A 83-88

ACADEMY OF SCIENCES OF THE USSR
P. N. LEBEDEV PHYSICAL INSTITUTE

I.E. Tamm Department of Theoretical Physics

High Energy and Cosmic Rays Physics

CERN LIBRARIES, GENEVA

CM-P00066983

Preprint E 88

ON THE NEW DEFINITION OF OFF SHELL
EFFECTIVE ACTION

B.S. Fradkin and A.A. Tseytlin

Moscow 1983

(March)

On the new definition of off shell effective action

E.E.Fradkin and A.A.Tseytlin

Department of Theoretical Physics,

P.N.Lebedev Physical Institute, USSR Academy of Sciences,

Leninsky Pr. 53, Moscow 117924, USSR

Abstract

We analyze the recently proposed definition of off shell gauge invariant, gauge independent effective action $\tilde{\Gamma}$, utilizing an invariant metric on the field space. It is shown how to establish correspondence between $\tilde{\Gamma}$ and the standard effective action, calculated in some particular (Landau-type) gauge. Several examples are explicitly discussed, including Yang-Mills theory, effective potential in scalar QED and one-loop quantum gravity. Generalization to the case of super-invariant theories (e.g. super Yang-Mills and supergravity) is also presented.

1. Introduction and general framework

Recently Vilkovisky proposed a novel definition of effective action in gauge theories [1]. It is reparametrization invariant, gauge invariant and gauge independent even off the effective "mass shell" and thus may find applications in dynamical symmetry breaking, external field (or source) problems and any especially in theories with non-trivial configuration space metric, including quantum (super)gravity. Here we discuss the relation of the new effective action to the standard one and illustrate general statements on a number of well-known field theories. We also generalize the definition on the case of theories with fermions including supersymmetrical ones.

Let us first review the Vilkovisky's definition in slightly modified manner, clarifying some important points. Let $M = \{\varphi\}$ be a configuration (boson field) space and $S(\varphi)$ be a scalar on M and an invariant of some global symmetry group G , i.e. $S'(\varphi') = S(\varphi)$, $\varphi' = f(\varphi)$, and $S(\tilde{\varphi}) = S(\varphi)$, $\tilde{\varphi} = g(\varphi)$. Then the standard definition of the euclidean effective action (generator of irreducible vertices)¹⁾

$$\Gamma(\phi) = W(J(\phi)) - J(\phi)\phi, \quad \frac{\delta W}{\delta J} = \phi, \quad e^{-W} = \int d\varphi \mu(\varphi) e^{-S(\varphi) + J\varphi}$$

$$e^{-\Gamma(\phi)} = \int d\varphi \mu(\varphi) e^{-S(\varphi) + (\phi - \varphi) \frac{\delta \Gamma(\phi)}{\delta \phi}} = \int d\eta e^{-S(\phi + \eta) + \eta \frac{\delta \Gamma(\phi)}{\delta \phi}} \quad (1)$$

does not in general provide an off shell ($\Gamma, \varphi \neq 0$) reparametrization invariant and G -invariant result. Namely, $\Gamma'(\phi') \neq \Gamma(\phi)$ and $\Gamma(\tilde{\phi}) \neq \Gamma(\phi)$, where $\Gamma(\phi)$ and $\Gamma'(f(\phi))$ are effective actions, calculated according to (1) starting with $S(\varphi)$, and

1) Here ϕ and $\eta = \varphi - \phi$ are the "background" and "quantum" fields and we use the "condensed notations" of ref. [2] Eq.(1) is equivalent to other definitions of Γ within the background field method [3-5]. For simplicity we shall include contributions of local ($\log \mu \sim S(0)$) measures in the symbol $d\eta$.

$S'(f(\phi))$ respectively. Suppose now that M is endowed with some symmetrical G-invariant connection Γ^i_{jk} . If we take $\phi^i = \dot{\phi}^i(\lambda)$ to be the tangent vector at ϕ to the geodesic, connecting ϕ and φ , $\ddot{\phi}^i + \Gamma^i_{jk}(\varphi(\lambda))\dot{\phi}^j\dot{\phi}^k = 0$, $\varphi(\lambda) = \phi$, $\varphi(\lambda) = \varphi$ (λ is an affine parameter), then it will be a vector under transformations of ϕ and a scalar under transformations of φ and the following relations will be true

$$\begin{aligned} \phi^k \nabla_k \phi^i &= -\phi^i, \quad \nabla_k = \frac{\partial}{\partial \phi^k} + \Gamma^i_{jk}(\phi), \\ \dot{\phi}^i &= \varphi^i - \phi^i = \phi^i - \frac{1}{2!} \Gamma^i_{j_1 j_2}(\phi) \phi^{j_1} \phi^{j_2} - \dots - \frac{1}{n!} \Gamma^i_{j_1 \dots j_n}(\phi) \phi^{j_1} \dots \phi^{j_n}, \\ \phi^{j_1} \dots \phi^{j_n} + \dots, \quad \Gamma^i_{j_1 \dots j_n} &= \Gamma^i_{(j_1 \dots j_{n-1}, j_n)} - (n-1) \Gamma^i_{m(j_1 \dots j_{n-1})} \Gamma^m_{j_n}, \\ \phi^i(\varphi, \phi) &= \dot{\phi}^i + \frac{1}{2} \Gamma^i_{j_1 j_2} \dot{\phi}^{j_1} \dot{\phi}^{j_2} + \frac{1}{6} (\Gamma^i_{j_1 j_2 j_3} + \Gamma^i_{j_1 j_2} \Gamma^m_{j_3}) \dot{\phi}^{j_1} \dot{\phi}^{j_2} \dot{\phi}^{j_3} + \dots \end{aligned} \quad (2)$$

$$\phi^i(\varphi, \phi) = \dot{\phi}^i + \frac{1}{2} \Gamma^i_{j_1 j_2} \dot{\phi}^{j_1} \dot{\phi}^{j_2} + \frac{1}{6} (\Gamma^i_{j_1 j_2 j_3} + \Gamma^i_{j_1 j_2} \Gamma^m_{j_3}) \dot{\phi}^{j_1} \dot{\phi}^{j_2} \dot{\phi}^{j_3} + \dots \quad (3)$$

Thus the new effective action

$$e^{-\tilde{S}(\phi)} = \int d\varphi \exp \left[-\tilde{S}(\varphi + \phi) + \phi^i(\varphi + \phi) \tilde{\Gamma}^i_{j_1 \dots j_n}(\varphi) \right], \quad (4)$$

generating the same S-matrix as Γ in (1), will be off shell reparametrization and G-invariant. The next (purely technical, possible already in (1)) step is to change the quantum variable $\eta \rightarrow \phi$ obtaining a manifestly "covariant" perturbation theory by noting that $\tilde{S}(\eta + \phi) = \tilde{S}(\phi) + \sum_{n=1}^{\infty} \frac{1}{n!} (\tilde{\Gamma}^i_{j_1 \dots j_n}(\phi) \phi^{j_1} \dots \phi^{j_n}) \phi^i$. The construction of $\tilde{\Gamma}$ in (4) depends (off shell) on the choice of the connection. For example, in the case of a riemannian manifold G-model, $\tilde{S} = \int d^4x \frac{1}{2} g_{ij}(\varphi) \partial_\mu \varphi^i \partial_\mu \varphi^j$, it is natural to take Γ^i_{jk} to be the Christoffel connection for g_{ij} . It is in this particular case where an off shell invariant effective action analogous to (4) (with the accent on the use of ϕ -perturbation theory) was previously discussed in

literature [7-9] . ^{2/} The expression (4) in its general form first appeared in [1].

