theory. If G is the global symmetry group (anomalies being taken into account) and H_0 is the invariance group of the vacuum, the Goldstone bosons lie in G/H_0 . If small masses or "weak" gauge couplings (g_c, g, g') break explicitly G , some of the Goldstones acquire a small mass (pseudo Goldstones).

The new feature implied by SUSY is that (pseudo) Goldstones must lie in chiral supermultiplets as long as SUSY is unbroken. Each multiplet has a complex scalar (i.e., a scalar S and pseudoscalar P) and a Weyl fermion. The total number n_G of real Goldstones (i.e., the number of broken generators) is given by :

$$n_G = 2 n_{S+P} + n_S + n_P = \# \text{ of broken generators} \quad (1)$$

where n_{S+P} is the number of Goldstone boson pairs that fill both spin zero components of a supermultiplet while n_S (n_P) is the number of Goldstones appearing as the scalar (pseudoscalar) component of a multiplet. In each such multiplet there is a Weyl fermion, called Goldstone partner (GP), so that the total number n_{GP} of GP's is :

$$n_{GP} = n_{S+P} + n_S + n_P \leq n_G \quad (2)$$

We see that n_{GP} is usually smaller than n_G . The inequality sign, which will hold in our example, is sometimes crucial in order that GP's form a subset of all the light fermions required by the anomaly matching constraints¹⁾.

This more precise counting is also related to the welcome property that GP's can be chiral under the global symmetry group H_0 even if the Goldstones themselves are real. The point here is that, in the case of an $S+P$ pair of

Goldstones, $S + iP$ and $S - iP$ can belong to a pair $\underline{r} + \bar{\underline{r}}$ of complex conjugate representations. The left-handed partner of $S + iP$ is also in $\underline{r}$, but the partner of $S - iP$ (in $\bar{\underline{r}}$) is just its charge conjugated right-handed partner. If $\underline{r}$ is a complex representation this just means that we are dealing with chiral fermions ^{*)}.

The next point is the fulfilment of 't Hooft's anomaly matching conditions. Examples of solutions have been recently constructed by Taylor ⁸⁾ by use of the complementarity idea ⁹⁾. What is the relation between these 't Hooftons and the previously discussed GP's ? It turns out that GP's may contribute to the anomaly matching conditions (though more fermions are usually needed) and thus some composite fermions play a dual rôle with mass protected by chirality and by SUSY.

We thus see that the interplay of 't Hooft and Goldstone + SUSY constraints is far from trivial and actually suggests a possibly exciting structure (hierarchy ?) of masses for the resulting composite fermions. This we now illustrate on a specific example.

3. - The model is based on the hypercolour gauge group $SU(6)_H$ acting on 6+6 chiral supermultiplets :

$$\Psi_\alpha^a \equiv \begin{pmatrix} \varphi_\alpha^a \\ \psi_\alpha^a \end{pmatrix}, \quad \chi_a^\alpha \equiv \begin{pmatrix} \eta_a^\alpha \\ \chi_a^\alpha \end{pmatrix} \quad \begin{array}{l} \alpha = 1, \dots, 6 \\ a = 1, \dots, 6 \end{array} \quad (3)$$

which transform as $6+\bar{6}$. Besides the ordinary gauge interaction terms, we can add a SUSY mass term of the form

$$\mathcal{L}_m = - \sum_{a=1}^6 \left[m_a \psi_\alpha^a \bar{\chi}_a^\alpha + \text{h.c.} + m_a^2 |\varphi_\alpha^a|^2 + m_a^2 |\eta_a^\alpha|^2 \right] \quad (4)$$

The maximal global symmetry is reached for $m_a = 0$ and no "weak" gauging and is given by :

$$G \xrightarrow[m \rightarrow 0]{g \rightarrow 0} G(0,0) = SU(6)_L \times SU(6)_R \times U(1)_V \times U(1)_X \quad (5)$$

*) Another more trivial reason for lack of reality is the possible presence of an R symmetry inside H_0 (see again our example) under which Goldstones and partners transform differently.

