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Optical Model Analysis of Intermediate Energy p-⁴He Scattering*

J.M. Greben and R. Courishankar

Physics Department, University of Alberta
Edmonton, Alberta, Canada T6G 2J1

Abstract: Recent Wolfenstein R-parameter data are used to explain and resolve previous problems with optical model descriptions of p-⁴He elastic scattering at 500 MeV. An essential component in this optical model analysis is a qualitative interpretation of different features of the elastic data in terms of the Born approximation. First we show that the R-data require the real spin-orbit potential to have certain geometrical properties which were missing in previous analyses. We can then show that the fast fall-off of the cross-section for small angles, together with the rapid increase and subsequent decrease of the polarization, establishes the need for an attractive tail in the real central potential. Other properties of the potentials can also be inferred from this qualitative analysis in particular a strong reduction of the spin-orbit potential. Our final potential gives a reduction of the χ^2 /datapoint by about 20 in comparison to previous potentials, and underlines the usefulness of the qualitative Born analysis.

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I. Introduction

In many respects ⁴He is an ideal target for studying microscopic theories of elastic scattering. It is the smallest magic nucleus, has no lowlying collective states, and provides a good laboratory for studying 1/A effects, as A is relatively small. Therefore, it is not surprising that many experimental and theoretical studies of ⁴He have been performed. Early theoretical work^{1,2)} was aimed at describing cross-section data around 1 GeV³⁾. Unfortunately, this theoretical work was hampered by the lack of knowledge of nucleon-nucleon amplitudes and the absence of polarization data. More recent measurements in this energy range^{4,5)} do in fact show that the large np spin-flip amplitudes used in this early work are inconsistent⁶⁾ with the polarization data. The situation in the 1 GeV region is also obscured by the incompatibility of different cross-section measurements.

At lower energies the experimental situation is more favourable. Neutron-proton scattering has been investigated⁷⁾ in detail between 200 and 500 MeV, and extensive phase shift analyses are available up to 800 MeV⁸⁾. Cross-section and polarization data for p-⁴He scattering are available over the whole angular range at 200, 350 and 500 MeV⁹⁾ and the Wolfenstein spin-rotation parameter R has been measured as well¹⁰⁾. The theoretical situation is also more favourable in a number of respects: not only are there smaller uncertainties in the NN-amplitudes, but relativistic and Δ -isobar effects⁶⁾ should also be less important in this energy range. Therefore, the data present a strong challenge to any microscopic theory, as the input for the theory can not easily be adjusted to fit the data. Despite these advantages very few microscopic calculations have been reported in this energy range since Kujawski¹⁾ published his results for 630 MeV in 1970. This might indicate that the simplest Glauber or multiple scattering calculations have so

far met with little success and that higher-order terms are too important to be neglected. For heavier nuclei, nuclear medium effects have been shown to be important in this energy range¹¹⁾, so that these effects may also complicate the description of ⁴He in this energy range.

In view of the likely complexity of the microscopic calculations it seems appropriate if one first acquires a better understanding of the process using methods which are closer to the data, in particular now that R-parameter data are available at 500 MeV. Clearly, a phase-shift analysis is rather impractical at these energies. An amplitude analysis appears more feasible, however, even if the R-data were highly accurate (which they certainly are not), such an analysis would not be able to determine the overall (angular-dependent) phase of the amplitude, and would be hampered by one undetermined relative sign. A physically more interesting and also less ambiguous way to describe the data is through an optical potential. An optical model description automatically satisfies unitarity and imposes further quite reasonable constraints on the amplitudes by requiring that the potential does not vary rapidly in its argument(s) and only changes slowly with energy. It also enables one to account for Coulomb effects in a simple way. One problem in such a description is the parameterization of the potential: it should be flexible enough to enable a good description of the data, yet not so flexible as to violate the constraints above or to result in a number of undetermined and/or dependent parameters. Previous optical model fits have been quite unsuccessful^{10,12,13)}, which probably indicates that the parameterization in terms of a local optical potential with Fermi formfactors is too restrictive. Since the parameterization of non-local potentials is highly arbitrary (perhaps with the exception of a $(-1)^{\ell}$ dependent exchange force), and since one expects that the medium and long range components of the potential can

reasonably well be represented by a local form, we will restrict ourselves to a local potential. Evidence for non-local effects can probably best be obtained by studying the (equivalent) local potential: oscillatory features or a strong increase of the potential in the interior may hint at these effects. Since we limit ourselves to the angular range over which R has been determined ($\theta_{c.m.} < 70^\circ$), we do not expect to be sensitive to non-local effects of very short-range, although our analysis could reveal non-local effects of intermediate range.

In order to avoid overparameterization and the problem of discrete ambiguities, which could seriously curtail the significance of numerical fits, we have first analyzed the data in a qualitative way. The qualitative analysis was performed by using the Born approximation. Within this approximation one can link important features of the data directly to properties of the different components of the potential. These associations appear to persist in the full calculation, so that the Born approximation proved to be a very good medium for interpreting the data. The qualitative analysis does not only provide a proper starting point for the quantitative analysis, but it also determines through a chain of rigorous arguments which features of the data play an essential role in determining certain properties of the potential, i.e. it enables one to determine the experimental signatures for these properties.

