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NOETHER COUPLING OF MASSIVE GRAVITINOS
TO $N = 1$ SUPERGRAVITY

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ABSTRACT

A new method is presented to determine which supergravity models can have a superHiggs effect with at least one surviving local supersymmetry. The basic ingredient is the coupling of the massive spin-3/2 multiplet to supergravity.

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Nowadays, much effort is devoted to spontaneous supersymmetry breaking of supergravity models. These models are either matter-coupled models or pure supergravity models. In this letter, we propose a new method, applicable to both classes, to determine what models can break a local N extended supersymmetry down to a local M extended supersymmetry with $M \geq 1$. Up to now, one starts at the massless end of the theory, that is to say, one couples some massless matter multiplets to a supergravity action¹⁾, or considers a pure supergravity action. By using then the freedom in the coupling^{1),2)} (due to the presence of scalars) or geometrical approaches [such as multivalued fields in higher dimensions³⁾ or Kaluza-Klein techniques⁴⁾] one can arrive at a superHiggs effect with or without a cosmological constant.

We will start at the end-point, that is to say, we consider the massive matter multiplets which are present in the spontaneously broken theory, with M extended supersymmetry, and try to couple them directly to the M extended gauge action. In this way one can discover which couplings are possible and which are not, without actually having to go through the rather complicated computations involving Weyl rescalings, chiral redefinitions and knowledge of tensor calculus. In particular, one may thus discover no-go and yes-go theorems. How one realizes a spontaneous supersymmetry breaking will not become clear by our method; that may be done either in four dimensions, by studying the potential, or by going to higher dimensions¹⁾⁻⁴⁾. What our method does permit us to determine is the most general spontaneously broken model with at least one local supersymmetry surviving.

It is clear that if one or more supersymmetries are broken, one or more gravitinos will acquire a mass, and thus one will need as an ingredient the massive spin-3/2 multiplet of M extended rigid supersymmetry. Depending on whether one ends up with a cosmological constant or not, one needs the massive spin-3/2 multiplets of rigid Poincaré or de-Sitter supersymmetry. In this letter, we will only consider spontaneously broken models without a cosmological constant, but one can extend our results to the de-Sitter case straightforwardly. Since the action and transformation rules of the massive $N = 1$ spin-3/2 Poincaré multiplet of rigid supersymmetry are not known, we will give them below. It should be stressed that our method can only be applied to cases where there is some local supersymmetry left after the breaking; in other words, we can study breaking from any N down to M with $N > M \geq 1$, but we have nothing to say about breaking of all supersymmetries.

As an example of our new approach, we will consider the breaking of $N = 2$ supergravity coupled to a massless $N = 2$ vector multiplet down to $N = 1$ supergravity. As we shall explain, these are the two basic building blocks for any other model. Moreover, in this model, the $N = 1$ end products are unique.

The massless $N = 2$ supergravity multiplet contains the following helicity states

$$\lambda = \left(\pm 2, \pm \frac{3}{2}, \pm \frac{3}{2}, \pm 1 \right) \quad (1)$$

The simplest $N = 2$ massless matter multiplet to couple to is the vector multiplet with helicity content

$$\lambda = \left(\pm 1, \pm \frac{1}{2}, \pm \frac{1}{2}, 0^+, 0^- \right) \quad (2)$$

For reasons to be explained, we take the overall parity of (2) to be negative. A superHiggs effect will require that only one spin-1/2 gets eaten by a gravitino, while the other gravitino must remain massless in order to end up with an unbroken supersymmetry. The massive spin-3/2 multiplet of rigid Poincaré supersymmetry has the following spin content

$$J = \left(\frac{3}{2}, 1^+, 1^-, \frac{1}{2} \right) \quad (3)$$

Comparing Eqs. (3) with (2) and (1), one notices that the pseudoscalar must be eaten by the axial vector in (2) and the scalar by the vector in (1) by an ordinary Higgs effect, while the remaining spin-1/2 must have the same mass as those massive spin-1 fields and the massive gravitino. In fact, the general $N = 2$ supergravity action coupled to matter will contain vector and scalar multiplets with helicity content $(\pm 1/2, \pm 1/2, 0^+, 0^+, 0^-, 0^-)$, but the above discussion shows that one can never break local supersymmetry to $N = 1$ if one does not have at least one vector multiplet. Compare this with the breaking of $N = 1$ to $N = 0$ ¹⁾ which can proceed in the presence of only $N = 1$ massless scalar multiplets.