Now let G contain local gauge transformations (g.t.), $\delta\varphi^i = R^i_\alpha(\varphi)E^\alpha = R^i(\varphi)$. Then if $\Gamma_{,i} \neq 0$, $\hat{S} = S - \int \Gamma_{,i}$ in (1) is not invariant under quantum g.t. ($\delta\varphi = R(\varphi)$, $\delta\phi = 0$) and thus no gauge fixing is formally needed off shell. However, the resulting perturbation theory for Γ will be singular in the limit $\Gamma_{,i} \rightarrow 0$. That is why, the standard attitude is to give sense to (1) by choosing some gauge χ thus making the limit regular. The price which is paid is an off shell gauge dependence of Γ , clear from the fact that one is averaging non-gauge-invariant functional $e^{i\Gamma}$. This observation prompts another way to regulate the limit: one is to substitute $\int \Gamma_{,i}$ by $\int \epsilon^i(\varphi)\Gamma_{,i}$ invariant under quantum g.t., thus restoring quantum gauge invariance of $\hat{S}$ in (1) off shell and as a result providing independence of subsequent gauge fixing. Also, we are to preserve the invariance under background g.t. ($\delta\varphi = 0$, $\delta\phi = R(\phi)$) spoiled by $\int \Gamma_{,i}$ if generators R^i_α are non-linear. A hint to construction of ϵ^i is contained in observation that a change of a gauge is simply a change of a parametrization of the space of group orbits $\mathcal{M} = M/G$ [2, 12-14] . Integrating over $\mathcal{M}$ we are to preserve reparametrization invariance and thus, in view of the

2) it was also proposed in ref. [10] to use analogous methods in the case of gravity considered as a non-linear realization of affine and conformal groups [11]. This approach, while stressing the "chiral" aspect of gravity, neglects the "gauge" one and thus is different from that of ref. [1] (see below).

previous discussion, need a connection on $\mathcal{M}$. It can be readily constructed using the natural metric on $\mathcal{M}$ [12, 13, 1]. If

g_{ij} is some G -invariant metric on M , i.e.,

$$\partial_i R_{j\alpha} + \partial_j R_{i\alpha} = 0, \quad R_{i\alpha} = g_{ij} R^j_\alpha, \quad \partial_i = \partial_i + \{i, j\} \quad (5)$$

then

$$\tilde{g}_{ij} = g_{ij} - R_{i\alpha} N^{\alpha\beta} K_{j\beta}, \quad N_{\alpha\beta} = R^i_\alpha g_{ij} R^j_\beta, \quad N^{\alpha\beta} = (N_{\alpha\beta})^{-1} \quad (6)$$

measures the distance between two orbits G_r and G_{r+dr} .

Thus we define $(\tilde{g}^{im} = g^{ik} g^{mj} \tilde{g}_{kj})$

$$\tilde{\Gamma}^i_{jk} = \frac{1}{2} \tilde{g}^{im} (\partial_{mj,k} + \partial_{mk,j} - \partial_{jk,m}) = \Gamma^i_{jk} + R^i_\alpha K^{\alpha}_{jk},$$

where

$$\Gamma^i_{jk} = \{i, j, k\} + T^i_{jk}, \quad B^i_\alpha = N^{\alpha\beta} R_{i\beta}, \quad (7)$$

$$T^i_{jk} = -2 B^i_\alpha \partial_{jk} R^\alpha + R^\alpha_\alpha \partial_j R^\beta_\beta B^i_\alpha B^\beta_k$$

For simplicity we avoid explicit parametrization of $\mathcal{M}$ and formally assume $\tilde{\Gamma}^i_{jk}$ to be defined on the whole M (for a rigorous argument see [1]). Defining $\tilde{G}^i$ as in (2) one can prove that (1) terms $\sim R^i$ in $\tilde{\Gamma}^i_{jk}$ drop from the

product $\tilde{G}^i \tilde{\Gamma}_{i,j}$ if $\tilde{\Gamma}$ is background invariant, i.e. $R^i(\phi) \tilde{\Gamma}_{i,j} = 0$ and thus (7) can be taken as the final form of connection on $\mathcal{M}$ [1]; (ii) transformation rules under background and quantum g.t. are (recall that $\tilde{G}^i$ is a vector under $\delta\phi$ and a scalar under $\delta\psi$)

$$\delta \tilde{G}^i /_{\delta\phi = R(\phi)} = [R^i_\alpha(\phi), \tilde{G}^i + R^i_\beta(\phi) \Sigma^\beta_\alpha(\psi, \phi)] \tilde{E}^\alpha \quad (8)$$

$$\delta \tilde{G}^i /_{\delta\psi = \psi(\psi)} = R^i_\beta(\phi) \Omega^\beta_\alpha(\psi, \phi) \tilde{E}^\alpha,$$

providing both quantum and background invariances of $\tilde{G}^i \tilde{\Gamma}_{i,j}$ if $R^i(\phi) \tilde{\Gamma}_{i,j} = 0$. It is worth stressing that (8) is valid only if

3/ If $\mathcal{M}$ is coordinatized by embedding $\mathcal{M} \rightarrow \mathcal{M}' \subset M$ with the
(cont. next pag.)

the gauge algebra is closed, i.e. $E_{\alpha\beta}^{+i} = 0$ in

$$R_{\rho,j}^i R_{\alpha}^j - R_{\alpha,j}^i R_{\rho}^j = R_{\rho}^j C_{\alpha\rho}^{+j} + S_{\alpha,j} E_{\alpha\rho}^{+i} \quad (9)$$

To illustrate this point we note that according to (3) and (7)

$$\delta \epsilon^i / \delta \phi = [R_{\alpha,j}^i \epsilon^j - R_{\alpha}^i(\phi) - \nabla_k R_{\alpha}^i \epsilon^k \dots] \epsilon^{\alpha}, \quad \delta \epsilon^i / \delta \psi = [R_{\alpha}^i(\phi) + R_{\alpha}^i \epsilon^k \dots] \epsilon^{\alpha},$$

where

$$\nabla_j R_{\alpha}^i = -R_{[\alpha}^k R_{\rho],k} B_j^{\beta} = \frac{1}{2} (R_{\rho}^i C_{\alpha\rho}^{+j} + S_{\alpha,k} E_{\alpha\rho}^{+k}) B_j^{\beta} \quad (10)$$