In order to demonstrate the validity of the qualitative arguments we have to go through a fairly lengthy analysis in Section 3. The potentials, obtained after the quantitative search are discussed in the subsequent section and compared to those obtained in a previous fit¹⁰⁾. In the final section we summarize our main conclusions and discuss the importance of the R-data and the Born approximation in the analysis.

II. General Formulation

The general expression for the scattering amplitude in spin 1/2-spin 0 scattering is

$$M = f(q) + \vec{\sigma} \cdot \vec{n} g(q) \quad (1)$$

where q is the momentum transfer: $q = 2k \sin(\theta/2)$, with k the center-of-mass (c.m.) momentum and θ the c.m. angle. The vector $\vec{n}$ is the normal to the scattering plane:

$$\vec{n} = \frac{\vec{k} \times \vec{k}'}{|\vec{k} \times \vec{k}'|} \quad (2)$$

The elastic observables are:

$$\frac{d\sigma}{d\Omega} = \left(\frac{m}{2\pi\hbar^2} \right)^2 I_0, \quad I_0 = |f|^2 + |g|^2, \quad (3)$$

$$P = 2\text{Re}(fg^*)/I_0, \quad Q = 2\text{Im}(fg^*)/I_0, \quad (4)$$

$$R = \cos \theta_R (|f|^2 - |g|^2)/I_0 + 2 \sin \theta_R \text{Im}(fg^*)/I_0, \quad (5)$$

and

$$A = 2 \cos \theta_R \text{Im}(fg^*)/I_0 - \sin \theta_R (|f|^2 - |g|^2)/I_0, \quad (6)$$

where θ_R is given in Appendix A and reduces to the lab angle in the non-relativistic limit. The cross-section, polarization and R-parameter are expressed in terms of four quantities: $|f|^2$, $|g|^2$, $\text{Re}(fg^*)$ and $\text{Im}(fg^*)$.

Since $\text{Re}(fg^*)^2 + \text{Im}(fg^*)^2 = |f|^2 |g|^2$, only three of the four are independent. Therefore, cross-section, polarization and R together do not only determine $|f|^2 + |g|^2$ and $\text{Re}(fg^*)$, but also $\text{Im}(fg^*)$ and $|f|^2 - |g|^2$, although for the latter quantities two discrete solutions are found. Similar discrete ambiguities occur if a different trio of observables is analyzed. The discrete ambiguity can only be removed by measuring a fourth observable.

The local optical potential is written as follows:

$$U = V_{\text{Coul}}(r) - V(r) - iW(r) + 2 \left(\frac{\hbar}{m c} \right)^2 \vec{k} \cdot \vec{s} \frac{1}{r} \frac{d}{dr} \{ V_{\text{SO}}(r) + iW_{\text{SO}}(r) \}. \quad (7)$$

In Born approximation we obtain (we ignore the Coulomb interaction)¹⁴⁾:

$$f(q) = -V(q) - iW(q), \quad (8)$$

and

$$g(q) = -i \left(\frac{\hbar k}{m c} \right)^2 \sin \theta \{ V_{\text{SO}}(q) + iW_{\text{SO}}(q) \}. \quad (9)$$

The Fourier transform is indicated by the same symbol as the untransformed function and is defined by

$$V(q) = \int d\vec{r} \int_0^\infty (q \cdot \vec{r}) V(r) = \frac{4\pi}{q} \int_0^\infty dr r \sin(qr) V(r). \quad (10)$$

Notice that the Fourier transform of the spin-orbit formfactor, not the transform of the spin-orbit potential, appears in $g(q)$! In the qualitative analysis we will frequently refer to the explicit expressions for the observables in Born approximation:

$$\begin{aligned} \frac{d\sigma}{d\Omega}(\theta) &= \left(\frac{m}{2\pi\hbar^2} \right)^2 I_0 \\ &= \left(\frac{m}{2\pi\hbar^2} \right)^2 \{ V(q)^2 + W(q)^2 + \left(\frac{\hbar k}{m c} \right)^4 \sin^2 \theta \{ V_{\text{SO}}(q)^2 + W_{\text{SO}}(q)^2 \} \}, \end{aligned} \quad (11)$$

$$P(\theta) = 2 \left[\frac{\hbar k}{m c} \right]^2 \sin \theta \{ -V(q)W_{\text{SO}}(q) + W(q)V_{\text{SO}}(q) \} / I_0, \quad (12)$$

and

$$\begin{aligned} R(\theta) &= \cos \theta_R \{ V(q)^2 + W(q)^2 - \left(\frac{\hbar k}{m c} \right)^4 \sin^2 \theta \{ V_{\text{SO}}(q)^2 + W_{\text{SO}}(q)^2 \} \} / I_0 \\ &\quad - 2 \left[\frac{\hbar k}{m c} \right]^2 \sin \theta_R \sin \theta \{ V(q)V_{\text{SO}}(q) + W(q)W_{\text{SO}}(q) \} / I_0. \end{aligned} \quad (13)$$

The analysis in terms of the Born approximation is simplified by the availability of analytic expansions for the Fourier transform of the Fermi formfactor and of the other formfactors used in this analysis: the Gaussian formfactor and

the parabolic Fermi formfactor. These expressions, together with the formulae for the volume integrals and r.m.s. radii, are given in Appendix B.

