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PRIMORDIAL COSMOLOGICAL INFLATION VERSUS LOCAL SUPERSYMMETRY BREAKING
IN SUSY GUTS COUPLED TO $N = 1$ SUPERGRAVITY

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A B S T R A C T

We present a model for a SUSY GUT coupled to $N = 1$ supergravity in which local supersymmetry breaks down in the gauge singlet sector. The constraints for the model to be physically acceptable are incompatible with inflation. The simultaneous breaking of local supersymmetry and gauge symmetry is proposed as a good prospect for inflation.

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The possibility that supersymmetric GUTs were able to contain an inflationary Universe [1] has recently been considered by Ellis, Nanopoulos, Olive and Tamvakis in a global supersymmetric model [2]. These authors have shown that the main requirement -- in order to achieve adequate inflation -- consists in some technically possible adjustments on the parameters of the superpotential. Furthermore, they find that the model becomes more natural (i.e. tuning needs to be less accurate) as the scale of the phase transition approaches the Planck mass M_P . Thus they are led to give up the idea of the SU(5) phase transition being the one responsible for inflation. In fact in ref. [2], phase transition occurs through a singlet field whose v.e.v. was of the order of M_P . This situation is called primordial inflation.

Any description of primordial inflation should take into account gravitational effects, as we are considering energy scales of the order of M_P . The first important step in this direction has been taken by Nanopoulos, Olive, Srednicki and Tamvakis [3]. These authors study the problem of primordial inflation in the context of an exact (unbroken) $N = 1$ supergravity. The required amounts of inflation and scale-independent density fluctuations are obtained there without imposing too stringent constraints on the parameters of the model. However, local SUSY-breaking effects may affect low-energy phenomenology when a supersymmetric grand unified theory (SUSY GUT) is coupled to $N = 1$ supergravity. If $N = 1$ supergravity is broken at a scale $m_s = (\mu M)^{\frac{1}{2}}$, $M = M_P / \sqrt{8\pi} = 2.4 \times 10^{18}$ GeV, the would-be goldstino is eaten by the gravitino, which acquires a mass of order μ via the super-Higgs effect [4]. Then global SUSY is explicitly (softly) broken by mass terms proportional to the gravitino mass [5]. On the other hand, local supersymmetry breaking is commonly believed to occur because of v.e.v.'s of order M of the scalar fields. This provides a chance for a primordial cosmological inflation to appear. The aim of the present paper is to analyse this scenario in a simple model. We will show that there is no phase transition and therefore no inflation occurs. In other words, when the Universe cools down, the temperature effects prefer to drive the Universe into the local SUSY-breaking phase with a vanishing

cosmological constant. Therefore any possibility of inflation at this level disappears.

As is well known [5], the action of a Yang-Mills theory "minimally" coupled to $N = 1$ supergravity depends only on a real function G of the scalar fields z^a ; G is given in terms of the superpotential f by

$$G(z^a, z_a) = z^a z_a + \log |f|^2. \quad (1)$$

From now on we will be working in M units. The over-all scalar potential is

$$V(z^a, z_a) = e^G \left\{ G^a G_a - 3 \right\} + \frac{1}{2} D_A^2, \quad (2)$$

where $G^a = \partial G / \partial z_a$, $G_a = \partial G / \partial z^a$, and $D_A = g G_a (T_A)^a_b z^b$. The goldstino is a linear combination of chiral fermion fields χ^a (superpartners of z^a) and gauginos λ_A :

$$\eta = e^{-G/2} G_a \chi^a + \frac{i}{2} D_A \lambda_A. \quad (3)$$

In this work we shall adopt the simplest point of view, namely: local SUSY-breaking (super-Higgs effect) takes place in a gauge singlet sector z . Then the goldstino itself is a gauge singlet and remains decoupled from the rest of the fields χ^a . The equality $D_A = 0$ is always satisfied as we have $G_a = 0$.