We now present the action and transformation rules of the massive spin-3/2 multiplet of $N = 1$ rigid supersymmetry. The action reads

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \bar{\Psi}_\mu \gamma^{\mu\nu\rho} \partial_\nu \Psi_\rho - \frac{m}{2} \bar{\Psi}_\mu \gamma^{\mu\nu} \Psi_\nu - \frac{1}{4} F_{\mu\nu}^2 \\ & - \frac{m^2}{2} A_\mu^2 - \frac{1}{4} G_{\mu\nu}^2 - \frac{m^2}{2} B_\mu^2 - \frac{1}{2} \bar{\lambda} \not{\partial} \lambda - \frac{m}{2} \bar{\lambda} \lambda \end{aligned} \quad (4)$$

where $\phi_\mu, A_\mu, B_\mu, \lambda$ are the fields corresponding to (3), and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $(\gamma^{\mu_1 \dots \mu_n}) = \gamma^{\mu_1} \dots \gamma^{\mu_n}$ totally antisymmetrized with strength one). Although (4) is the most obvious candidate action, we can write down other off-diagonal couplings; however, most of them can be eliminated by field redefinitions.

Requiring (4) to be invariant under field variations fixes the transformation rules up to four constants. These are fixed by requiring the usual $N = 1$ supersymmetry algebra

$$[\delta(\epsilon_1), \delta(\epsilon_2)]\phi = 2(\bar{\epsilon}_2 \gamma^m \epsilon_1) \partial_m \phi \quad (5)$$

where ϕ is any of the fields. The result is ^{*})

$$\begin{aligned} \delta \phi_\mu = & \frac{1}{4} \gamma^{\alpha\beta} \gamma_\mu \bar{\epsilon} \mathbb{F}_{\alpha\beta} \epsilon - \frac{1}{2} \gamma_\mu \gamma^{\alpha\beta} \bar{\epsilon} \mathbb{F}_{\alpha\beta} \epsilon \\ & + \frac{1}{6m} \partial_\mu (\gamma^{\alpha\beta} \bar{\epsilon} \mathbb{F}_{\alpha\beta} \epsilon) + m \left(\frac{2}{3} A_\mu - \frac{1}{3} \gamma_{\mu\nu} A^\nu \right) \\ & + \frac{2}{3} \partial_\mu (\gamma^\alpha A_\alpha \epsilon) \end{aligned}$$

$$\begin{aligned} \delta(A_\mu + i B_\mu) = & \bar{\epsilon} (1 - \gamma_5) \gamma_\mu \lambda - \frac{1}{\sqrt{3}} \bar{\epsilon} (1 + \gamma_5) \gamma_\mu \lambda \\ & - \frac{1}{\sqrt{3}m} \partial_\mu (\bar{\epsilon} (1 + \gamma_5) \lambda) \end{aligned} \quad (6)$$

$$\delta \lambda = \frac{1}{2\sqrt{3}} \gamma^{\alpha\beta} \bar{\epsilon} \mathbb{F}_{\alpha\beta} \epsilon - \frac{m}{\sqrt{3}} \gamma^\alpha A_\alpha \epsilon$$

$$A_\alpha = A_\alpha + i \gamma_5 B_\alpha$$

^{*}) In our conventions $\{\gamma^\mu, \gamma^\nu\} = 2\delta^{\mu\nu}$ with $\mu = 1, 4$ and $\gamma_5 \gamma_5 = +1$ with γ_5 purely imaginary. All γ^μ and γ^5 are Hermitian.