III. Optical Model Analysis

In this section we will discuss our qualitative and quantitative analysis of the cross-section (Fig. 1), polarization (Fig. 2) and R-data (Fig. 3). In the qualitative analysis it is important to interpret the most conspicuous features of the data first, since they probably have the simplest and most unique interpretation. Once some general features of the potential are determined we have a better chance at interpreting finer details of the data, and can pursue a more quantitative analysis. To keep the analysis to a reasonable length we have occasionally suppressed arguments which strictly speaking are necessary to establish the conclusion(s). The arguments presented are the most important ones, emerging after the full analysis and its numerical verification.

In comparing the different observables we see that polarization and R have more structure than the cross-section, and that their structure also starts at lower momentum transfer. This structure is likely to be associated with zeros in the momentum space formfactors, and therefore will provide information on the geometry of the potentials. Consider the first zero in P at 30° . This zero coincides with a local maximum in R, while the cross-section has no structure in this region. In Born approximation this zero requires VW_{SO} to be equal to W_{SO} . In order to draw further conclusions from this equality we will assume that $|W(q)| \gg |V(q)|$ for small q. All optical model analyses at intermediate energies have indicated that the volume integral of the imaginary potential is much larger than that of the real potential, so that this is a reasonable assumption. We have to make such an assumption because the observables calculated in Born approximation are invariant under an overall phase

change of the potential so that the data can not establish this phase. Since the full Schrödinger equation is sensitive to the overall phase, the assumption will later be verified in the full calculation, and is found to be justified. Given the dominance of the imaginary potential, VW_{SO} can only equal W_{SO} if either $W(q)$ or $V_{SO}(q)$ is close to a zero. The behaviour of R near 30° tells us which possibility applies! Near 30° R is dominated by the first term in Eq. (13) since it is proportional to $\cos \theta_R$. As long as $W(q)$ is large this term will roughly behave like $\cos \theta_R$, however, as soon as $W(q)$ goes through zero the spin-orbit terms start to dominate ($V(q)$ is small) and it will behave more like $-\cos \theta_R$. Since R does not fall off near 30° , but rather has a local maximum there, the zero in P can not be due to $W(q) = 0$, and consequently must be due to $V_{SO}(q) = 0$. In previous fits^{10,12,13}, where R-data were not included in the analysis, the solution $W(q) = 0$ was preferred because this gives a better description of the small angle data if one employs standard Fermi formfactors for the potentials. Consequently, our new interpretation of the data near 30° will also lead to a reinterpretation of the small angle data.

The next important feature is the second zero of P at 39° , which coincides with a minimum in R and $d\sigma/d\Omega$. A zero in $W(q)$ can explain all three features: the minimum in $d\sigma/d\Omega$ because a zero in the dominant formfactor is likely to affect this observable; the zero in P because W_{SO} is small and can equal VW_{SO} ; and the minimum in R because the first term in R drops off rapidly when $W(q) \rightarrow 0$. Since $\sin \theta_R$ is not so small for $\theta = 39^\circ$, we may also want to consider the second term in R (Eq. 13). This term also will decrease rapidly when $W(q) \rightarrow 0$, because the dominant contribution $-W(q)W_{SO}(q)$ is positive and will vanish in this limit.

For $\theta > 39^\circ$ $|W(q)|$ will increase rapidly and therefore the first term in Eq. (13) will return to a positive value. However, the second term $-W(q)W_{SO}(q)$

will become large and negative unless $W_{SO}(q)$ changes sign, as well. Experimentally, R returns rapidly to a positive value, so that we conclude that W_{SO} changes sign around 39° , as well. Finally, one can attribute the maximum in P around 45° partly to the presence of a zero in $V(q)$ at this point.

Having established the position of the zeros of the formfactors, and thereby an important aspect of the geometry of the potentials, we can now proceed to look at the less conspicuous features at smaller angles. The behaviour at small angles is mainly determined by the volume integrals and r.m.s. radii of the potentials. For a typical potential (e.g. with Fermi shape) the position of the zero of the formfactor (abbreviated by q_0) is roughly inversely proportional to the radius of the potential ($q_0 \langle r^2 \rangle^{1/2} \approx 5 + 1.5$). This condition already imposes certain constraints on the r.m.s. radii. The first property we have to reproduce is the fast fall-off of $d\sigma/d\Omega$ for small angles. In previous fits^{10,13} this behaviour was implemented by having $W(q)$ change sign at relatively small q , although the resulting r.m.s. radii were large in comparison to the size of ${}^4\text{He}$ (2.1 fm in Ref. 10, and 2.2 fm in Ref. 13). However, we just determined that $W(q)$ does not change sign at $\theta = 30^\circ$, but rather at 39° , so that $W(q)$ will fall-off considerably slower. In addition, the real central formfactor does not change sign until $\theta = 45^\circ$, which normally would mean that it falls off even slower than $W(q)$. It thus appears that we have to generalize the standard description in a rather drastic way to account for the small angle behaviour. Let us therefore consider this behaviour in some more detail.

