Ref.TH.3566-CERN

A MINIMAL $SU(2) \times U(1) \times U(1)$ ELECTROWEAK MODEL
WITH MIRROR FERMIONS

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A B S T R A C T

We present a minimal extension of the standard electroweak model, which accommodates mirror fermions, based on $SU(2) \times U(1) \times U(1)$. Mirror mixing happens through sterile neutrino states and induces radiative mixing for charged leptons. Quarks and mirror quarks are not mixed with each other, consistent with the suppression of flavour changing neutral currents. Higgs sector, fermion masses and neutral currents are discussed. In this scheme there can be a second Z boson as light as 0.2 TeV.

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1. - INTRODUCTION

The curious one-sided chiral character of Nature as seen in the weak interactions has remained a fundamental enigma ever since the parity violation was discovered. It was, however, emphasized already by Lee and Yang in their classical paper¹⁾ that an over-all left-right symmetry in a broad sense can be maintained if one conjectures the existence of mirror particles ("right-handed particles"). Recently, mirror fermions have attracted plenty of renewed attention for the reason that they seem to be unavoidable in many post SU(5) grand unified gauge theories aimed at family unification^{2),3)}. Among the multitude of new particles these models usually predict, mirror fermions are extraordinary for their relatively low mass. They are expected to be lighter than 250 GeV because their mass originates in the $SU(2)_W \times U(1)_Y$ breaking. Some of them might well lie in a range not much above the present experimental lower limit 20 GeV provided by PETRA. Mirror fermion models hence have the virtue of being experimentally verifiable or falsifiable in the very near future, probably in LEP.

It is, therefore, an interesting question to ask: what would a low-energy model that incorporates ordinary and mirror fermions look like? Up to now, no such model has really been worked out in detail, although several authors have considered the low-energy end of different family unified models⁴⁾. In this paper we propose a minimal extension of the Glashow-Weinberg-Salam (GWS) model which accommodates also mirror fermions. Since we are interested here only in the testable low-energy phenomena, we do not try to embed our model into a framework of any GUT, thus avoiding prejudices coming along with such scenarios.

The need to extend the standard electroweak gauge symmetry arises from the fact that otherwise one could construct direct gauge invariant mass terms that couple fermions with their mirror counterparts, clearly a prospect not to be cherished. The $SU(2)_W \times U(1)_Y$ invariant mass terms (e.g., $\bar{e}_R E_L + \text{h.c.}$, where E stands for mirror electron) are possible, because the left-handed ordinary fermion transforms similarly to the right-handed component of the corresponding mirror fermion, and vice versa [e.g., $e_R \sim E_L \sim (1, -2)$]. The solution we propose is to add to the GWS model an extra gauge symmetry $U(1)$ which prevents these terms, i.e., our electroweak model is based on $SU(2)_W \times U(1)_Y \times U(1)_U$. We will assume that fermion assignment under $SU(2)_W \times U(1)_Y$ is the same as in the GWS model, and consequently the electric charge is given by $Q = T_3 + Y/2$ as usual.

There are, of course, many different ways of defining the generator U of the extra $U(1)$ in order to forbid the appearance of the undesired mass terms. It is enough just to assign the fermion and its mirror counter part with unequal

U charge [e.g., $U(e_R) \neq U(E_L)$]. Limitations, however, arise from the need of cancelling the triangle anomalies⁵⁾ associated with the new hypercharge. One could, of course, postulate some very heavy fermions that take care of this cancellation, but a more attractive situation is that anomalies already vanish within the low mass world of ordinary fermions and mirror fermions. The relevant anomaly conditions are

$$\begin{aligned} \sum T_3^2 (U_L - U_R) &= 0, & \sum Q^2 (U_L - U_R) &= 0, \\ \sum Q (U_L^2 - U_R^2) &= 0, & \sum (U_L^3 - U_R^3) &= 0, \end{aligned} \quad (1)$$

where sums run over chiral fermions, and $T_{3L(R)}$ and $U_{L(R)}$ are the neutral generator matrix of $SU(2)_W$ and the generator matrix of $U(1)_U$, respectively, for the left(right)-handed fermions.