Thus we come to the following definition of reparametrization invariant, gauge invariant and gauge independent off shell effective action due to Vilkovisky [1] ^{4/}

$$e^{-\tilde{F}(\phi)} = \int [d\eta]_x \exp[-S(\phi+\eta) + \epsilon^i(\phi+\eta, \phi) \tilde{F}(\phi)_i] \quad (11)$$

$$[d\eta]_x = d\eta \delta(x(\phi, \eta)) \det Q(\phi, \eta), \quad Q_{\alpha\beta} = \frac{\delta \chi_{\alpha}}{\delta \eta^{\beta}} R_{\beta}^i(\phi+\eta) \quad (12)$$

where χ_{α} is an arbitrary (not necessarily background

help of gauge condition $\chi_{\alpha}(\phi) = 0$, then (6) can be extended on the whole M [1]: $\tilde{g}_{ij} = \bar{g}_{ij} + \chi_{\alpha,i} \chi_{\beta,j} C^{\alpha\beta}$ ($C_{\alpha\beta}$ is some group metric). The restriction on $\tilde{g}_{ij}$ is $\tilde{g}_{ij} = P g P$

$P_j^i = \delta_j^i - g^{ik} \chi_{\alpha,k} \Delta_{\alpha\rho}^{-1} \chi_{\beta,j} C^{\alpha\rho}$, $\Delta_{\alpha\beta} = \chi_{\alpha,i} \chi_{\beta,j} g^{ij}$. Note that $\det \tilde{g} = (\det Q_{\alpha\beta})^2 \times (\det C_{\alpha\beta})^{-1} \times (\det N_{\alpha\beta})^{-1} = \det(\tilde{g})_{m'} \times \det \Delta_{\alpha\beta} \times (\det C_{\alpha\beta})^{-1}$. Thus integrating over $\tilde{g}_{ij}$ with the invariant measure $(d\psi)_{m'} \sqrt{\det \tilde{g}_{m'}}$ we obtain the result of standard covariant quantisation.

^{4/} here ϵ^i is constructed according to (2), (3), (7) and thus $\tilde{F}$ depends on the choice of G -invariant metric $\tilde{g}_{ij}$ on M . If G does not include supersymmetry (see below), then $\tilde{g}_{ij}$ is assumed to be local, i.e. independent of derivatives of ψ , so that transformation (3) is local in the limit $R_{\alpha}^i \rightarrow 0$.

covariant) gauge. Eq.(11) is to be contrasted to the standard definition (cf. [3-6])

$$e^{-\Gamma_X(\phi)} = \int [d\psi]_X \exp[-S(\phi+\psi) + i \int \Gamma_X, :], \quad (13)$$

which gives off shell gauge invariant (but X -dependent) effective action only if (i) generators R^i_α are linear (what is not true, e.g., in supergravity) and (ii) the gauge X is covariant under simultaneous $\delta\psi = R(\psi)$, $\delta\phi = R(\phi)$ transformations (and $X(\psi=\phi) = 0$).

2. Gauge fixing and relation to the standard effective action.

In view of gauge independence of $\tilde{\Gamma}$ we are free to choose some distinguished one, namely the "orthogonal" gauge

$$R^i_\alpha(\phi) \tilde{g}_{ij}(\phi) \tilde{\sigma}^j(\psi, \phi) = 0 \quad (14)$$

Introducing the notations $\tilde{\sigma}_{1,n} = \Pi_{1,n}(\psi) \tilde{\sigma}$, $\Pi_{n,j}^i = R^i_\alpha \tilde{B}^{\alpha j}_n$, $\Pi_{1,j}^i = \tilde{g}^{im} \tilde{g}_{jm}$, $\tilde{\sigma}_{1,n}^i = R^i_\alpha(\psi) \tilde{f}^{\alpha n}(\psi, \phi)$, we conclude (cf.(8)) that under quantum g.t. $\delta\tilde{\sigma}_1 = 0$, $\delta\tilde{f}^{\alpha n} = \Omega^{\alpha n}_j \tilde{e}^j$, i.e. $\tilde{\sigma}_1^i$

is a physical, while $\tilde{f}^{\alpha n}$ is a spurious variable, which drops from the exponent in (11). Thus $\tilde{f}^{\alpha n} = 0$ or (14) is really a natural gauge. Noting that according to (7) $T^i_{jk}(\phi) \tilde{\sigma}_1^j \tilde{\sigma}_1^k = 0$, we can put (assuming (14)) $T^i_{jk} = 0$ in (2) and thus in (3). Changing the variable $\tilde{\eta} \rightarrow \tilde{\sigma}$ we get

$$e^{-\tilde{\Gamma}(\phi)} = \int d\tilde{\sigma} \delta(R_{i,\alpha}(\psi) \tilde{\sigma}^i) \det \tilde{Q}(\psi, \phi) e^{-\tilde{S}(\psi, \phi)} \quad (15)$$

$$\tilde{S} = S(\phi) + \sum_{n=1}^{\infty} (\partial_{j_1} \dots \partial_{j_n} S) \tilde{\sigma}^{j_1} \dots \tilde{\sigma}^{j_n} = \tilde{\sigma}^i \tilde{F}_{i,\alpha}^{\alpha},$$

$$\tilde{Q}_{\alpha\beta} = R_{i,\alpha}(\psi) R^i_\beta(\psi + \psi(\phi, \phi)) = R_{i,\alpha} (R^i_\beta + R^i_{\beta,j} \tilde{\sigma}^j + \partial_\alpha (R^i_{\beta,j})) \times \tilde{\sigma}^j \tilde{\sigma}^j + \dots \quad \det \tilde{Q} = \left| \det \frac{\partial \tilde{S}}{\partial \tilde{\sigma}} \right| = \det \tilde{Q}.$$

For example, at one loop

$$\tilde{F}(\phi) = S(\phi) - \frac{1}{2} \log \frac{\det N_{\alpha\beta}}{\det \Delta_{\alpha\beta}}, \quad \Delta_{\alpha\beta} = \Pi_{\alpha} \Delta \Pi_{\beta}, \quad (16)$$

$$\Delta_{ij} = \partial_i \partial_j S = S_{,ij} - \left\{ \begin{matrix} k \\ ij \end{matrix} \right\} S_{,k}$$

To establish correspondence with the standard definition (13), another (Landau-De Witt or LD) gauge appears to be useful

$$R_{i\alpha}(\phi) \zeta^i = 0, \quad \text{i.e. } \zeta^i_{||} = 0 \quad (17)$$

Then from (3) and (7) we get

$$\begin{aligned} \sigma^i = & \zeta^i + \frac{1}{2} \left\{ \begin{matrix} i \\ jk \end{matrix} \right\} \zeta^j \zeta^k + \frac{1}{6} \left(\left\{ \begin{matrix} i \\ j_1 j_2 \end{matrix} \right\} \zeta^{j_1} \zeta^{j_2} + \right. \\ & \left. + \left\{ \begin{matrix} i \\ m j_1 \end{matrix} \right\} \left\{ \begin{matrix} m \\ j_2 j_3 \end{matrix} \right\} - 3 \partial_{j_1} R^i_{\alpha} B^{\alpha}_m \left\{ \begin{matrix} m \\ j_2 j_3 \end{matrix} \right\} \right) \zeta^{j_1} \zeta^{j_2} \zeta^{j_3} + \dots \end{aligned} \quad (18)$$