For small q one can expand the cross-section in the Born approximation according to

$$\left(\frac{d\sigma}{d\Omega}\right)_B \sim [V(0)^2 + W(0)^2] \left[1 - \frac{1}{6} q^2 \langle r^2 \rangle_{\text{eff}}^2 + O(q^4)\right], \quad (14)$$

where the effective r.m.s. radius is given by

$$\langle r^2 \rangle_{\text{eff}}^2 = \frac{V(0)^2 \langle r^2 \rangle_V^2 + W(0)^2 \langle r^2 \rangle_W^2 - \frac{3}{2} \left(\frac{\hbar k}{m c}\right)^2 \{V_{SO}(0)^2 + W_{SO}(0)^2\}}{V(0)^2 + W(0)^2}. \quad (15)$$

The r.m.s. radii of real and imaginary central potential are denoted by $\langle r^2 \rangle_V^{1/2}$ and $\langle r^2 \rangle_W^{1/2}$, respectively. Notice that the spin-orbit potentials reduce the value of $\langle r^2 \rangle_{\text{eff}}^2$. In order to reproduce the experimental fall-off, which corresponds to $\langle r^2 \rangle_{\text{eff}}^{1/2} = 1.9$ fm, we can first of all try to increase $\langle r^2 \rangle_W^{1/2}$ without changing the position of the zero at 39° . For a Fermi formfactor this can be accomplished by reducing the radius and increasing the diffuseness in a controlled fashion. However, such a formfactor has various undesirable features (small secondary maximum at 46° ; R too low for $\theta > 40^\circ$), so that we found it necessary to introduce a mixture of two formfactors: a Fermi formfactor for the large angle features and Gaussian formfactor for modifying the small angle characteristics. Since the value of $\langle r^2 \rangle_W^{1/2}$ is still limited by the physical size of ${}^4\text{He}$ and by q_0 , these modifications can only bring about a moderate increase in $\langle r^2 \rangle_{\text{eff}}^2$. Obviously, we have to improve on the other terms as well. We can increase the value of $\langle r^2 \rangle_V^{1/2}$ considerably by introducing an attractive tail. If the configuration space potential changes sign then $\langle r^2 \rangle_V^{1/2}$, as defined by the expansion of the formfactor, can no longer be considered as the physical r.m.s. radius, and in fact can take on any value between $-\infty$ and $+\infty$, depending on the potential. For a suitable attractive tail $\langle r^2 \rangle_V^{1/2}$ can thus become much larger than the radius of ${}^4\text{He}$, and thereby improve $\langle r^2 \rangle_{\text{eff}}^2$. Finally we can increase $\langle r^2 \rangle_{\text{eff}}^2$ by reducing the spin-orbit potentials.

We will now consider the polarization to see whether these modifications are acceptable. For a strong enough attractive tail, $V(q)$ will be positive for small q . Under these circumstances the two terms in the Born approximation for P (Eq. 12) will be constructive. In order to obtain the same polarization

we will thus have to reduce the spin-orbit potentials when an attractive tail is present. This is exactly the change we needed to improve $\langle r \rangle_{\text{eff}}^2$. The reduction of the spin-orbit potentials also enables us to improve the description of data at larger angles, where the spin-orbit formfactors become more important because of the $\sin \theta$ - dependence (e.g. the minimum in the cross-section at 39° would be filled up by the real spin-orbit contribution if this formfactor is too large). We can also turn this argument around and observe that a steep slope of the experimental polarization near 0° could signal the necessity of an attractive tail. Such evidence would even be stronger if we also observe that the polarization reaches its first zero at relatively small q : near this zero $V(q)$ will be negative again and the destructive nature of the two terms in P will shift the zero of P below that of $V_{SO}(q)$. In the present case the small angle data together with some properties of the spin-orbit potentials provide ample evidence for an attractive tail. We have parameterized this tail with a parabolic Fermi formfactor (wine-bottle Wood-Saxon), but the specific shape of the formfactor is not so essential.

On the basis of our present understanding we could continue to look at more detailed features and spell out further evidence for our interpretation of the data. However, since we have already drawn the main conclusions we will now move on to discuss the numerical analysis. Since we have obtained analytical expansions for all formfactors (Appendix B) we could perform the Born approximation calculations with great ease. The different contributions to $d\sigma/d\Omega$, P and R (Eqs. 11-13) could be analyzed separately so as to obtain a better insight in the sensitivities of these terms to various parts of the potentials. This possibility is unique for the Born approximation. In the full calculation all contributions are mixed up and the role of different components of the potentials is much harder to establish.