The equations for a ground state with a vanishing cosmological constant can be written for an arbitrary superpotential $f(z, x^a)$ as [6]

$$\begin{aligned} f_z &= (\sqrt{3} - z) f, & f_{z_a} &= (\sqrt{3} - z) f_a \\ f_{z z} &= [(\sqrt{3} - z)^2 - 1] f, & f_a &= -f x_a, \end{aligned} \quad (4)$$

where eqs. (4) are for real v.e.v.'s.

If we consider a superpotential $f(z, x^a)$ of the simple form

$$f(z, x^a) = v(z) + \omega(x^a) \quad (5)$$

equations (4) can be written as [7]

$$z = \sqrt{3} \quad , \quad v_z = 0 \quad , \quad f = -v_{zz} \quad . \quad (6)$$

Since z must take a v.e.v. of the order of the Planck mass and $\langle x^a \rangle$ is of the order of the GUT scale, we will study first the solution of eqs. (6) in the symmetric GUT phase, i.e. when $x^a = 0$. In that case, eqs. (6) read

$$z = \sqrt{3} \quad , \quad v_z = 0 \quad , \quad v = -v_{zz} \quad , \quad (7)$$

where we used the condition $\langle \omega \rangle = 0$. As we will show later on, this condition is a good approximation as far as $\langle \omega \rangle \ll \langle v \rangle$. Let us take an SU(5) model in order to see the implications of this restriction, i.e. $\omega = [(\lambda/3) \text{Tr } \Sigma^3] + [(m/2) \text{Tr } \Sigma^2]$, where $\langle \Sigma \rangle \simeq \theta(m_x)$ and $\lambda \simeq \theta(m/m_x)$. Now the above inequality will read $m m_x^2 \ll \mu^2 M^2$ (i.e. $m \ll 10^8$ GeV for $m_x \simeq 10^{15}$ GeV). The particular case $m \simeq \theta(m_W)$ has been recently proposed by Ellis, Nanopoulos, Olive and Tamvakis [2] as a GUT model able to describe the postinflationary epoch.

We choose an easy-running model and take a cubic superpotential $v(z)$:

$$v(z) = a + a_1 z + \frac{1}{2} a_2 z^2 + \frac{1}{3} a_3 z^3 \quad . \quad (8)$$

We would like to emphasize here that models with some of the a_i 's equal to zero lead to trouble. The scalar potential $V(z, z^*)$ for a real field z is

$$V(z) = e^{z^2} \left\{ a_1^2 - 3a_2^2 + (3a_2^2 - a_1^2 + 2a_1 a_3) z^2 + 2(2a_3 + a_1) a_1 z^3 + (2/3 a_1 a_3 + a_3^2 + a_1^2 + 3a_2^2) z^4 + 2a(a_1 + a_3) z^5 + (a^2 + a_3^2/3 + 2/3 a_1 a_3) z^6 + 2/3 a a_3 z^7 + a_3^2/9 z^8 \right\} \quad . \quad (9)$$

Using eqs. (7), the parameters are given by

$$a_1 = \sqrt{3} \left(\frac{\mu}{2} - a \right) \quad , \quad a_2 = 2a \quad , \quad a_3 = -\frac{1}{\sqrt{3}} \left(\frac{\mu}{2} + a \right) \quad , \quad (10)$$

where $\mu = \langle v \rangle$ is a parameter which gives essentially the gravitino mass. Notice that $a_2 = 2a$ [eq. (10)] eliminates the linear term in eq. (9) and guarantees that the origin $z = 0$ is a stationary point. On the other hand, the ground state at

present has a vanishing cosmological constant and therefore it corresponds to $z = \sqrt{3}$. The idea is that inflation happens during the roll-over of z from the false vacuum $z = 0$ to the present ground state. Now we analyse the conditions for this scenario to occur. Once we know that the points $z = 0$ and $z = \sqrt{3}$ are stationary points of the potential, we will impose on them the necessary conditions for them to become minima.

