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One of the problems in constructing viable supersymmetric models, both global and local, is that they require a supersymmetry breaking scale M_{SUSY} much less than the Planck mass. The constraint that there should be a Higgs scalar with mass $\lesssim 1$ TeV requires $M_{\text{SUSY}} \sim 10^{11}-10^{13}$ GeV [1,2]. Although models built with M_{SUSY} input satisfying this bound can then generate a reasonable mass spectrum stable against radiative corrections [4,5] it seems one is left with the question why M_{SUSY} should be much less than M_{Planck} . In this letter we show a mechanism which automatically explains this. It relies on the fact that M_{SUSY} is related (in globally supersymmetric theories) to the value of the scalar potential at the minimum. The minimisation automatically favours the smallest possible value for M_{SUSY} and in a class of models this value is much less than the fundamental scale M in the scalar potential.

Supersymmetric grand unified theories (SUSY-GUTs) rely on F type supersymmetry breaking [4,5,6] and there is a great deal of freedom in choosing the form of the scalar potential generating this breaking. Relatively straightforward potentials are possible by simply omitting various terms in the superpotential and relying on the nonrenormalisation of F terms [7] to ensure that such terms are not generated in higher perturbative order. However many people dislike such "supernatural" potentials and argue that one should use a "natural" superpotential ie the most general one consistent with the symmetries of the theory (including R symmetries). It is possible to build global SUSY-GUTs which are natural in this sense but this leads to a great deal of complexity in the scalar sector [8]. In this note we re-examine this question in the context of local SUSY-GUTs. We find

A GEOMETRIC HIERARCHY FOR THE SUPERSYMMETRY BREAKING SCALE

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Abstract. We consider F type supersymmetry breaking through O'Raifeartaigh-Fayet potentials. It is shown how a class of models gives rise to a supersymmetry breaking scale reduced relative to the fundamental scale M of the potential by powers of (M/M_{Planck}) . The role of R invariance in such potentials is discussed.

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it very easy to construct natural F type SUSY breaking models and, moreover, in these models M_{SUSY} is often expected to be much less than M_{Planck} .

The standard examples of O'Raifeartaigh [6] breaking has the superpotential P in the form

$$P = \lambda_1(A^2 - M^2)B + \lambda_2 A C M \quad (1)$$

where M is an input mass scale and A, B and C are singlet chiral supermultiplets. The only symmetry constraining P is the R symmetry under which $P(\phi_i)$ transforms as

$$P(\phi_i) \rightarrow P(\phi_i) e^{i\theta_i} = e^{2i\alpha} P(\phi_i) \quad (2)$$

The phases θ_i for the chiral superfields A, B, and C are

$$\theta_A = 0; \theta_B = \theta_C = 2\alpha \quad (3)$$

Eq(1) is "supernatural" in the sense that it omits terms, such as $A^2 C$, allowed by the symmetries of the potential, but is easy to include these terms while still breaking supersymmetry. However R invariance ensures 'ino masses are zero [9], probably in conflict with experiment. To avoid this one must break the R invariance, either softly or spontaneously. The former leads to phenomenologically acceptable theories although it is somewhat unmotivated [5]. Here we explore the latter possibility.

To spontaneously break R invariance the form of eq(1) must be modified because, as may be seen from eq(3), the field A which develops as nonzero vacuum expectation value (vev) has $\theta_A = 0$ and the model remains R invariant. An alternative form which does violate R invariance spontaneously is given in eq(4)

$$P = \lambda_1(XY - M^2)B + \lambda_2 X^3 \quad (4)$$

Under R transformation the singlet fields X, Y and B transform as in eq(2) with

$$\begin{aligned} \theta_X &= 2\alpha/3 \\ \theta_Y &= -2\alpha/3 \\ \theta_B &= 2\alpha \end{aligned} \quad (5)$$

Clearly eq(4) is the most general R invariant form.