The quantitative determination of the potential parameters was done with the search program SEEK¹⁵⁾, which was extended to account for the additional formfactors¹⁶⁾. The χ^2/point is considerably better than in the previous fits. For the cross-section the χ^2/point is 5 (119 in Ref. 10, 227 in Ref. 17), for the polarization it is 2 (31 in Ref. 10, 144 in Ref. 17) and for the R-parameter it is .9 (125 in Ref. 10). The similarity of the full calculation and the Born approximation calculation can perhaps be understood through a crude unitarity argument. In the K-matrix approximation one writes the partial wave amplitude as $(V+iW)/\{1-i\pi\rho(V+iW)\}$, where ρ is a phase space factor which might also account for absorption. Since W is dominant (and positive) the major modification of the Born amplitude ($V+iW$) would be a reduction of magnitude. Since a renormalization of the amplitudes does not affect the polarization or the shape of the cross-section this argument may explain why the Born approximation predicts the trends in the data so well. We should caution, however, that this argument ignores specific spin-orbit effects and oversimplifies the transition between the partial wave and full amplitudes.

IV. Discussion of Optical Potentials

First we discuss the central potentials which are shown in Fig. 4. The real potential consists of a repulsive component with Fermi shape, and an attractive tail, parameterized by a parabolic Fermi (PF) formfactor. The relevant strength and geometrical parameters are given in Table I. The volume integral J_R of the real potential is quite sensitive to the cancellation between the repulsive and attractive components and is now negative (-124 MeV fm^3), rather than positive as in previous analyses (e.g. 99 MeV fm^3 in Ref. 10). Since the magnitude and nature of the attractive tail is determined by the small angle data (where the imaginary potential dominates) and by the spin-orbit potentials (which are more constrained by large angle data),

the accuracy with which J_R is determined depends very much on the overall quality of the fit. In previous analyses such an overall fit was not accomplished. For example, in a series of fits¹⁷⁾ between 560 and 1240 MeV, the polarization at small angles was consistently underestimated, while the first zero in this observable occurred at too large a momentum transfer. According to our previous analysis this behaviour could be cured by introducing a stronger attractive tail, so that we expect that the J_R given in Ref. 17 are too positive over the whole energy range. In order to obtain an error estimate of J_R one would have to use a completely unbiased parameterization of the potentials and perform a statistical analysis. Such a technical analysis, which does not emphasize the qualitative physical features of the potential, lies outside the scope of the present paper.

Because of the cancellation between attractive and repulsive components in the real potential it is rather meaningless to talk about an r.m.s. radius of the total potential. For 85 MeV the real potential was still attractive for all r and an r.m.s. radius of 2.1 fm was found¹³⁾. This value seems in reasonable agreement with the attractive component in our potential if we imagine that it follows the ${}^4\text{He}$ -density in shape. A problem in previous fits^{10,12,13)} using a single Fermi-formfactor was the small diffuseness parameter (e.g. $a_R = .008$ fm in Ref. 13). This is probably a reflection of the fact that in some sense the square well potential is the best approximation to the true potential within the constraints of Fermi-formfactors.

The imaginary central potential is shown in Fig. 4b. Since this potential dominates the cross-section at small angles, the volume integral J_I is not much different from previous results (-456 MeV fm³ vs. -483 MeV fm³ in Ref. 10 and -510 MeV fm³ in Ref. 13). The r.m.s. radius of the imaginary potential is 1.71 fm, considerably smaller than in previous analyses (2.07 fm in

Ref. 10 and 2.22 fm in Ref. 13). We note that the reaction cross-section, which is primarily sensitive to the imaginary potential, is only slightly reduced with respect to the earlier fits (9.4 fm² vs. 10.3 fm² in Ref. 10 and 11 fm² in Ref. 13).

Both the real and imaginary potential are large in the interior of the nucleus. Whether this behaviour persists at very short range is not determined by our analysis, since we limited ourselves to $\theta < 70^\circ$. The rapid increase of the potential in the interior may well signal non-local effects, just as in the NN-potential. Recent studies¹⁸⁾ of microscopic nucleon-nucleus potentials also indicate the presence of strong core components in the phase-equivalent local potentials. If this interpretation is correct then it means that a local representation of the short-range potential becomes less meaningful, so that there is little justification for extending the local optical model analysis to larger angles.