The eigenvalues of the mass matrix

$$m^2(z, z^*) = \begin{pmatrix} \frac{\partial^2 V}{\partial z \partial z^*} & \frac{\partial^2 V}{\partial z^2} \\ \frac{\partial^2 V}{\partial z^{*2}} & \frac{\partial^2 V}{\partial z^* \partial z} \end{pmatrix} \quad (11)$$

should be positive at both points. For $z = 0$ it implies

$$a^2 - \frac{\mu^2}{4} > 0 \quad (12)$$

whilst for $z = \sqrt{3}$ we get

$$-4 \left(\frac{\mu}{2} \right)^2 < \frac{\mu}{2} \left(\frac{\mu}{2} + a \right) < 0. \quad (13)$$

Conditions (12) and (13) are satisfied provided that

$$a = -\rho \frac{\mu}{2}, \quad 1 < \rho < 5 \quad (14)$$

and the value of the potential at the minimum $z = 0$ is

$$V_0 = \frac{3}{4} \mu^2 (1 + 2\rho). \quad (15)$$

In order to produce inflation, we need z to be confined near the origin. Thus we look for a local maximum of the potential in $z \ll 1$. To this end, we expand eq. (9) up to fourth-order terms in z and look for a stationary point $0 < z \ll 1$, which must be a local maximum at least along the real axis. We find

$$z_1 = \frac{2(\rho^2 - 1)}{\sqrt{3} \rho (1 + 5\rho)} \quad (16)$$

so that $\rho-1$ measures the proximity of z_1 to 0. In the limit $\rho = 1$ both points coalesce in a saddle point.

The probability for tunnelling from the local minimum at $z = 0$ to the local maximum at $z = z_1$ is given by [8]

$$P \simeq 4 \left(\frac{\mu}{2} \right)^4 (\rho^2 - 1)^2 e^{-B}, \quad (17)$$

where B is the Euclidean gravitational action

$$B \simeq 24 \pi^2 \frac{V_1 - V_0}{V_0^2} \quad (18)$$

for $V_1 - V_0 \ll V_0$. We find

$$B \simeq \frac{64 \pi^2}{27} \left(\frac{\mu}{2} \right)^{-2} \frac{(\rho-1)^3}{\rho^2 (1+5\rho^2)^2 (1+2\rho)^2}. \quad (19)$$

On the other hand, the time τ spent by the field z to move from the local maximum at $z = z_1$ to the global minimum at $z = \sqrt{3}$ can be characterized by

$$\tau = \frac{3 H}{\mu^2 (\rho^2 - 1)}, \quad (20)$$

where H is the Hubble constant $H^2 = V_0/3$. Using the above results we find

$$H \tau = \frac{3}{4} \frac{1+2\rho}{\rho^2 - 1}. \quad (21)$$

We now require $B \gg 1$ [8,9], so that phase transition is unlikely to happen and the Universe continues in the de Sitter state ($z = 0$) until it makes a transition to $z = z_1$. On the other hand, we require $H\tau > 65$ in order to obtain the desired amount of inflation [1]. The former condition can be satisfied very easily because μ^{-2} (in M units) is a very big number $\mu^{-2} \sim 10^{32}$, but the latter condition is satisfied only if ρ is very close to one. In particular, writing $\rho = 1 + \frac{1}{2}\epsilon$ we find

$$\mu^{2/3} \ll \epsilon < 0.04 \quad (22)$$

and this gives for the gravitino mass the comfortable bound $\mu \ll 4 \times 10^{-3}$.

Concerning density fluctuations [10] $\delta d/d$, we have for $\rho \gtrsim 1$

$$\frac{\delta d}{d} = (2\pi)^{-3/2} \mu \log^2 \left(\sqrt{3} \frac{\mu}{2} \kappa^{-1} \right) \quad (23)$$

then the Harrison-Zel'dovich bound [11] $\delta d/d \lesssim 10^{-4}$ gives $\mu < 6 \times 10^{-7}$, which is in agreement with the bound (22).

The shape of the potential is the following: It has a local minimum at $z = 0$, then a local maximum at $z = z_1$, and again a local minimum at $z = \sqrt{3}$. After the latter there is a local maximum and a global minimum with very negative values of the potential. The barrier between the minimum at $z = \sqrt{3}$ and the (negative) global minimum is a function of ρ . For $(\rho-1) \rightarrow 0$ the barrier is very small, and the system would easily decay into the big anti de Sitter vacuum. This is a physically undesirable situation. When ρ increases, the barrier grows up very rapidly and it becomes insurmountable even for modest values of ρ ($\rho \gtrsim 2$). So the system will be stable at $z = \sqrt{3}$. In figs. 1 and 2 we show the shape of the potential for the maximum value of ρ allowed by inflation ($\rho = 1.03$) and for some other values of ρ compatible with a minimum at $z = \sqrt{3}$.