We note that there are terms in (6) which are m independent, but are absent in the massless model. They are needed to cancel m independent variations due to using m^{-1} laws in mass terms. Since the m^{-1} terms in (6) are all gauge variations, the varied action does not contain m^{-1} terms. However, there are also gauge variations which are m independent. The occurrence of m^{-1} terms in variation laws has been observed before in the massive $N = 1$ vector multiplet⁵⁾. We found these results as follows. At the order m level, the need for the second terms in $\delta\varphi_\mu$ became apparent, but since it undid the cancellation at the order m^0 level, we found the third term in $\delta\varphi_\mu$.

We must now couple the action in (4) to $N = 1$ supergravity. It is interesting that we need once more an $N = 1$ Noether coupling technique in order to obtain results concerning an $N = 2$ model. In fact, by this method the first extended supergravity, namely $N = 2$ supergravity, was constructed⁶⁾. The easiest method for finding the Noether current is to use (6) into (4) but with x dependent supersymmetry parameters $\epsilon(x)$, and to identify the Noether current as j_N^μ in $\int \delta\mathcal{L} = \int \partial_\mu \bar{\epsilon}(x) j_N^\mu$. In this way we obtain

$$J_{\mu N} = \left(\mathbb{F}_{\mu\rho} + \frac{1}{2} \epsilon_{\mu\rho\alpha\beta} \mathbb{F}^{\alpha\beta} \gamma_5 \right) \varphi^\rho + m A_\sigma \gamma_\mu^{\sigma\nu} \varphi_\nu + \frac{1}{2\sqrt{3}} \left(\mathbb{F}^{\alpha\beta} \gamma_{\alpha\beta} - 2m \tilde{A}^\alpha \gamma_\alpha \right) \gamma_\mu \lambda \quad (7)$$

The coupling of the massive spin-3/2 multiplet to $N = 1$ supergravity is now given to order ψ_μ by

$$\mathcal{L} = \mathcal{L}_{S.G.} + \mathcal{L}(\text{in. eq. (4)}) + \kappa \bar{\Psi}^\mu J_{\mu N} \quad (8)$$

We will not investigate here whether the linearized result in (8) can be extended to a full non-linear coupling. Our main task was in fact to illustrate the general approach that one should follow in possible searches for spontaneously broken supergravity theories.

We will now consider the minimal multiplet structure that one should consider for higher N theories.

For higher N the situation is more interesting because one can envisage different possibilities. First let us consider theories with $N \leq 4$. In these theories one has the possibility of adding matter multiplets so the situation is similar to the $N = 2$ case with the exception that now central charges can be present for massive multiplets of the residual unbroken supersymmetries.

In fact, from simple counting arguments we can break $N = 4$ down to $N = 2$ in two different ways depending on whether the massive $N = 2$ gravitino multiplets have vanishing central charges or not. We first consider the situation with no central charges.

In this case the $N = 2$ massive gravitino multiplets have the following spin content⁷⁾

$$\bar{J} = \frac{3}{2}, (4) \bar{J} = 1, (5+1) \bar{J} = \frac{1}{2}, (4) \bar{J} = 0 \quad (9)$$

and the number in parentheses denote the multiplicity of these spin states. Note that these multiplicities, for N extended massive multiplets, are given by antisymmetric $USp(2N)$ representations. [This explains the notation in Eq. (9)].