We define the U charge as follows:

$$\begin{aligned} U &= +Y && \text{for ordinary fermions,} \\ U &= -Y && \text{for mirror fermions.} \end{aligned} \quad (2)$$

For this U, the anomalies cancel within ordinary fermions and within mirror fermions separately (in each generation), as can be checked from (1). As a matter of fact, one can show that U proportional to Y is the only such solution of (1), if one requires (as we want to do) that the $SU(2)_W \times U(1)_Y$ sterile neutrino ν_R and its mirror counterpart N_L are neutral also in respect to $U(1)_U$ ⁶⁾. (For example, another evident way to forbid terms like $\bar{e}_R E_L$ is to define $U = +Y$ for the right-handed fermions and mirror fermions and $U = -Y$ for the left-handed ones, but it is easy to see that this would cause an U^3 anomaly.)

The classification of the electron family under $SU(2)_W \times U(1)_Y \times U(1)_U$ is:

$$\begin{aligned} \begin{pmatrix} \nu \\ e \end{pmatrix}_L &\equiv f_L \sim (2, -1, 1), & e_R &\sim (1, -2, -2), & \nu_R &\sim (1, 0, 0), \\ \begin{pmatrix} N \\ E \end{pmatrix}_R &\equiv F_R \sim (2, -1, -1), & E_L &\sim (1, -2, 2), & N_L &\sim (1, 0, 0). \end{aligned} \quad (3)$$

The states ν_R and N_L are optional since they do not play any rôle in the cancellation of anomalies. Their existence will, however, allow for the following interesting scenario. Since ν_R and N_L are neutral under the enlarged gauge group, we can still have a direct mass term connecting them with each other

$$m_1 \bar{\nu}_R N_L + \text{h.c.} \quad (4)$$

Thus mixing between ordinary and mirror fermions (mirror mixing) is not completely prevented by the imposed $U(1)_U$ symmetry. It is intriguing that as a consequence the model may yield a kind of hierarchical picture of mirror mixings suggested by a phenomenological analysis^{7),8)}. According to such studies, while large mirror mixings are possible for neutrinos, mixing of charged leptons is more restricted (a typical mixing angle ≤ 0.2 rad), whereas quark mirror mixing is because of the limits for the flavour changing neutral currents, stringently constrained ($\leq 10^{-4}$ rad). In our model the term (4) may cause considerable neutrino mixing and allows, as will be seen, for a radiative mixing between e and E . In contrast, the quark mirror mixing is forbidden to all orders in perturbation theory.

The only direct mass terms one can construct in addition to (4) are the non-mixing Majorana mass terms

$$m_2 \bar{\nu}_L^c \nu_R + m_3 \bar{N}_R^c N_L + \text{h.c.} \quad (5)$$

Allowing both (4) and (5) leads to an extended version of the so-called see-saw mechanism⁹⁾ for neutrino masses. With reasonable assumptions we find $m_{\nu_e} \approx 14$ eV, $m_{\nu_\mu} \approx 0.6$ MeV and $m_{\nu_\tau} \approx 170$ MeV.

We will show that the model is consistent with the neutral current (NC) data, provided $U(1)_U$ is broken at the scale $\gtrsim 0.5$ TeV. The mass of the new neutral gauge boson Z' can be as low as 0.2 TeV. There is also another possibility, to which we will refer occasionally, namely that the second Z boson is much lighter than $W^\pm$ and Z . In this case the gauge coupling constant of the $U(1)_U$ subgroup has to be suppressed in order to make the model consistent with the NC results.

2. - HIGGS SECTOR

Let us now go to the details of our model. The Higgs sector consists of three multiplets,

$$\phi_f \sim (2, 1, 1), \quad \phi_F \sim (2, 1, -1), \quad \phi_S \sim (1, 0, 1). \quad (6)$$

The isosinglet ϕ_S is introduced to break $U(1)_U$ at slightly higher energies than where ϕ_F (and ϕ_f) breaks $SU(2)_W \times U(1)_Y$ and to give a high enough mass for the corresponding gauge boson Z' . If $\langle \phi_S \rangle \ll \langle \phi_F \rangle$, an alternative picture results, in which the second Z remains very light [a similar situation occurs also in some supersymmetric models¹⁰⁾].