We would now like to emphasize a point about the situation we have been considering, namely a ground state with a vanishing cosmological constant and a gravitino mass characterized by μ . The point is that this situation must be true at energies lower than the GUT scale. This fact, which we neglected in eqs. (7) to (10), may cause severe modifications in our scheme. In other words, it is not $\langle v \rangle$ which is equal to μ , as it was considered in eq. (10), but

$$\{ \langle z \rangle, \langle x^a \rangle \} = \langle v \rangle + \langle \omega \rangle = \mu. \quad (24)$$

This amounts to replacing the z -independent term (a) in $v(z)$ by $a - \langle \omega \rangle$ but keeping eq. (10) unchanged. We remember that after eqs. (7) we made the approximation $\langle \omega \rangle = 0$ and discussed its feasibility. Now the minimum moves from $z = \sqrt{3}$ owing to the substitution $a \rightarrow a - \langle \omega \rangle$ and the cosmological constant becomes negative. We take $\langle \omega \rangle$ [$\langle \omega \rangle = (\mu/2) x$] as a new parameter and consider the changes in previous

results. We write $a = -(\mu/2) (\rho + x)$, with $x = 0$ ($m_{\mathbf{x}}^2/\mu M^2$), and we then have $m \simeq x \times 10^8$ GeV for $\mu = 10^2$ GeV and $m_{\mathbf{x}} = 10^{15}$ GeV. In fig. 3 the variation of the potential (at $T = 0$) with x is shown. There we see that the cosmological constant varies with the value of x . The changes of x^a , from the value 0 to the v.e.v. calculated from eq. (4), mimic the variation of the z sector of the effective potential during the GUT phase transition after the Planck epoch. This process is shown by fig. 3. When x grows and becomes larger than one, the modifications become more and more dramatic. Then $m_{\mathbf{x}} > 10^8$ GeV and surpasses the temperature of the SU(5) phase transition. The cosmological implications of this situation are well known [2,12].

The inflationary scenario can apply only when the thermal energy gives a negligible contribution to the energy density of the early Universe [1]. However, finite temperature effects have been neglected, up to now, in our scheme. The inclusion of these effects changes the effective potential in [13]

$$V(z, z^*, T) = -\frac{\pi^2}{90} N(T) T^4 + \frac{1}{8} \text{tr } m^2(z, z^*) T^2 + V(z, z^*), \quad (25)$$

where $N(T)$ is the effective number of light degrees of freedom at temperature T . For the gauge (non-singlet) sector this means that the Universe must be in the most symmetric phase at very high temperatures, i.e. when the T^4 term dominates [14]. The Universe cools down in this phase, and it continues to do so until the temperature reaches the scale of confinement for the GUT model under consideration. There, we begin to have the possibility of a transition to a broken phase with more degrees of freedom than the unbroken, but confining, phase. For the singlet sector the situation is completely different. In this sector the particles are light for the range of values of $\langle z \rangle$ we are considering, i.e. $m^2(z, z^*) \simeq \theta(\mu^2)$. Thus the T^4 is constant and does not prejudice which value of z corresponds to the high-temperature ground state. Therefore, the T^2 term governs both position and evolution of the ground state during the "cool down" of the Universe. For real v.e.v.'s we can write:

$$\delta V(T) = \frac{\partial^2 V}{\partial z^2} \frac{T^2}{8} = \frac{T^2}{8} e^{z^2} \left\{ 4a_1 a_3 + 12a(2a_3 + a_1)z + (54a^2 + 6a_1^2 + 12a_3^2 + 38a_1 a_3)z^2 + 4a(24a_3 + 17a_1)z^3 + (96a^2 + 14a_1^2 + 28a_3^2 + 40a_1 a_3)z^4 + a(52a_1 + 88a_3)z^5 + (38a^2 + 170/9 a_3^2 + 20a_1 a_3 + 4a_1^2)z^6 + a(28a_3 + 8a_1)z^7 + \left(\frac{46}{9}a_3^2 + 4a^2 + \frac{8}{3}a_1 a_3\right)z^8 + \frac{8}{3}a a_3 z^9 + \frac{4}{9}a_3^2 z^{10} \right\} \quad (26)$$