The potential following from eq(4) is

$$V = |\lambda_1(XY - M^2)|^2 + |\lambda_1 X B|^2 + |\lambda_1 Y B + 3\lambda_2 X^2|^2 \quad (6)$$

Eq(6) favours a nonzero vacuum expectation value for Y and from eq(5) this implies R invariance will be broken. Eq(6) has a supersymmetric minimum given by

$$\begin{aligned} \langle XY \rangle &= M^2 \\ \langle X \rangle &= 0 \\ \langle B \rangle &= 0 \end{aligned} \quad (7)$$

This minimum requires $\langle Y \rangle = \infty$, perhaps not a physically reasonable value. To see what could constrain $\langle Y \rangle$ we must consider the corrections to eq(6). We know that perturbative corrections do not change V if supersymmetry is unbroken at tree level. What about gravitational corrections? If these limit $\langle Y \rangle \neq 0(M_{\text{Planck}})$ then eq(6) will not be zero and supersymmetry will be violated. With $\langle Y \rangle = M_P$ the minimum has $\langle X \rangle \approx M^2/M_{\text{Planck}}$ and $\langle B \rangle \approx -3\lambda_2 X^2/\lambda M_{\text{Planck}}$. Then the value of V at the minimum is only $(9\lambda_2^2 M^{12}/M_{\text{Planck}}^8)$ corresponding to $M_{\text{SUSY}} \approx \sqrt{\lambda} M^3/M_{\text{Planck}}^2$.

If the $N = 1$ model is an effective field theory resulting from a constituent theory (extended supergravity?) at a scale M_{Planck} there would be expected to be composite mass scales (eg pseudogoldstone boson masses), smaller than M_{Planck} but related to M_{Planck} through the couplings in the theory. If in eq(4) M is such a scale we see that the supersymmetry breaking

scale will be much smaller than either M_{Planck} or M .

To see if gravitational effects will indeed cutoff the value of $\langle Y \rangle$ consider an explicit model coupling of the $N=1$ chiral supermultiplets to gravity. The effective potential of eq(4) is modified to read [10,11]

$$V = \frac{1}{e} \{ |X|^2 + |Y|^2 + |B|^2 \} X^2 + [|\lambda_1(XY - M^2) + B^* P \kappa|^2 + |\lambda_1 X B + Y^* P \kappa|^2 + |\lambda_1 Y B + 3\lambda_2 X^2 + X^* P \kappa|^2 - 3|B^2 \kappa^2|] \quad (8)$$

where $\kappa = \sqrt{8\pi}/M_{\text{Planck}}$ and P is as defined in eq(4) and the standard kinetic term for the chiral fields are preserved. If the vacuum expectation values (vev) of X, Y and B are small eq(8) reduces approximately to eq(6) and drives the vevs towards their supersymmetric values given by eq(7). However as $\langle Y \rangle$ approaches M_{Planck} the additional terms present in eq(8) may no longer be neglected. For large $\langle Y \rangle$, V has the form

$$V \approx e^{\kappa^2 \langle Y \rangle^2} \left\{ \left| -\frac{3}{\langle Y \rangle^2} + \frac{\kappa^2}{\langle Y \rangle^2} \right|^2 - 3\kappa^2 \left| \frac{1}{\langle Y \rangle^3} \right|^2 + |\lambda_2 M^6|^2 \right\} \quad (9)$$

This has a minimum with $\langle Y \rangle^2 \approx \frac{6}{\kappa^2}$, and each term is of order $\lambda^2 \frac{M_{\text{Planck}}^{12}}{M_{\text{Planck}}}$ as anticipated above.

At tree level V is negative at the minimum but radiative corrections occur at the same order and for a suitable fine tuning of parameters may be made to vanish*. One can also arrange this cancellation at tree level by including in P new terms of dimension greater than three and involving further couplings and fields. Since the effective theory described by eq(8) is not

* For example the coupling $\lambda_1 B X Y \kappa^2$ following from eq(8) breaks global supersymmetry and gives rise at one loop level to terms of the form $|Y P^* \kappa|^2$. For large Y this behaves as $\lambda_2 M^6 \left| \frac{\kappa^2}{Y^2} \right|^2$ comparable with the terms of eq(8). The coefficient of this term depends on $|\lambda_1|^2$ and must be regulated.

renormalisable there is no reason why such terms should not be included.