Fermion and mirror fermion masses are generated by ϕ_f and ϕ_F , according to the Yukawa couplings

$$\mathcal{L}_Y = g_Y \bar{f}_L \phi_f e_R + g'_Y \bar{f}_L \phi_f^c \nu_R + \tilde{g}_Y \bar{F}_R \phi_F E_L + \tilde{g}'_Y \bar{F}_R \phi_F^c N_L, \quad (7)$$

where $\phi_{f(F)}^c = i\tau_2 \phi_{f(F)}^*$ and the analogous quark terms are disregarded for simplicity. We can see from (7) that no mirror mixing is produced through the spontaneous symmetry breaking.

The most general Higgs potential is

$$V(\phi_f, \phi_F, \phi_S) = V_0 + V_1, \quad (8)$$

where

$$V_0 = \sum_{f, F, S} \left[\mu_i^2 \phi_i^\dagger \phi_i + \lambda_i (\phi_i^\dagger \phi_i)^2 \right] + \lambda_3 |\phi_f^\dagger \phi_F|^2 + \lambda_4 (\phi_f^\dagger \phi_f)(\phi_F^\dagger \phi_F) + [\beta_1 \phi_f^\dagger \phi_f + \beta_2 \phi_F^\dagger \phi_F] \phi_S^\dagger \phi_S, \quad (9)$$

$$V_1 = \beta_0 \left[\phi_f^\dagger \phi_F \phi_S^2 + \phi_F^\dagger \phi_f (\phi_S^\dagger)^2 \right].$$

One readily recognizes the automatic global symmetry

$$\phi_f \rightarrow e^{i\alpha_f} \phi_f, \quad \phi_F \rightarrow e^{i\alpha_F} \phi_F, \quad \phi_S \rightarrow e^{\frac{i}{2}(\alpha_f - \alpha_F)} \phi_S \quad (10)$$

obeyed by this potential. By suitably defining the transformation properties of the fermion fields, this is also a symmetry of the Yukawa Lagrangian (7). Owing to this, one can always rotate away the phases from the VEVs of ϕ_f and ϕ_F . Therefore, the VEVs yielding the desired breaking pattern can be chosen as

$$\langle \phi_f \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_f \end{pmatrix}, \quad \langle \phi_F \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_F \end{pmatrix}, \quad \langle \phi_S \rangle = e^{i\theta_S} \frac{v_S}{\sqrt{2}}. \quad (11)$$

Note that the potential $V(\phi_f, \phi_F, \phi_S)$ depends on the phase θ_S only through v_1 .

The stationary conditions for the vacuum are obtained by minimizing the potential (8). The outcome of the minimization is that

$$\sin 2\theta_S = 0. \quad (12)$$

This is a CP conserving solution, as can be verified by using arguments similar to those given in Ref. 11). If we wish to have spontaneous CP violation we should thus add more Higgses to our model.

For a finite range of parameters, (11) and (12) give a minimum of the potential (8). It is possible to have solutions where

$$v_f \ll v_F < v_S. \quad (13)$$

On phenomenological grounds v_S need not be much larger than v_F ; as we will see later, $v_S \geq 0(2)v_F$ may be enough. There are thus practically speaking two mass scales in the model: the one where the electroweak symmetry is broken down to $U(1)_{em}$, which at the same time is the mass scale of mirror fermions, and the other one associated with the masses of the ordinary fermions. We consider it as an advantage of our scenario that the hierarchy between fermion masses and the weak gauge boson (and mirror fermion) masses comes about naturally, without the artificial suppression of Yukawa coupling constant by several orders of magnitude, needed for example in the standard GWS model. Of course, our model does not provide any fundamental explanation as to why the parameters of the Higgs potential (9) happen to be such that the hierarchy $v_f \ll v_F$ results, i.e., that the weak gauge bosons are much heavier than ordinary fermions.

To ensure the existence of the hierarchy (13), we need to impose the condition $|\beta_0| \leq v_f^3/(v_F v_S^2)$. This is, however, a natural condition in 't Hooft's sense¹²⁾, since by letting $\beta_0 \rightarrow 0$ we increase the global symmetry of the Higgs sector from $U(1)^2$ to $U(1)^3$.

The smallness of β_0 has some interesting phenomenological consequences. There are four neutral Higgs scalars in the theory, along with a charged one (and its antiparticle). Two of the neutrals are very heavy with masses of the order of v_F and v_S . The remaining two in turn are relatively light, their

masses are proportional to $(-\beta_0 v_S^2 v_F / v_f)^{1/2} \approx O(v_f)$ if $|\beta_0| \approx O[v_f^3 / (v_F v_S^2)]$. This may set a lower limit on v_f . In Ref. 13), where a hierarchy of VEVs in a two Higgs version of the GWS model was assumed, a lower bound of 6 GeV was found for the light Higgs. As our model resembles this model we expect the lower bound also in our case to be of the order of few GeVs.