We now discuss the spin-orbit potentials, shown in Fig. 5. The quantities determining the strength of the spin-orbit formfactors for small angles are $(\hbar k/m_\pi c)^2 V_{SO}(0)$ and $(\hbar k/m_\pi c)^2 W_{SO}(0)$. These quantities are given in Table I under the heading of volume integrals. For larger angles, where the spin-orbit formfactors play a more important role in the cross-section, the strength would better be characterized by the maximum value of $k q V_{SO}(q)$ (cf. Eq. (9)). This definition does not lead to a simple expression, although it can reasonably well be approximated by $k V_{SO} R^2$ for a typical Fermi formfactor. In the literature¹⁹⁾ one often sees the volume integral of the spin-orbit potential defined as the volume integral of $\frac{1}{r} \frac{dV_{SO}}{dr}$. Although convenient to use (its value for the Fermi formfactor is simply $V_{SO} R_{SO}$), its physical meaning is not clear as it does not take into account the spin-orbit operator. For this reason Nadasen et al.¹⁹⁾, who made a survey of the spin-orbit volume

integrals, found it necessary to characterize the strength of the potential by a quantity different from the volume integral. They suggested the value of the potential at the surface, which is also proportional to $kV_{SO} R_{SO}^2$, and is less energy-dependent than the standard volume integral. Despite their limited relevance we have included the values of the standard volume integral and radius of the spin-orbit potential in the table, so as to facilitate the comparison with other surveys.

The spin-orbit potentials obtained in the present study are considerably weaker than those obtained in the previous analysis¹⁰⁾. As noted before, this reduction is related to the presence of the attractive tail in the real central potential. The range of the spin-orbit potentials is smaller than those of their central counterparts, which is probably due to the short-range nature of the nucleon-nucleon spin-orbit force.

Unfortunately there are no modern microscopic calculations to compare with. Our results are in broad qualitative agreement with the central microscopic potential of Kujawski¹⁾ at 630 MeV, however, in detail there are many differences, in particular in the spin-orbit potentials. An interesting result of his calculation is that the real central potential is still very attractive at 630 MeV. This could be seen as further evidence against the survey in Ref. 17, however, the shortcomings in his calculation (old NN amplitudes, severe approximations in the higher-order terms) make it hard to draw strong conclusions.

V. Discussion

Previous optical model fits^{10,12,13)} to proton-⁴He scattering at intermediate energies were rather unsuccessful. The fits to cross section and polarization data were of poor quality and required rather extreme geometries, while the predictions for R were in total disagreement with the data. The situation was not much different if the Dirac equation was used

instead of the Schrödinger equation²⁰⁾. The present study shows that the Wolfenstein R-parameter data can be of great value in understanding the true nature of the optical potential. Using a qualitative analysis based on the Born approximation, we could identify the problem in the previous fits. The poor predictions for R were due to the erroneous association of the first zero in the polarization with the diffraction minimum of the imaginary central potential rather than with that of the real spin-orbit potential. The R-data were also helpful in determining the diffraction minima of other components of the potential, like that of the imaginary spin-orbit potential. In summary, the R-data provide a lot of information on the geometries and the differences in geometry of the various components of the potential. This knowledge is critical in determining other qualitative aspects of the potential, as we now discuss.

The reduction of the r.m.s. radius of the dominant imaginary potential necessitated a reinterpretation of the sharp fall-off of the cross section at small angles. We showed that this fall-off, together with the steep increase and subsequent rapid decrease of the polarization at small angles, provided strong qualitative evidence for an attractive tail in the real central potential. Although there exists theoretical and experimental²¹⁾ evidence for an attractive tail, the present study appears to be the first one in which the experimental signatures for an attractive tail are identified explicitly, and the attractive tail is inferred from the data, rather than being a successful working hypothesis. Our Born approximation analysis shows that the presence of an attractive tail in turn leads to a strong reduction in the spin-orbit potentials, in our case by about 50%. The same phenomenon can be seen in a study of ⁴⁰Ca²²⁾. These results clearly demonstrate that in intermediate energy proton physics the shape of the potentials plays an important role, and that any study based on

pure Fermi-shapes should be considered with suspicion.

An interesting phenomenon of our central potentials is their large magnitude in the interior. This phenomenon has not yet been seen in phenomenological analysis for larger nuclei, however, this may be due to artificial constraints on the optical potentials. We suggest that this behaviour is caused by non-localities in the optical potential. Microscopic calculations are required to determine whether this core behaviour can be described in terms of standard multiple scattering theory, or whether delta-isobar or more exotic degrees of freedom are called for.

Since R-data provide mainly information on the geometry of the potentials, they are less essential for obtaining a good fit in those cases where the shape of the potentials are already known qualitatively. Apparently, this was the case for ^{40}Ca , where a fit to cross section and polarization²²⁾ lead to a good prediction of the rotation function Q. However, we disagree with the conclusion in Ref. 22, that the spin-rotation data contain little new information. The extent of information contained in data cannot be determined reliably from a constrained fit. It can be inferred from a qualitative analysis, such as we performed in Section 3, or in a statistical analysis of an unbiased fit²³⁾, where the mean square uncertainty in the potential is linked to the data set. Our qualitative analysis shows that R-data contain a lot of information. The importance of R-data is also stressed in a recent analysis of low energy nucleon-nucleus scattering²⁴⁾. The authors of that paper claim that only R-data can reduce the large (and often unrecognized) uncertainty in optical potentials.