Let us summarise what this example has shown. Constructing a simple $N=1$ globally supersymmetric potential to spontaneously violate R invariance led us to a form which was supersymmetric in the limit one of the fields Y acquired infinite vacuum expectation value. We argued that this is physically unreasonable and that gravitational effects would limit the vev of Y to $O(M_{\text{Planck}})$ and showed that an explicit $N=1$ locally supersymmetric model indeed did this. As a result supersymmetry is broken but the resultant vev of Y is $O(\lambda^2 \frac{M_{\text{Planck}}^{12}}{M_{\text{Planck}}})$ giving rise to a supersymmetry breaking scale.

$$M_{\text{SUSY}} = O(\lambda^2 \frac{M_{\text{Planck}}^3}{M_{\text{Planck}}^2}) \quad (10)$$

For a (composite) scale M less than M_{Planck} , M_{SUSY} will be much less than M_{Planck} ; for example $M = 10^{-2} M_{\text{Planck}}$ gives $M_{\text{SUSY}} \approx 10^{-11}$ GeV, consistent with the bounds on M_{SUSY} necessary to have a light Higgs doublet.

The underlying reason a low value for M_{SUSY} results is quite general. If a potential with fundamental scale M has a supersymmetric minimum with an infinite vev we expect gravitational effects (not necessarily local $N=1$ supersymmetry) to limit it to $O(M_{\text{Planck}})$ and the resultant violation of supersymmetry to be of $O(\frac{M}{M_{\text{Planck}}})^n$. Our example is just the simplest possibility realising this idea. Although motivated by the desire to break R invariance spontaneously, this is not an essential ingredient and the effect persists in models in which R invariance is exact. If R invariance is spontaneously broken there will be a Goldstone boson. It is given by the combination

$$[\sum_i \frac{1}{\phi_i} \sum_j R_i (\langle \phi_i \rangle^* \phi_j - \langle \phi_j \rangle^* \phi_i)]^\dagger \quad (11)$$

where R_i is the R charge (cf eq(5)) and the sum runs over all scalars. Thus the goldstone boson is principally along the Y direction.

The coupling of this goldstone boson is $\frac{1}{M} \partial_\mu J_\mu^R$ where J_μ^R is the R current. If M is close to M_{Planck} it will almost decouple, and will not affect low energy phenomenology, although it might have an effect in cosmology. It is possible gravitational effects explicitly violate the R invariance giving a mass to this state.

Finally can this supersymmetry breaking mechanism be used to build a realistic model? This is straightforward and proceeds by coupling the supersymmetry breaking to the gauge non-singlet sector either through a Yukawa coupling [5] of the form

$$\lambda_3 \phi_1 \phi_2 X \quad (12)$$

where ϕ_1 and ϕ_2 are, for example, a 5 and $\bar{5}$ of SU(5) with R phases β and $-\beta + \frac{4\alpha}{3}$ respectively, or through gravitational corrections in a locally symmetric version of the model [11,12]. These give reasonable spectra for the gauge non-singlet fields, although in the latter case M_{SUSY} must be $0(10^{11} \text{ GeV})$ to give adequate splitting between supersymmetric states (splitting $\sim M_{\text{SUSY}}^2/M_{\text{Planck}}$). In the former case any value of $M_{\text{SUSY}} \gtrsim 10^3 \text{ GeV}$ is viable depending on the magnitude of the Yukawa couplings.

However the suppression of eq(9) is probably insufficient to get such a low value for M_{SUSY} . It is easy to see that this suppression factor may be arbitrarily increased by altering the form of the superpotential. For example replace eq(4) by

$$P = \lambda_1 (XY - M^2)B + \lambda_2 X^{3+n}/M_{\text{Planck}}^n \quad (13)$$

The term $\sim X^{3+n}$ is a non-renormalisable term, but as we are including gravitational corrections in eq(5) we must allow for such term in P. P has an R invariance with $\theta_X = \frac{2\alpha}{(3+n)}$, $\theta_Y = -\theta_X$, $\theta_B = 2\alpha$. Eq(12) together with the modified form of eq(8) show supersymmetry (and R symmetry) is broken with

$$M_{\text{SUSY}} = \sqrt{\lambda(n+3)} \left(\frac{M_{\text{Planck}}}{M} \right)^{3+n} M_{\text{Planck}} \quad (14)$$

This can give an arbitrarily large suppression of M_{SUSY} relative to M_X .

In conclusion we have demonstrated a mechanism to generate a geometric hierarchy relating M_{SUSY} to M and M_{Planck} . This naturally allows for a small supersymmetry breaking scale consistent with the bounds on M_{SUSY} . Once this is achieved one may construct supersymmetric theories with large scale mass hierarchies stable against radiative corrections.

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