Comparing (11) and (13) one can prove the following statement: if there exists a parametrization in which g_{ij} is field independent, i.e. $\left\{ \begin{matrix} i \\ jk \end{matrix} \right\} = 0$ (and thus $\sigma^i = \zeta^i$ if $\zeta^i = 0$) then in this parametrization $\tilde{F}(\phi) = \Gamma_{LD}(\phi)$. Therefore in order to obtain $\tilde{F}$, e.g., for a Yang-Mills plus linear matter multiplets theory one need only to calculate the standard effective action (13) in the LD-gauge (17). If, however, g_{ij} is non-trivial (as it is in the case of gravity) then $\tilde{F} \neq \Gamma_{LD}$ already at one-loop: fluctuation operators Δ differ by the local "correction term" $\left\{ \begin{matrix} k \\ ij \end{matrix} \right\} S_{,k}$ (cf. (16)). Starting with two loops there are also differential corrections to vertices (18). The resulting expressions for $\tilde{F}$ in the gauges (14) and (17) are of course the same. Let us now illustrate calculation of $\tilde{F}$ on several characteristic examples.

1. Yang-Mills theory

In the ordinary parametrization we have: $SA^a_{\mu} = \partial_{\mu}^{ab} \epsilon^b$,

$$\partial_\mu^{ab} = \delta^{ab} \partial_\mu + f^{abc} A_\mu^c, \quad \delta_{ab} = f_{acd} f_{bcd} \quad \text{and thus}$$

$$i \rightarrow (x_i, a_i, \mu_i), \quad \alpha \rightarrow (x_\alpha, a_\alpha), \quad g_{ij} \rightarrow \delta_{\mu_i \mu_j} \delta^{a_i a_j} \delta(x_i - x_j),$$

$$\{j_k\} = 0, \quad R_{\mu_i}^{\alpha} \rightarrow \partial_{\mu_i}^{a_i a_\alpha} \delta(x_i - x_\alpha), \quad N_{\alpha\beta} \rightarrow -(\partial_{x_\alpha}^2)^{a_\alpha a_\beta} \delta(x_\alpha - x_\beta),$$

$$N_{\alpha\beta}^{-1} = (-\partial_{x_\alpha}^2)^{-1}_{\alpha\beta}$$

i.e. (7) takes the form

$$\Gamma_{jk}^i \rightarrow \delta_{\mu_i \mu_j} f_{a_i a_j b} N_{bc}^{-1}(x_j - x_k) \partial_{\mu_k}^{c a_k} \delta(x_i - x_j) +$$

$$+ \frac{1}{2} \partial_{\mu_i}^{d b} f_{a_i d c} N_{c\mu}^{-1}(x_i - x_k) \partial_{\mu_k}^{p a_k} N_{c\mu}^{-1}(x_i - x_j) \partial_{\mu_j}^{c a_j} + (j \leftrightarrow k) \quad (19)$$

In the LD-gauge (17), i.e. $\partial_\mu(B) A_\mu = 0$, we have $\tilde{\Gamma}(B) =$

$$= \Gamma_{LD}(B) = -\log \left\{ \det A \delta(\partial_\mu(B) A_\mu) \det [\partial_\mu(B) \partial_\mu(A+B)] \exp[-S(A+B) + A \frac{\delta \tilde{\Gamma}}{\delta B}] \right\}$$

This gauge is the $\alpha \rightarrow 0$ limit of $\mathcal{L}_g = \frac{1}{2\alpha} (\partial_\mu(B) A_\mu)^2$ and thus can be used for explicit calculations (cf. [15]). At one loop

$$\tilde{\Gamma}(B) = \Gamma_{\alpha=0} = S(B) - \frac{1}{2} \log \frac{\det(-\partial^2)}{\det \Delta_{11}} \quad (20)$$

where $\Delta_{1\mu\nu} = -\delta_{\mu\nu} \partial^2 - 2F_{\mu\nu}$ and $\Pi_{1\mu\nu} = \delta_{\mu\nu} - \partial_\mu \partial_\nu \partial^2$. Only if $J_\mu = \partial_\mu F_{\mu\nu} = 0$ does (20) coincide with the standard expression in the $\alpha = 1$ gauge, because $\det \Delta_1 = \det \Delta_{11} \det(-\partial^2) \times \det(\delta_{\alpha\mu} + \partial_{\alpha\beta}^{-2} f_{\beta\gamma} J_\mu^\gamma \Delta_{1\mu\nu}^{-1} \partial_\nu^{p\mu})$ (for $\alpha=0$ gauge result (20) see also [16]).^{5/} in (20) can be calculated, e.g., for a constant non-abelian B_μ^a with $\partial_\mu F_{\mu\nu} = 0$ and thus applied to the problem of vacuum as a minimum of $\tilde{\Gamma}$. Here it may be useful to clarify the meaning of reparametrisation invariance of $\tilde{\Gamma}$ (11):

5/ Note that if one starts calculating $\tilde{\Gamma}$ without fixing any gauge but separating $A_\mu = A_{1\mu} + \partial_\mu(B)\rho$, then at one loop $\delta^2 S = A_1 \Delta_1 A_1 + A_1 K_1 \rho + \rho K_2 \rho$. Adding the correction term $\sim A A T \partial F$, with $K_2 \rho$ given by (19), it is easy to check that it cancels the ρ -dependent terms restoring quantum gauge invariance. Separating $\int d\rho$, e.g., by the gauge $\rho = 0$ we again get (20).

if we start with some unconventional variables, $A'^a_\mu = f(A)_\mu^a$, then g_{ij} is no longer trivial, $\{j'_k\} \neq 0$, and we are to use the general expression (11), e.g. in the gauge (17), giving some $\tilde{F}'(B')$. Then reparametrization invariance implies $\tilde{F}'(B') = \tilde{F}(f^{-1}(B'))$.