where $U(1)_V$ is the preon number and $U(1)_X$ turns out to be an R symmetry ⁴⁾ (acting on hypergluinos as well as on hyperquarks). Consistently with the gauging of the physical low energy group $G_W = SU(3)_C \times SU(2)_L \times U(1)_Y$ we take, in Eq. (4) :

$$m_a = m, \quad a = 1, 2, 3; \quad m_b = 0, \quad b = 4, 5, 6 \quad (6)$$

The condition $m_4 = 0$, which is not needed for the weak gauging, will be related to the masslessness of the neutrino ^{*}). At $m \neq 0$ the global symmetry group becomes :

$$G(m, 0) = SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_V \times U(1)_X' \quad (7)$$

where $U(1)_B$ will play the rôle of baryon number and $U(1)_X'$ is again an axial R symmetry. At this point, the identification of the low energy group as a subgroup of (7) is specified by the way the preons transform under the over-all gauge group $SU(6)_H \times SU(3)_C \times SU(2)_L \times U(1)_Y$, i.e. :

$$\begin{aligned} \Psi_{\alpha}^{a=1,2,3} &: (6, 3, 1, 1/6) \quad , \quad \chi_{a=1,2,3}^{\alpha} : (\bar{6}, \bar{3}, 1, -1/6) \\ \Psi_{\alpha}^{a=4} &: (6, 1, 1, -1/2) \quad , \quad \chi_{a=4,5}^{\alpha} : 2 \times (\bar{6}, 1, 1, 1/2) \\ \Psi_{\alpha}^{a=5,6} &: (6, 1, 2, 0) \quad , \quad \chi_{a=6}^{\alpha} : (\bar{6}, 1, 1, -1/2) \end{aligned} \quad (8)$$

After weak gauging, the global symmetry group becomes ^{**)}

$$G(m, g) = SU(2)_R' \times U(1)_B \times U(1)_V \times U(1)_A \times \widetilde{U(1)}_e \quad (9)$$

where $SU(2)_R'$ acts on $\chi_{4,5}^{\alpha}$ and a combination of $\widetilde{U(1)}_e$ and τ_{3R}' will be the lepton number.

^{*}) We could also consider the case in which all $m_a = 0$ in association with the $SU(6)_H$ group broken down to $SU(4)_H \times SU(2)_H$. In such a case $SU(4)_H$ would play the rôle of the binding interaction, $SU(2)_H$ that of technicolour and the broken generators would act as extended technicolour interactions. We shall discuss this case in more detail elsewhere.

^{**)} Anomalies due to the "weak" gauging are neglected throughout the paper. They might provide tiny masses to some otherwise massless composites as shown some time ago by Weinberg ¹⁰⁾.

At this point we have to make a dynamical assumption about the bosonic symmetry group H_0 of the vacuum. We shall choose :

$$H_0 = SU(4)_V \times SU(2)_V \times \widetilde{U(1)}_V \times U(1)_X \quad (10)$$

and assume supersymmetry not to be spontaneously broken. Note that, at $m_a = 0$, such H_0 is generated by elementary field expectation values of the form

$$\langle \varphi_a^\alpha \rangle = \langle \eta_a^\alpha \rangle = \begin{pmatrix} A & & & & & \\ & A & & & & \\ & & A & & & \\ & & & A & & \\ & & & & B & \\ & & & & & B \end{pmatrix} \quad (11)$$

which break $SU(6)_H$ without breaking SUSY. Using complementarity⁹⁾, one could try to work in such a Higgs picture and then reinterpret the results in the confined picture. However, at $m \neq 0$, VEV's (11) break supersymmetry by giving a non-vanishing vacuum energy. We shall then assume the breaking from $G(0,0)$ to H_0 to be rather induced by gauge invariant order parameters, e.g., by :

$$\langle \varphi_a^\alpha \eta_b^\alpha \rangle = \begin{pmatrix} A^2 & & & & & \\ & A^2 & & & & \\ & & A^2 & & & \\ & & & A^2 & & \\ & & & & B^2 & \\ & & & & & B^2 \end{pmatrix}; \langle \det \varphi \rangle = \langle \det \eta \rangle = A^4 B^2 \quad (12)$$

which, at least naively, does not break supersymmetry even at $m \neq 0$.

One should note, however, that the arguments of Ref. 5), while allowing VEV's as in (12) for $m_a = m \rightarrow 0$ ($a = 1, \dots, 6$), do seem to forbid a smooth $m \rightarrow 0$ limit in the case of the mass assignment (6). It is possible that (12) is incompatible with SUSY at some deeper level in which case either different order parameters have to be found to produce our H_0 or H_0 has to be changed, or, finally, with masses as in (6) SUSY gets broken^{*)}.