Finally, we want to stress that the Born-approximation analysis is very useful in unravelling the nature of the potential. The qualitative features predicted by the Born approximation persisted in the full calculation as long as $\theta \leq 40^\circ$.

We also find that our qualitative statements apply equally well to the case of ^{40}Ca , e.g. in the explanation of the reduction of the spin-orbit potential when an attractive tail is present²²⁾. The large value of Q for small angles in the Schrödinger calculation of Ref. 22 can also be easily understood in Born approximation, and is due to the absence of an attractive tail.

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Appendix A Relativistic Formulation of R in Terms of the Scattering Amplitudes

We generalize Hoshizaki's discussion for the NN-system²⁵⁾ to the nucleon-nucleus case. First we give the non-relativistic definition of R.

Given the following vectors in the lab-system: $\hat{p}_i^L$, $\hat{p}_f^L$, $\hat{n} = \hat{p}_i^L \times \hat{p}_f^L / |\hat{p}_i^L \times \hat{p}_f^L|$, $\hat{s}_i = \hat{n} \times \hat{p}_i^L$ and $\hat{s}_f = \hat{n} \times \hat{p}_f^L$ (see Fig. 6) one defines R via the equation:

$$\langle \hat{O}_f^{(1)} \rangle_f = P \hat{n} + R \hat{p}_i^L \hat{s}_f + R' \hat{p}_i^L \hat{p}_f^L, \quad (A1)$$

where P is the polarization normal to the scattering plane and $\hat{p}_i^L$ is the polarization of the original beam in the direction $\hat{s}_i$. For $P_i = 1$ one obtains the following relations:

$$\left. \begin{aligned} R &= (1 - P^2)^{1/2} \cos(\theta_L - \beta), \\ \text{and} \\ R' &= (1 - P^2)^{1/2} \sin(\theta_L - \beta), \end{aligned} \right\} \quad (A2)$$

where β is the angle of rotation of the polarization vector in the scattering

plane. In order to calculate R non-relativistically one determines†

$$\left. \begin{aligned} K_x^{x'} &= \text{Tr}(M_{S_i}^{(1)} N_{S_f}^{(1)}) / \text{Tr}(NM^{\dagger}), \\ K_x^{z'} &= \text{Tr}(M_{S_i}^{(1)} M_{p_f}^{(1)}) / \text{Tr}(NM^{\dagger}), \end{aligned} \right\} \quad (\text{A3})$$

and

in the c.m. frame, makes the simple rotation to the lab-system and arrives at Eq. (5) with $\theta_R = \theta_L$.

According to Hoshizaki²⁵⁾ the spin-operator of the non-relativistic treatment can be equated to the relativistic spin operator, if the latter is defined in the particle rest-frame. Since $K_x^{x'}$ and $K_x^{z'}$ are calculated in the c.m. frame and R is measured in the lab-system we have to perform three consecutive Lorentz transformations to connect the particle rest-system related to the c.m. frame with the particle rest-system related to lab-frame.

For such successive Lorentz transformations with non-parallel velocities the spin operator is rotated over an angle which is smaller than the angle between the two velocities $\vec{p}_f$ and $\vec{p}_L$ (where $\vec{p}_f$ is the final c.m. momentum). Therefore, we have to determine the shift Ω :

$$\Omega = \theta - \theta_L - \mathcal{V}, \quad (\text{A4})$$

where $\mathcal{V}$ is the angle between the lab' component vector $\vec{p}_L^{(1)}$ and $\vec{p}_f$ as seen in the c.m. frame of the outgoing particle, and θ is the c.m. angle. For equal mass particles $\mathcal{V} = \theta_L$, but this is no longer true for unequal mass particles. Apart from this the discussion of Hoshizaki is not changed and we find:

† An extensive non-relativistic discussion of the polarization transfer coefficients is given in Ref. 26. The expressions for K_z^x and K_z^z in Table I in that reference appear to have the wrong sign.

$$\tan \mathcal{V} = \frac{\sin \theta}{\gamma(\cos \theta + \beta/\beta_{\text{cm}})}, \quad \gamma = (1 - \beta^2)^{-1/2}, \quad (\text{A5})$$

where β is the velocity of the outgoing particle in the c.m. frame and β_{cm} is the velocity of the c.m. frame in the lab. β_{cm} can easily be calculated as the difference of the velocity of the incoming particle in the lab (β_L^i) and the c.m. (β):

$$\beta_{\text{cm}} = \frac{\beta_L^i - \beta}{1 - \beta_L^i \beta}. \quad (\text{A6})$$

The angle θ_R in Eq. (5) is now defined by the equation

$$\mathcal{V} = \theta - \theta_R. \quad (\text{A7})$$

For completeness we also give the expression for the lab angle:

$$\tan \theta_L = \frac{\sin \theta}{\gamma_{\text{cm}}(\cos \theta + \beta/\beta_{\text{cm}})}. \quad (\text{A8})$$

Appendix B Analytic Expansions of Momentum Space Formfactors.