4. Scalar electrodynamics

The aim of our next example - one-loop effective potential in scalar QED - is to illustrate the cancellation of gauge dependence in $\tilde{\Gamma}$ (and also to show that calculation of $\tilde{\Gamma}$ in any gauge other than the LD one (17) is very cumbersome). Let us take the following class of gauges

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (\partial_\mu \varphi)^2 + \frac{1}{4!} (\varphi^4 \varphi^2)^2, \\ \mathcal{L}_g = \frac{1}{2\alpha} (\partial_\mu A_\mu + \beta g \varepsilon_{ab} \phi_a \lambda_a)^2, \quad \lambda^a = \varphi^a - \phi^a, \quad (21)$$

where $\partial_\mu \varphi^a = \partial_\mu \varphi^a + g \varepsilon^{ab} \phi_b A_\mu$ and assume that the background fields are $B_\mu = 0$, $\phi^a = \text{const.}$ At one loop (11) gives

$$\tilde{\Gamma}(\phi) = S(\phi) - \frac{1}{2} \log \det \Delta + \log \det \Delta_{gh}, \quad (22)$$

$$\Delta_{ij} = (S + S_g)_{,ij} - \Gamma^k_{ij} S_{,k}, \quad \Delta_{gh\alpha\beta} = \chi_{\alpha,i}(\phi) R^i_{\beta}(\phi).$$

The construction of connection is straightforward: $\delta A_\mu = -\partial_\mu \xi$,

$$\delta \varphi^a = g \varepsilon^{ab} \phi_b \xi, \quad g_{ij} \rightarrow \{g_{\mu\nu}, \delta^{ab}\}, \quad R^i_{\alpha} \rightarrow \{-\partial_{\alpha}^i, g \varepsilon_{a,b} \phi_b\} \delta(x_i - x_a),$$

$$N_{\alpha\beta} \rightarrow (-D + g^2 \phi^2) \delta(x_\alpha - x_\beta),$$

$$\text{and thus } \Gamma^a_{\mu\nu} \mathcal{L}_{,a} = -\frac{\lambda g^2}{6} \phi^\nu \partial_\mu N_{\nu\mu}^{-2} \partial_\nu \text{ etc.,}$$

$$\text{where } \mathcal{L}_{,a} = \frac{1}{6} \phi^a \phi^2. \text{ As a result, } \Delta_{\mu\nu} = \kappa^2 \delta_{\mu\nu} + \dots,$$

$$\Delta_{a\mu} = i g \varepsilon_{ab} \phi_b C k_\mu, \quad C = 1 - \beta/\alpha + \frac{1}{6} \phi^2 \kappa^2 N^{-2}, \quad (23)$$

$$\Delta_{ab} = (\kappa^2 + \frac{1}{2} \phi^2) \Pi_{ab}^n + B \Pi_{ab}^1, \quad B = \kappa^2 + (\frac{1}{6} + \frac{\beta g^2}{\alpha}) \phi^2 -$$

$$- \frac{\lambda g^2 \phi^4}{6} (2 - g^2 \phi^2 N^{-1}) N^{-1}, \quad N^{-1} = \frac{1}{\kappa^2 + g^2 \phi^2}, \quad \Pi_{ab}^n = \frac{\phi^a \phi^b}{\phi^2}.$$

Integrating over $\hat{z}^a$, we get

$$\bar{\Delta}_{\mu\nu} = \Delta_{\mu\nu} - \Delta_{\mu\alpha} \Delta^{-1}_{\alpha\beta} \Delta_{\beta\nu} = (k^2 + g^2 \phi^2) \Pi_{\mu\nu}^\perp + E \Pi_{\mu\nu}^\parallel, \\ \Pi_{\mu\nu}^\parallel = \frac{k_\mu k_\nu}{k^2}, \quad E = \alpha^{-1} k^2 + g^2 \phi^2 + \frac{18 g^2 \phi^4}{6} N k^{-2} - \frac{g^2 \phi^2 k^2}{B} C, \quad (24)$$

and finally $(\tilde{\Gamma} = \int d^4k [\frac{1}{4!} \phi^4 + \tilde{V}_1(\phi)], \quad dk = \frac{d^4k}{(2\pi)^4})$

$$\tilde{V}_1 = -\frac{1}{2} \int d^4k [\log(k^2 + \frac{1}{2} \phi^2) + 3 \log(k^2 + g^2 \phi^2) + \log B + \log E - \\ - 2 \log \alpha^{-1/2} (k^2 + \beta g^2 \phi^2)] = -\frac{1}{2} \sum_n \int d^4k \log(k^2 + M_n^2) = \frac{\gamma}{64\pi^2} \phi^4 \log \phi^2 + \dots \quad (25)$$

The coefficient γ has the following gauge dependent value if calculated according to the standard recipe (13) (i.e. omitting N^{-1} -terms in B, C, E in (25)) (cf. [17] for $\beta=0$)

$$\gamma = 3g^4 + \frac{5}{18} \lambda^2 + \frac{2\beta - \alpha}{3} \lambda g^2. \quad (26)$$

To calculate γ for the "corrected" expression (25) one is to observe that $\gamma \sim \sum_n M_n^4$ and thus integrating $\int d^4k \log(k^2 + M_n^2) + A_1 k^{2m-2} + A_2 k^{2m-4} + \dots$ we only need to now $A_1^2 - 2A_2$. As a result, all α and β -dependent terms cancel and we find

$$\tilde{\gamma} = 3g^4 + \frac{5}{18} \lambda^2 + \frac{2}{3} \lambda g^2. \quad (27)$$

One could avoid the above involved calculations by working in the LD gauge (17), i.e. $\partial_\mu A_\mu + g E_{ab} \phi_a \hat{z}_a = 0$, where $\tilde{\Gamma} = \Gamma_{LD}$, thus obtaining (27) as the $\alpha=0, \beta=1$ limit of (26). It is interesting to note that it is possible to calculate V_1 according to (1) without fixing any gauge: we get (24) with $E \rightarrow (\lambda/6) g^2 \phi^2 / (k^2 + \frac{1}{6} \phi^2)$ and thus $V_1 = -\frac{1}{2} \int d^4k \cdot \{ \log(k^2 + \frac{1}{2} \phi^2) + 3 \log(k^2 + g^2 \phi^2) + [\log(k^2 + \frac{1}{6} \phi^2) + \log \frac{1/6 g^2 \phi^2}{k^2 + \frac{1}{6} \phi^2}] \}$, coincides with the result in the unitary gauge ($\gamma = 3g^4 + \lambda^2/4$) after cancelling the infinite ($\sim \int d^4k \log \phi^2$) term by the local

measure. Repeating the calculation with the "correction" term $\Gamma_{jk}^i S_{,k}$ (22), we see that it annihilates the longitudinal part of $\bar{\Delta}_{\mu\nu}$, restoring quantum gauge invariance and thus making necessary to fix some gauge, e.g. $\partial_\mu A_\mu = 0$ one; then $\bar{V} = -\frac{1}{2} \cdot \int dk \{ \log(k^2 + \frac{1}{2} p^2) + 3 \log(k^2 + g^4 p^2) + \log B_0 \}$, $B_0 = B(p=0)$ (cf. (23)). The result for γ is again (27). Thus the result of new "corrected" recipe is similar to the R-gauge one and is different from the that of the $\bar{U}$ -gauge (or gauge-non-fixing) prescription.