Due to the illustrative character of our example, we shall proceed by ignoring the above important questions and thus will develop the (mainly group theoretical) consequences of assuming (12) and unbroken SUSY. The interlocking of the different symmetry groups is shown in the Figure, which is inspired by Weinberg's work¹¹⁾.

*) Most of the above discussion can also be repeated for the gauging of $SU(2)_L$ since the associated D term breaks SUSY with the VEV's given in (11).

Let us start with the limit $m, g \rightarrow 0$. There should be 52 real Goldstone bosons in this case. They span the coset space $G(0,0)/H_0$ and thus transform under the $SU(4)_V \times SU(2)_V$ subgroup of H_0 as :

$$(4,2)_{S+P} + (\bar{4},2)_{S+P} + (15,1)_P + (1,3)_P + (1,1)_P + (1,1)_S \quad (13)$$

with the subscripts S, P standing for scalar, pseudoscalar.

At $m \neq 0$, the surviving group is

$$G(m,0) \cap H_0 = SU(3)_c \times SU(2)_V \times U(1)^2 \quad (14)$$

under which the Goldstones in (13) transform as

$$(3+1,2)_{S+P}^L + (\bar{3}+1,2)_{S+P}^R + (8+3+\bar{3}+1,1)_P + (1,3)_P + (1,1)_P + (1,1)_S$$

where the extra index R, L indicates transformation under $SU(2)_L \times SU(2)_R \supset SU(2)_V$. Out of these bosons the ones which remain massless are :

$$(1,2)_{S+P}^L + (1,2)_{S+P}^R + (1,1)_P + (1,3)_P + (1,1)_P + (1,1)_S \quad (15)$$

After gauging of $SU(3)_c \times SU(2)_L \times U(1)_Y$ ($g \neq 0$) the only unbroken global symmetry reduces to just $U(1)_B \times U(1)_L$ (baryon times lepton number) and eight bosons remain massless. These are :

- i) the $SU(2)_V$ triplet $(1,3)_P$ which is eaten up by $W^\pm, Z^0$;
- ii) three singlets under $SU(3)_c \times SU(2)_L \times U(1)_Y$ having also $B = L = 0$;
- iii) the neutral complex scalar in $(1,2)_{S+P}^R$ whose partners will be identified as the right-handed neutrino (and its antiparticle).

We now turn to the fermions. The partners of the Goldstones in (13) are Weyl spinors transforming as

$$(4,2)^L + (\bar{4},2)^R + (15,1) + (1,3) + (1,1) + (1,1) \quad (16)$$

While $(4,2)$ and $(\bar{4},2)$ find all their complex bosonic components in (13), the remaining 20 fermions need an extra scalar or pseudoscalar partner to form a supermultiplet. The structure of fermions in (16) illustrates clearly the point made in the introduction : under $SU(4) \times SU(2)_L \times SU(2)_R$ the left-handed fermions in (16) lie in

$$(4, 2, 1) + (\bar{4}, 1, 2) + \text{real representations} \quad (17)$$

hence in a complex representation. The bosons in (13), on the other hand, are real since $S+iP$ lie in $(4, 2, 1) + (\bar{4}, 1, 2)$ while $S-iP$ lie in $(\bar{4}, 2, 1) + (4, 1, 2)$ together with the (right-handed) antiparticles of (17). Note also that the number of left-hand fermions is 36 while the number of Goldstone bosons is 52, in agreement with the inequality (2).

For unbroken SUSY and $m = g = 0$ all the fermions in (16) are massless. The components $(4, 2)^L + (\bar{4}, 2)^R$ have the right quantum numbers to describe one family of quarks and leptons (plus a right-handed neutrino). At $m, g \neq 0$, only the partners of the true Goldstones are kept massless. These include the right-handed neutrino and three more neutrino-like fermions with $B = L = 0$. The ordinary (left-handed) neutrino is not the partner of a real Goldstone : it is, however, forced to be massless by the absence of any suitable partner that can couple to it via a B and L conserving mass term ^{*)} :

In order to understand further the light fermionic spectrum, the discussion of the Goldstone phenomenon has to be supplemented by the analysis of the anomaly matching conditions for the unbroken chiral symmetries.