Since we must have two multiplets as given by (9) the $N = 4$ theory must contain, beyond the $N = 4$ graviton multiplet with helicities

$$\lambda = \pm 2, (4) \lambda = \pm \frac{3}{2}, (6) \lambda = \pm 1, (4) \lambda = \pm \frac{1}{2}, (2) \lambda = 0 \quad (10)$$

at least three massless vector multiplets with helicity content

$$\lambda = \pm 1, (4) \lambda = \pm \frac{1}{2}, (6) \lambda = 0 \quad (11)$$

Given the multiplets in (10) and (11) we see that they can give rise to the $N = 2$ gravity multiplet given by (1), two massive gravitinos multiplets given by (9) and one $N = 2$ scalar multiplet with helicity content

$$(2) \lambda = \pm \frac{1}{2}, (4) \lambda = 0 \quad (12)$$

Note that the two massive gravitinos can have different masses because they do not carry central charges. This is in contrast with the other choice that we will discuss in a moment. However, with the minimal multiplet assignment discussed now, we cannot further break $N = 2$ down to $N = 1$ since there is no massless vector multiplet left to provide the superHiggs effect of $N = 2$ down to $N = 1$.

We now consider the different possible assignments of $N = 2$ massive gravitinos, namely to massive $J = 3/2$, $N = 2$ multiplets with central charges³⁾.

In this case the two massive gravitinos of the $N = 4$ theory broken to $N = 2$ form a single $N = 2$ multiplet with central charge given by

$$2 (\bar{J} = \frac{3}{2}, (2) \bar{J} = 1, \bar{J} = \frac{1}{2}) \quad (13)$$

The central charge transformation rotates these two multiplets into each other. In this case, one can check that the minimal Higgs structure of the $N = 4$ theory is given by the gravity multiplet of Eq. (10) and a single $N = 4$ massless vector multiplet given by (11). With these two multiplets the $N = 2$ unbroken theory contains the $N = 2$ gravity multiplet given by (1), the complex massive gravitino multiplet given by (13) with two gravitinos degenerate in mass, and two further $N = 2$ vector multiplets given by Eq. (2).

In contrast to the previous case with no central charges, this set of multiplets is suitable for breaking $N = 2$ down further to $N = 1$, since one of the vector multiplets left can act as superHiggs multiplet to give an $N = 1$ supersymmetric mass to a third gravitino contained in the $N = 2$ gravity multiplet given by (1). This analysis can be extended to any higher N theory and can give necessary conditions for all patterns of supersymmetry breaking in these theories. For $N > 4$ the breaking pattern is even more restricted since we cannot add any longer independent matter multiplets and the goldstinos can only come from the spin-1/2 states of the gravity multiplet itself.

As an illustration, let us consider the most interesting case, namely the maximally extended $N = 8$ theory. In Ref. 3) it was shown that this theory can undergo spontaneous supersymmetry breaking with $N = 6, 4, 2$ or 0 supersymmetries left unbroken⁸⁾. The reason why only an even number of supersymmetry is left unbroken is due to the fact that massive gravitinos are assigned to massive multiplets with central charges so they must be pairwise degenerate in mass.

This situation is unavoidable if we break $N = 8$ down to $N = 6$ or 4 since in these cases, massive $J = 3/2$ multiplets can only exist with a non-vanishing central charge. However, if one breaks the $N = 8$ theory down to $N = 2$ it is possible to contemplate a different multiplet structure for the $N = 2$ massive gravitinos. They can be assigned in this case to $N = 2, J = 3/2$ massive multiplets with no central charges.

In this case, if we start with the $N = 8$ gravity multiplet with helicity content given by

$$\lambda = \pm 2, (8) \lambda = \pm \frac{3}{2}, (28) \lambda = \pm 1, (56) \lambda = \pm \frac{1}{2}, (70) \lambda = 0 \quad (14)$$

the $N = 2$ multiplets which are left are the following:

- a) The $N = 2$ gravity multiplet [Eq. (1)] and six $N = 2, J = 3/2$ massive multiplets [Eq. (9)].
- b) $(3-k) N = 2, J = 1$ massless vector multiplets [Eq. (2)], $k N = 2, J = 1$ massive vector multiplets ($J = 1, (4) J = 1/2, (5) J = 0$) and $(4-k)$ scalar multiplets [Eq. (12)] with $0 \leq k \leq 3$.