5. Quantum gravity

In the standart parametrization $g_{\mu\nu} = E^\lambda \partial_\lambda g_{\mu\nu} + g_{\mu\lambda} \partial_\lambda E^\lambda + g_{\nu\lambda} \partial_\lambda E^\lambda$, $R_{\mu\nu\rho} = 2 g_{\rho(\mu} D_{\nu)} \delta(x_\mu - x_\rho)$, $D_\mu = \partial_\mu + \frac{1}{2} \Gamma_{\mu\nu}^\lambda$. The invariant (5) local (no $\partial_\lambda g_{\mu\nu}$) field space metric is unique up to a real parameter $[2, 14]$ $(1 + \gamma)$

$$ds^2 = \int dx^\mu G^{\mu\nu\alpha\beta} dg_{\mu\nu} dg_{\alpha\beta}, \quad G^{\mu\nu\alpha\beta} = \frac{1}{2} \sqrt{g} (g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha}) \quad (28)$$

Its Christoffel connection is

$$\left\{ \begin{matrix} \mu\nu & \lambda\kappa \\ \rho\sigma \end{matrix} \right\} = -\frac{1}{2} \left[\delta_{\rho\sigma}^{(\mu} g^{\nu\lambda)} \delta_{\sigma}^{(\kappa} \delta_{\sigma}^{\lambda)} + \delta_{\rho\sigma}^{(\mu} g^{\nu\lambda)} \delta_{\sigma}^{(\kappa} \delta_{\sigma}^{\lambda)} - \frac{1}{2} g^{\mu\nu} \delta_{\rho\sigma}^{\lambda\kappa} \right] - \frac{1}{2} g^{\lambda\kappa} \delta_{\rho\sigma}^{\mu\nu} + \frac{1}{2(1+\gamma)} g_{\rho\sigma} g^{\mu\lambda} g^{\nu\kappa} + \frac{1}{2(1+\gamma)} g_{\rho\sigma} g^{\mu\nu} g^{\lambda\kappa} \quad (29)$$

The operator $N_{\alpha\beta}$ in (6) and the LD gauge (17) have the form

$$N_{\alpha\beta} = 2 \left[-g_{\alpha\beta} D^2 - R_{\alpha\beta} - (1+2\gamma) D_\alpha D_\beta \right] \sqrt{g} \delta(x_\alpha - x_\beta) \quad (30)$$

$$R_{\alpha\beta\gamma} G^{\alpha\beta\mu\nu} h_{\mu\nu} = 0, \text{ i.e. } D_\rho (h_\alpha^\rho + \gamma \delta_\alpha^\rho h) = 0. \quad (31)$$

The expression for $\tilde{\Gamma}$ is then given by (11), or at one loop by (16), where now $\Delta^{\mu\nu\alpha\beta} = \hat{\Delta}^{\mu\nu\alpha\beta} - \left\{ \begin{matrix} \mu\nu & \alpha\beta \\ \rho\sigma \end{matrix} \right\} \frac{\delta S}{\delta g_{\rho\sigma}}$, $\hat{\Delta} = \frac{\delta^2 S}{\delta g^2}$.

If $S = -\int d^4x (R - 2\Lambda_0) \sqrt{g}$, then $h \Delta h = \frac{1}{2} h \Delta_0 h - [D_\mu (h^\mu_\alpha - \frac{1}{2} S^\mu_\alpha h)]^2$, where

$\Delta_0 = -G_0 D^2 + X_0$, $X_{0\mu\nu\rho} = 2C_{\mu\nu\rho} + \frac{R}{6} (g_{\mu\alpha} g_{\nu\rho} - \frac{1}{2} g_{\mu\nu} g_{\alpha\rho}) - 2\Lambda_0 G_{0\mu\nu\rho}$ (here $C_{\mu\nu\rho}$ is the Weyl tensor and $G_0 = G(\lambda = -\frac{1}{2})$). Using

(29) to establish the correction term and employing the condition (31) we get $\Delta = \frac{1}{2} (-\hat{G} D^2 + \hat{X})$, $\hat{G} = G(\hat{\lambda})$, $\hat{\lambda} = -\frac{1}{2} - 2(\lambda + \frac{1}{2})^2$, $E_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

$$\hat{X}^{\mu\nu} = X_0^{\mu\nu} + 2E_{\alpha\beta}^{\mu\nu} - \frac{1}{2} g^{\mu\nu} E_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} E^{\mu\nu} - R \left(\frac{\lambda + 1/2}{\lambda + 4\lambda} \right), \quad (33)$$

$$+ (S_{\alpha\beta}^{\mu\nu} - \frac{1}{2} g^{\mu\nu} g_{\alpha\beta} S).$$

For simplicity we shall take $\lambda = -\frac{1}{2}$ (this value of λ

in (28) was obtained in [1] by adding the condition $\hat{G} = G$).

Then $\hat{G} = G_0$ and the one-loop result for $\tilde{\Gamma}$ is

$$\tilde{\Gamma}(g) = S(g) - \frac{1}{2} \log \frac{\det(-g_{\mu\nu} D^2 - R_{\mu\nu})}{\det(\Delta_{\perp}^{\mu\nu}{}_{\rho\sigma})}, \quad (34)$$

where $\Delta_{\perp}$ is Δ subjected to (31). Let us now compare the divergences of $\tilde{\Gamma}$ with those of the standard effective action in the gauges $\mathcal{L}_g = \frac{1}{2\alpha} [D_\mu (h^\mu_\alpha + \lambda S^\mu_\alpha h)]^2$ known for $\Lambda_0 = 0$ [18] and for $\Lambda_0 \neq 0$ [19]. Introducing the notations:

$$\Gamma_\infty = -\frac{1}{2} \left(\frac{1}{2} L^4 B_0 + L^2 B_2 + B_4 \log \frac{L^2}{\mu^2} \right), \quad L \rightarrow \infty,$$

$$B_p = \frac{1}{(4\pi)^2} \int d^4x \sqrt{g} \mathcal{L}_p, \quad B_0 = 2, \quad B_2 = \beta_2 R + \beta_2 \Lambda_0, \quad (35)$$

$$B_4 = \frac{5\beta_1}{15} R^2 + \beta_2 E_{\mu\nu}^2 + \frac{1}{2} \beta_3 R^2 + \beta_4 R \Lambda_0 + \beta_5 \Lambda_0^2.$$

we have for $\lambda = -\frac{1}{2}$ and $\alpha = 1$ (De Donder gauge) and

$\alpha = 0$ (31) cases

	β_2	β_3	β_4	β_5	β_1	β_2	(36)
$\alpha = 1 :$	$7/20$	$23/40$	$-26/3$	20	$-23/3$	20	
$\alpha = 0 :$	$7/10$	$62/40$	-8	12	$-13/3$	12	

Observing that the only coefficient which changes after adding the correction term is β_2 we conclude that the infinite part of $\tilde{F}$ is given by (25) with $\beta_1, \beta_4, \beta_5, \rho_1, \rho_2$ being as in (36) for $\alpha=0$. The value of β_2 can be found by direct diagram computation or by using the algorithm for non-diagonal operators developed in [20]. Moreover, if the background geometry is of the Einstein type, $R_{\mu\nu} = \Lambda g_{\mu\nu}$ (i.e. $E_{\mu\nu} = 0$) then the correction term vanishes and $\tilde{F}$ coincides with the standard one in the $\alpha=0$, $\Lambda = -1/2$ gauge.