The Fourier transform of the Fermi formfactor

$$f_F(r) = [1 + \exp(\frac{r-R}{a})]^{-1}, \quad (\text{B1})$$

can be evaluated by means of a change of contours in the complex plane²⁷⁾.

The result is

$$f_F(q) = \frac{4\pi}{q} \phi(q), \quad (\text{B2})$$

where

$$\begin{aligned} \phi(q) &= \pi a R [-\cos(qR) + z \sin(qR) \coth(q\pi a)] / \sinh(q\pi a) \\ &\quad - 2a^3 q \sum_{n=1}^{\infty} \frac{n}{2^{2n} (n+q/a)^2} (-e^{-R/a})^n, \end{aligned} \quad (\text{B3})$$

and $z = \pi a/R$. For the parabolic Fermi formfactor

$$f_{PF}(r) = (r/R)^2 [1 + \exp(\frac{r-R}{a})]^{-1}, \quad (B4)$$

we obtain

$$f_{PF}(q) = \frac{4\pi}{q} \psi(q) = -\frac{4\pi}{qR^2} \phi(q), \quad (B5)$$

with

$$\begin{aligned} \psi(q) = & \pi a [- (1+3z^2) \cos(qR) + z(3+5z^2) \sin(qR) \coth(q\pi a) \\ & + 6z^2 \coth^2(q\pi a) \{ \cos(qR) - z \sin(qR) \coth(q\pi a) \}] / \sinh(q\pi a) \\ & - \frac{24a^5 q}{R^2} \sum_{n=1}^{\infty} \frac{n(n^2 - q^2 a^2)}{2^2 a^2} (-e^{-R/a})^n. \end{aligned} \quad (B6)$$

The volume integral of the Fermi formfactor

$$J_F = f_F(0) = \frac{4\pi R^3}{3} \{1 + z^2 + 6(\frac{a}{R})^3 e^{-R/a}\} + 0(e^{-2R/a}), \quad (B7)$$

and the r.m.s. radius

$$\langle r^2 \rangle_F = \frac{3}{5} R^2 \{1 + \frac{7}{3} z^2\} + 0(e^{-R/a}), \quad (B8)$$

are well-known. For the parabolic Fermi formfactor we find

$$J_{PF} = f_{PF}(0) = \frac{4\pi R^3}{5} \{ (1+z^2)(1 + \frac{7}{3} z^2) + 120(\frac{a}{R})^5 e^{-R/a} \} + 0(e^{-2R/a}), \quad (B9)$$

and

$$\langle r^2 \rangle_{PF} = \frac{5}{7} R^2 \frac{(1+6z^2 + \frac{31}{3} z^4)}{(1 + \frac{7}{3} z^2)} + 0(e^{-R/a}). \quad (B10)$$

Finally, for the Gaussian potential

$$f_G(r) = \exp\{-(r/b)^2\}, \quad (B11)$$

the Fourier transform is very simple:

$$f_G(q) = \pi^{3/2} b^3 \exp\{-(qb/2)^2\}. \quad (B12)$$

Its volume integral is $\pi^{3/2} b^3$, and its r.m.s. radius squared $3/2 b^2$.

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Table I

Parameters of the formfactors. The volume integrals of the spin-orbit formfactors include the factor $(\hbar k/m_\pi c)^2$. The numbers in brackets are the volume integrals (divided by $A^{1/3}$), and r.m.s. radii of the spin-orbit potentials.

Formfactor	Strength (MeV)	Radius (fm)	Diffuseness (fm)	J/A (MeV fm ³)	$\langle r^2 \rangle^{1/2}$ (fm)
V _F	-32.17	0.964	0.364	72.6	1.54
V _{PF}	12.27	2.167	0.415	-197	2.87
W _F	96.94	1.294	0.247	-299	1.36
W _G	20.50	1.80	-----	-166	2.20
V _{SO}	3.10	1.441	0.365	527 (70.7)	1.76(1.84)
W _{SO}	-3.51	1.273	0.247	-347 (-70.7)	1.35(1.49)

Figure Captions

- Fig. 1. Comparison of cross-section data⁹⁾ with present (—) and previous¹⁰⁾ (---) optical model calculations.
- Fig. 2. Comparison of analyzing power⁹⁾ with present (—) and previous¹⁰⁾ (---) optical model calculations.
- Fig. 3. Comparison of Wolfenstein R-parameter data¹⁰⁾ with present (—) and previous¹⁰⁾ (---) optical model calculations.
- Fig. 4. Real (a) and imaginary (b) central potential in MeV. Solid line is present potential made up from a Fermi formfactor (•••) and a parabolic Fermi or a Gaussian formfactor(---•••). Potential is compared to previous¹⁰⁾ Wood-Saxon potential(---).
- Fig. 5. Real (a) and imaginary (b) spin-orbit potentials (—). Result from previous analysis¹⁰⁾ (---) is shown as well.
- Fig. 6. Directions of polarization vectors in the scattering plane.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6