At $m = 0$ the $(U(1)_X)^3$ anomaly gives 37 at the preonic level [if we normalize to $q_X(\text{gluino}) = -1$]. The Goldstones have zero $U(1)_X$ charge, but since this is an R symmetry, the Goldstone partners have $q_X = 1$. Hence our 36 Goldstone partners in (16) fail to saturate this anomaly by one unit. Using the Higgs picture (11) plus complementarity (at $m = 0$) confirms that the Goldstone partners are saturating the $(U(1)_X)^3$ anomaly together with one extra total singlet particle [for a similar case, see Ref. 8]. It can be checked that this set of 37 massless fermions satisfies all 't Hooft's anomaly conditions. In particular, if $B \rightarrow 0$ [limit of good $SU(2)_L \times SU(2)_R$], the $(4, 2) + (\bar{4}, 2)$ fermions have to become massless because of $SU(2)_{L,R}^2 \times U(1)$ anomalies.

The information obtained so far on the fermions is collected in the Table which summarizes all the various protections acting on the fermions against acquiring masses of order Λ_H . It follows that the upper limit on the various

^{*)} Possible mass terms of order m connecting v_L with heavy ($\sim \Lambda_H$) neutral colourless fermions give rise to a mass matrix with vanishing determinant as long as L is a good quantum number.

fermion masses lighter than Λ_H is as in the last column of the Table. Taking $m, B \ll \Lambda_H$ we find fermion masses occurring at scales $0, g^2(mB/\Lambda_H), mB/\Lambda_H, m, gB$, i.e., an interesting pattern of mass scales for the neutrino, the charged lepton, the quark doublet, exotic particles (octet quarks) and the weak scale all below Λ_H . An especially amusing feature is the way this scheme manages to keep the neutrino massless (for unbroken SUSY).

We stress again, however, that this model should only be seen as an encouraging illustration of the general points made at the beginning and that more model-building efforts should be able to yield something better. At the same time, a deeper understanding of the symmetry properties of SUSY vacua appears to be essential for further progress.

Needless to say, a realistic particle spectrum requires a SUSY breaking mechanism that lifts scalar masses without altering significantly the gross features of the fermionic spectrum.

A possibility that we are exploring is that of giving an extra mass term to the scalar hyperquarks, as suggested in Ref. 5), and also implied by supergravity induced breaking¹²⁾. As a result of such soft breaking, the fermionic family mass is still protected by the chiralities achieved as m or B go to zero, while their scalar partners are only protected as Goldstones as $g \rightarrow 0$. It looks possible, at least at first sight, to give masses $O(m_W)$ to the G_W non-neutral scalars, while leaving our 16 fermions much lighter.

We plan to come back to this important point, possibly within a realistic model, elsewhere.

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Light fermions transforming under $SU(3)_C \times SU(2)_L$	Goldstone partners			't Hooftons		Upper limit on mass
	GP	GP at $g=0$	GP at $g=m=0$	$U(1)_X$ protection $m=0$	$SU(2)_L \times SU(2)_R$ protection $B=0$	
$(u_d)_L = (3,2)$ $u_R, d_R = (3,1)$			x	x	x	$\frac{mB}{\Lambda_H}$
$(\nu_\ell^*)_L = (1,2)$ $\ell_R = (1,1)$		x		x	x	$g^2 \frac{mB}{\Lambda_H}$
$\nu_R = (1,1)$	x			x	x	0
$(1,3)$	x			x	x	Eaten by Wino and Zino $gB = m_W$
$(8+3+\bar{3},1)$			x	x		m
$(1,1), (1,1), (1,1)$ $B=L=0$	x			x		0
$(1,1)$				x		m
*) $m_{\nu_L} = 0$ since no lepton number conserving mass term is possible in SUSY limit.						

TABLE - Summary of mass protection mechanisms acting on the light composite fermions of our specific model. The last column is a rough estimate of the upper limit implied by the presence of various simultaneous protections. The actual powers of g, m, B are only indicative and could be in principle estimated following the method of Ref. 11).

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FIGURE CAPTION

The various symmetry groups relevant to the model discussed in the text. Numbers indicate dimensionality of groups or coset spaces. Fermion mass protection by chirality is indicated in the relevant subgroup of H_0 , protection by SUSY (Goldstone partnership) is mentioned in the relevant coset spaces.