It should be understood that $G^{\mu\nu\alpha\beta}$ in (28) is not related to a particular structure of gravitational action (it can be used for construction of $\tilde{F}$ for any covariant action, e.g. of the $R + R^2$ - type). Thus $G^{\mu\nu\alpha\beta}$ does not actually govern the full dynamics of Einstein quantum gravity. At the same time, it is important to take into account the non-trivial geometry of $g_{\mu\nu}$ configuration space dictated by $G^{\mu\nu\alpha\beta}$ in quantization of this theory.

6. Supersymmetrical theories

Let us first formally generalize the definition of $\tilde{F}$ (11) on the case when fields and gauge parameters can be either bosons or fermions. We shall take all derivatives to be right ones and use two kinds of invariant contractions: $\{^k \}_k$ and $\{^k \}_k^k$, $\{^k \}_k = (-)^k \{^k \}_k$ (cf. [21, 22]). Then $ds^2 = dy^i g_{i\bar{j}} dy^{\bar{j}}$, $dS = S_{,2} dy^k, \delta y^i = R^i{}_2 E^{\bar{2}}, S_{,2} R^k{}_2 = 0$, and we get the analog of (7) in the form

$$\Gamma^i{}_{k\bar{m}} = \{^i{}_{k\bar{m}}\} + T^i{}_{k\bar{m}}, \quad \Gamma^i{}_{k\bar{m}} = (-)^k \Gamma^i{}_{k\bar{m}},$$

$$\{^i{}_{k\bar{m}}\} = \frac{1}{2} g^{i\bar{j}} [g_{k\bar{m},\bar{j}} + g_{\bar{m},k\bar{j}} (-)^{km} - g_{\bar{m},\bar{j}k} (-)^{m(\bar{j}+k\bar{m})}] \quad (37)$$

$$T^i_{\alpha\beta} = -R^i_{\alpha;\hat{m}} B^\alpha_{\hat{k}} (-)^{m\alpha} - R^i_{\alpha;\hat{k}} B^\alpha_{\hat{m}} (-)^{k(\alpha+m)} + \\ + \frac{1}{2} R^i_{\alpha;\hat{m}} R^n_{\hat{\beta}} B^\beta_{\hat{m}} B^\alpha_{\hat{k}} (-)^{dm} + \frac{1}{2} R^i_{\alpha;\hat{k}} R^n_{\hat{\beta}} \\ \times B^\alpha_{\hat{m}} B^\beta_{\hat{k}} (-)^{\beta(m+d)}$$

where $g_{m\hat{n}} g^{kn} = \delta_m^n$, $N_{\hat{\alpha}\hat{\beta}} = R^i_{\hat{\alpha}} g_{i\hat{m}} R^m_{\hat{\beta}} (-)^{i\alpha}$,
 $B^\alpha_{\hat{k}} = g_{k\hat{m}} R^m_{\hat{\beta}} N^{\beta\alpha} (-)^{k(\alpha+1)}$, $\tilde{g}^i_{\hat{m}} = \tilde{g}^i_{\hat{m}} + (-)^{ik} \tilde{g}^k_{\hat{m}} \{ \cdot \cdot \cdot \}$
 Constructing $\tilde{\sigma}^i = \tilde{\zeta}^i + \frac{1}{2} \tilde{\zeta}^m \Gamma^i_{m\hat{k}} \tilde{\zeta}^{\hat{k}} (-)^{im} + \dots$ as in (2), (3),
 we finally get (11) with $\tilde{\Gamma}^i_{\hat{k}} \tilde{\sigma}^{\hat{k}}$. The LD gauge (17) now is

$$R^i_{\hat{\alpha}} g_{i\hat{m}} \tilde{\zeta}^m (-)^{i\alpha} = 0 \quad (\text{for example, in the spinor QED:}$$

$$\partial_\mu A_\mu + ie a \bar{\psi}_{\text{gauge}} \psi_{\text{gauge}} + h.c. = 0, [a] = \text{cm}).$$

Consider now a theory which is supersymmetry invariant (or "Q-invariant"). To preserve off shell supersymmetry we need a Q-invariant metric on the field space. The construction of such metric in terms of component fields appears to be non-trivial (Q-algebra mixes $\mathcal{Y}$ and $\partial\mathcal{Y}$). For example, in the Wess-Zumino model ($\mathcal{Y}^i \rightarrow \varphi, \psi, F, \dots$) no local metric can be obtained starting with $\int d^4x (d\varphi d\varphi^* + a d\psi d\psi + \dots)$, while the correct Q-invariant metric is $ds^2_{wz} = \int d^4x (d\varphi dF - \frac{1}{2} d\psi d\psi + h.c. \dots)$ (for notations see [23]). Its field independence implies the equivalence of (1) and (4) and thus Q-invariance of $\tilde{\Gamma}(\varphi, \psi, F)$. At the same time, no analog of ds^2_{wz} exists for a component gauge multiplet $(A_\mu, \lambda, \bar{\lambda})$. Thus we confront a problem of construction of gauge-invariant, super-invariant component effective action, e.g., for a super-Yang-Mills theory. A natural solution is provided by superspace generalization of the definition (11), (37) of $\tilde{\Gamma}$: one simply substitutes space by superspace, fields by superfields (e.g. chiral or real),

gauge transformations by supergauge ones. Hence in order to get a supergauge-invariant, super-gauge independent (and super-field reparametrization invariant) off shell effective action $\tilde{\Gamma}_S$ we only need to find super-gauge-invariant (and super-Poincare invariant) metric on the space of superfields. Given $\tilde{\Gamma}_S$ in terms of background superfields we can choose a physical (WZ) background supergauge solving the above problem. More explicitly, in the WZ-model, $ds_{WZ}^2 = \int d\bar{z} (d\Phi d\Phi S(\bar{\theta}) + h.c.)$, $d\bar{z} = d\bar{z}'$, $d^2\theta d^2\bar{\theta}$, $\Phi(\bar{z}) = \varphi + \sqrt{2}\theta\psi + \theta\theta F$, etc. while in the N=1 super Yang-Mills case

$$ds_{SYM}^2 = \int d\bar{z} \text{tr} (de^V de^{-V}) = \int d\bar{z} g_{ab}(V) dV^a dV^b, \quad (38)$$

where $V^+ = V$, $V_{ij} = V^a t_{ij}^a$ (t^a are generators of gauge group G), $W_{ij} = -\frac{1}{2} \bar{D}^2 e^{-V} D_{ij} e^V$. This metric (and $S = \int d\bar{z} (W^a W_a S(\bar{\theta}) + h.c.)$) is invariant under super-gauge transformations $e^{V'} = e^{-i\Lambda^+} e^V e^{i\Lambda}$, $\delta V = i L_{V/2} [(\Lambda + \Lambda^+) + \coth L_{V/2} (\Lambda - \Lambda^+)]$, $L_A \dots = [A, \dots]_{i.e.}$, $\delta V^a = R_c^a(V) \Lambda^c + h.c.$. Given the group G , one first finds $g_{ab}(V)$ (being the metric on homogeneous space G_c/G_R) and $R_c^a(V)$, then calculates the connection (37) and finally establishes the form of the effective action $\tilde{\Gamma}_S$ (being gauge-independent, it can be evaluated either in WZ or in manifestly supersymmetric quantum super-gauges). It should be stressed that the use of non-trivial metric (38) is essential for construction of gauge invariant $\tilde{\Gamma}_S$ in view of non-linearity of R_c^a . At this point one observes the analogy between this recipe and the special

"background-quantum" splitting used in [24, 25] for particular super YM and supergravity models (cf. the relation of refs. [7] and [9]). The advantage of the new approach is in its generality, quantum gauge independence and reparametrization invariance.