where the parameters are those given in eqs. (10) and (14). One easily realizes from eq. (26) that $V(z, T)$ does not have a minimum at $z = 0$. This means that, unlike the non-singlet sector, finite temperature effects do not necessarily drive the field z into the minimum with vanishing v.e.v. when the Universe cools down. Taking the value $\rho = 4$, we show in fig. 4 how the finite temperature effects drive the field into the value $z = \sqrt{3}$ at low temperatures. This happens for the other values of ρ as well. Also, fig. 4 shows that the T^2 term becomes negligible for temperatures below the Planck mass. This is due to the different orders of magnitude which are involved, i.e. $V \approx \theta(\mu^2 M^2)$ whilst $m^2 \approx \theta(\mu^2)$. Hence the T^2 effects can be neglected as soon as we have $T < M$. However, the vacuum energy density is predominantly thermal, i.e. the T^4 term dominates for temperatures $T > (\mu^2 M^2)^{1/4} \approx 10^{10}$ GeV, which are well below the Planck mass! Therefore no trace of primordial inflation should be left even in the case where T^2 effects could not directly drive the vacuum into the $z = \sqrt{3}$ ground state. Figure 5 completes this analysis, showing the potential, including finite temperature corrections, for a value of $x = 2$.

Summarizing, we need a twofold achievement in order to have any chance of inflation: i) $V(z, z^*)$ must dominate the vacuum energy density and ii) the ground state of the high-temperature phase should be different from the low-temperature value $\sqrt{3}$. This could be handled by choosing values of ρ and x far larger than the ones considered before. However, we proved that the point $z = \sqrt{3}$ is not a minimum for $\rho \geq 5$. On the other hand, the GUT phase transition becomes incompatible with post-inflationary standard cosmology for $x > 1$ [2,11,14], as was already pointed out after eq. (24). In short, there is no room for primordial cosmological

inflation in a SUSY GUT coupled to $N = 1$ supergravity when local SUSY-breaking occurs in a gauge singlet sector, through a cubic superpotential.

A natural question which arises as a consequence of our analysis is the following: Could inflation exist in the case where the breaking of local supersymmetry occurs in a non-singlet sector? This situation is currently under investigation by the present authors, but some points could be put forward: i) the T^4 term will here play a different role from the one in the singlet case; ii) inflation should start at a temperature higher than the confinement scale of the GUT model; iii) in order to have an adequate value of the local SUSY-breaking, the scale m_s should be $m_s \simeq \langle \omega \rangle \simeq \mu M^2$ and $m \lesssim 10^8$ GeV. On the other hand, Ellis, Nanopoulos, Olive and Tamvakis [15] have found that SUSY GUTs with broken global supersymmetry would be good candidates for producing inflation in the GUT phase transition. Since $N = 1$ supergravity induces an explicit (soft) breaking of global SUSY with a mass splitting between mesons and fermions of the order of the gravitino mass, one could wonder if these results apply to the case of local supersymmetry as well.

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Figure captions

- Fig. 1 : Effective potential V at zero temperature for the maximum value of ρ compatible with inflation. The minimum at $z = 0$, the maximum at z_1 , and the minimum at $z = \sqrt{3}$ are not apparent.
- Fig. 2 : Effective potential V at zero temperature for different values of ρ in the range $[0,5]$.
- Fig. 3 : Influence of the non-singlet sector on the effective potential V at zero temperature and $\rho = 4$. The parameter x is explained in the text.
- Fig. 4 : Dependence of the effective potential $V + \delta V$ on the temperature T for $\rho = 4$ and $x = 0$.
- Fig. 5 : Dependence of the effective potential $V + \delta V$ on the temperature T for $\rho = 4$ and $x = 2$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5