With this multiplet assignment the six massive gravitinos can have all different masses and further, due to the presence of $N = 2$ vector multiplets, it is in principle possible to further break $N = 2$ down to $N = 1$, as in the first example of this paper.

The $N = 8$ theory, broken to $N = 1$, can have the following multiplet assignment

$$\begin{aligned} & (2, \frac{3}{2}) + (7) \left(\frac{3}{2}, (2) 1, \frac{1}{2} \right) + (14-K) (1, \frac{1}{2}) \\ & + K (1, (2) \frac{1}{2}, 0) + (28-K) \left(\frac{1}{2}, (2) 0 \right) \end{aligned} \quad (15)$$

$K=0, \dots, 14$

Note that for $k = 14$ the degeneracy of these multiplets corresponds to irreducible representations of the exceptional group G_2^* . One can also consider the breaking of $N = 8$ supergravity to $N = 2$ with central charge massive multiplets. This breaking pattern is explicitly realized in the models considered in Ref. 3) and discussed in Ref. 8). If one first decomposes the $N = 8$ multiplet into $N = 2$ massless multiplets one finds the following multiplet structure:

$$\begin{aligned} & (2, (2) \frac{3}{2}, 1) + 6 \left(\frac{3}{2}, (2) 1, \frac{1}{2} \right) + \\ & 15 (1, (2) \frac{1}{2}, (2) 0) + 10 \left((2) \frac{1}{2}, (4) 0 \right) \end{aligned} \quad (16)$$

The multiplicities 6, 15, 10 of these multiplets correspond to irreducible totally antisymmetric $SU(6)$ representations. If one assumes that all massive multiplets have non-vanishing central charges, a possible set of multiplets, after the breaking, is

$$\begin{aligned} & (\pm 2, (2) \pm \frac{3}{2}, \pm 1)_{m=0} + 6 \left(\frac{3}{2}, (2) 1, \frac{1}{2} \right)_{m \neq 0} \\ & + (14-2\ell) (1, (2) \frac{1}{2}, 0)_{m \neq 0} + (1+2\ell) (\pm 1, (2) \pm \frac{1}{2}, (2) 0)_{m=0} \quad (17) \\ & + 7 \left((2) \pm \frac{1}{2}, (4) 0 \right) \end{aligned} \quad 0 \leq \ell \leq 7$$

*) The group G_2 has also been found in connection with the spontaneous compactification of the 11 dimensional supergravity down to four dimensions⁴⁾.

Since central charges can only act on pairs of multiplets one has always at least one massless vector multiplet left^{*)}.

For $\ell = 1$ one reproduces the breaking pattern of Ref. 3). For $\ell = 0$ the multiplicities 6, 14 and 7 correspond to $USp(6)$ irreducible traceless antisymmetric representations T^i , T^{ij} , T^{ijk} (anti-self-dual) respectively. Whether the corresponding field theory, supposing it does exist, would have an internal $USp(6)$ symmetry, is an open question.

From our previous consideration, it is clear that our analysis, being purely algebraic, gives only necessary conditions for the pattern of possible hierarchical breaking of supersymmetries. Whether such breaking patterns are possible depends on the theory itself.

In extended theories with $N \leq 4$ it should be possible to have less restrictions since we can add independent matter multiplets to the pure supergravity Lagrangian. For $N > 4$ the breaking pattern is probably very restricted since the theories are self-coupled.

An interesting problem is the relation between the Kaluza-Klein approach⁴⁾ and the method discussed in the present article.

Since our approach, being purely algebraic, describes in principle the most general breaking such that at least one supersymmetry is left unbroken, it seems likely that the Kaluza-Klein approach only describes a subset of all possible breaking patterns.

^{*)} This massless vector multiplet is suitable to further break $N = 2$ down to $N = 1$ supersymmetry.

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