Finally, let us discuss the case of N=1 supergravity. Working in superspace, a natural candidate for super invariant metric is $\int d^2z (E_A^M E_B^{N\dots}) dE_M^A dE_N^B$. However, supervielbein E_M^A is a constrained variable and therefore dE implies a shift of H and ∂H , H^M being the unconstrained (dynamical) superfield of Ogievetsky-Sokatchev or Siegel-Gates. The desired super-invariant metric on the (H, ϕ) -space (ϕ is an auxiliary chiral superfield) can be taken in the form

$$ds_{SQ}^2 = \int d^2z [E dH^M g_{MN} (H, \partial H) dH^N + E^{1/2} d\phi d\phi \delta(\bar{\theta}) + h.c.] \quad (39)$$

where $g_{MN} = E_M^A \gamma_{AB} E_N^B (-)^{N(1+A)}$, $E = s \det E_M^A$,

and E_M^A

is built from H and ∂H according to the rules of ref. [26] :

$$E_A = E_A^M D_M, E_{\alpha} = e^{-H} D_{\alpha} e^H, H = H^M D_M, \text{ etc.}$$

A peculiar feature of supergravity is that metric necessarily depends on derivatives of fields. All the following construction of invariant action $\tilde{I}_S$ can be done as above. Note that

(39) is not an extension of a simplest Lorentz and general coordinate invariant metric for component fields :

$$ds^2 = \int d^4x \sqrt{g} \left\{ [e_{\mu}^{\alpha} e_{\nu}^{\beta} - \frac{1}{2}(1+t) e_{\mu}^{\alpha} e_{\nu}^{\beta} + t g^{\mu\nu} g_{\alpha\beta}] de_{\mu}^{\alpha} de_{\nu}^{\beta} + q_1 d\bar{\psi}_{\mu} d\psi_{\nu} g^{\mu\nu} + q_2 (ds ds + dP dP - dA_{\alpha} dA_{\alpha}) \right\}, (t\text{-parameter})$$

which is sufficient, if we do not demand supersymmetry of $\tilde{I}_S$, e.g., computing it in the gravitational sector (the correspond-

ing LD gauges (17) are: $D_\mu (\delta^\mu_\nu - \frac{1}{2} e^\mu_\nu) = 0$, $e^\mu_\nu \delta^\nu_\mu = 0$, $D_\mu \tau_\mu = 0$. Even in this case ϵ^c depends on all powers of quantum auxiliary fields and thus one cannot integrate them out from the very beginning.

In conclusion we remark that the new effective action may be useful in non-polynomial theories describing coupling of matter/^{with} (super) gravity. The non-trivial configuration space metric of the "pure" theory is then "extended" on the matter fields sector. We are grateful to Dr. G.A.Vilkovisky for informing us about his work prior to publication and useful discussions.

References

1. G.A. Vilkovisky, in: B. De Witt Sixtieth anniversary volume, edited by S.Christensen (Adam Hilger, Bristol, 1983); Phys. Lett., to be published
2. B.S.De Witt, Dynamical Theory of Groups and Fields (Gordon and Breach, N.Y., 1965); Phys. Rev. 162 (1967) 1195
3. G.'t Hooft, in Funct. and Probab. Methods in Quant. Field Theory, vol.I, Acta Univ. Wratisl. N 368 (1976)
4. B.S.De Witt, in Quantum Gravity II, ed. C.Isaacs (Oxford, U.P. Oxford, 1981)
5. D.G.Boulware, Phys. Rev. D23 (1981) 389
6. L.F.Abbott, Nucl. Phys. B185 (1981) 189
7. J.Honerkamp, Nucl. Phys. B36 (1972) 130; L.Tataru, Phys. Rev. D12 (1975) 3351
8. D.I.Kazakov, V.N.Pervushin and S.V.Pushkin, Teor. Mat. Fiz. 31 (1977) 169; J. Phys. A11 (1978) 2093
9. L.Alvarez-Gaume, D.Z. Freedman, S.Mukhi, Ann. Phys. 134 (1981) 85; D.G.Boulware and L.S.Brown, Ann. Phys. 138 (1982) 392.
10. V.N.Pervushin, Teor. Mat. Fiz. 27 (1976) 16
11. A.B.Borisov and V.I.Ogievetsky, Teor. Mat. Fiz. 21 (1974) 329
12. B.S.De Witt, Phys. Rev. 160 (1967) 1145
13. O.Babelon and C.M.Viallet, Phys. Lett. 85B (1979) 216; Commun. Math. Phys. 81 (1981) 515
14. B.S.De Witt, in Recent developments in gravitation,

- ed. M. Levy and S. Deser (Plenum, 1979)
15. D.M. Capper and A. Mac-Lean, Nucl. Phys. B203 (1982) 403
16. G.M. Gallatin, Phys. Rev. D26 (1982) 505
17. S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 1888;
R. Jackiw, Phys. Rev. D9 (1974) 1686.
18. R.E. Kallosh, O.V. Tarasov and I.V. Tyutin, Nucl. Phys. B137
(1978) 145
19. B.S. Fradkin and A.A. Tseytlin, Lebedev Inst. preprint
N 70 (1981)
20. O. Barvinsky and G.A. Vilkovisky, Univ. Texas at Austin
preprint (1983)
21. R. Arnowitt and P. Nath, Gen. Relat. Grav. 7 (1976) 89
22. P. van Nieuwenhuizen, Phys. Rept. C68 (1981) 189
23. J. Bagger and J. Wess, Supersymmetry and supergravity
(Princeton Univ. Press, 1983)
24. M.T. Grisaru, W. Siegel and M. Roček, Nucl. Phys. B159 (1979)
429
25. M.T. Grisaru and W. Siegel, Nucl. Phys. B187 (1981) 149;
Nucl. Phys. B201 (1982) 292
26. W. Siegel and S.J. Gates, Jr., Nucl. Phys. B147 (1979) 77.

Т - 07303

Подписано в печать 11 марта 1983 года

Заказ № 171. Тираж 100 экз. л/з 0.9

Отпечатано в Редакционно-издательском
отделе БИАИ СССР

Москва, В-312, Ленинский проспект, 53