In this context it is important to notice that we cannot impose the hierarchy (13) naturally if $v_f \ll \alpha v_F$, where α is a typical gauge coupling of the theory. No fine tuning is needed, however, if $v_f \gtrsim O(\alpha v_F)$. If we adopt this standpoint, we should have $v_f \gtrsim O(1)$ GeV.

The charged Higgs mass matrix is

$$\mathcal{M}_H = \begin{pmatrix} \lambda_3 v_f^2 & -\beta_0 \frac{v_F}{2v_f} v_s^2 & -\lambda_3 v_f v_F + \beta_0 \frac{v_s^2}{2} \\ -\lambda_3 v_f v_F + \beta_0 \frac{v_s^2}{2} & \lambda_3 v_f^2 - \beta_0 \frac{v_f}{2v_F} v_s^2 \end{pmatrix}, \quad (14)$$

which gives for the mass of the charged physical Higgs $m_{H^\pm}^2 \approx \lambda_3 v_F^2 \approx O(10^4)$ GeV².

The rich Higgs structure of our model will give rise to many interesting phenomena which we will discuss elsewhere. Here we will just mention that there will be no Higgs induced flavour changing neutral currents in the model, since all ordinary quarks of the same charge obtain their masses from one Higgs.

3. FERMION MASSES

The charged fermion masses (m_f) and mirror fermion masses (m_F) are obtained from the Yukawa Lagrangian (7). With $v_f \approx O(\alpha v_F)$ these are related by

$$m_f \approx \frac{\tilde{g}_Y}{g_Y} O(\alpha) m_F \approx 10^{-2} m_F, \quad (15)$$

if $\tilde{g}_Y \approx g_Y$. Recalling that $m_F \approx 20 - 100$ GeV, this immediately gives the correct order of magnitude for m_f . No tuning of Yukawa couplings is needed.

The case of neutrino masses is more complicated. We will allow for all possible $SU(2)_W \times U(1)_Y \times U(1)_U$ invariant mass terms. That is to say, we include the mirror mixing term (4), the Majorana terms (5) as well as the Dirac

masses from (7). Assuming that all the mass parameters in the direct terms (4) and (5) are equal, i.e., $m_1 = m_2 = m_3 = M$, the neutrino mass matrix takes in the $(\nu_L, \nu_L^C, N_L, N_L^C)$ basis the form

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & \mu_1 & 0 & 0 \\ \mu_1 & M & M & 0 \\ 0 & M & M & \mu_2 \\ 0 & 0 & \mu_2 & 0 \end{pmatrix}, \quad (16)$$

where μ_1 and μ_2 are Dirac masses for neutrino and mirror neutrino, respectively. The natural scales would be $\mu_1 \approx O(v_F)$, $\mu_2 \approx O(v_F)$ and $M \approx O(v_S)$, provided v_S is the largest symmetry breaking scale, as we are presuming.

The diagonalization of (16) leads to four Majorana neutrinos, of which three are heavy and one is very light. The masses are approximately

$$\begin{aligned} m_1 &\approx \frac{\mu_1^2}{M}, \\ m_2 &\approx m_3 \approx \frac{1}{\sqrt{2}} \mu_2 \left(1 - \sqrt{2} \frac{\mu_2}{M}\right), \\ m_4 &\approx 2M, \end{aligned} \quad (17)$$

where the hierarchy (13) is assumed. We identify the first state (ν_1) with the ordinary neutrino. It is primarily a left-handed neutrino of the standard model but has also a mirror component suppressed by $O(\mu_1/M)$. If we take $v_S \approx 1$ TeV, $\mu_1 = g_Y' v_F$ and assume $g_Y = g_Y'$, (i.e., $\mu_1 = m_e$) we find that $m_{\nu_e} \approx 0.25$ eV. In order to meet with the result of the Moscow experiment ($14 \text{ eV} < m_{\nu_e} < 46 \text{ eV}$)¹⁴⁾ we should take $g_Y \approx 10^{-1} g_Y'$. If we adjust this relation so that $m_{\nu_e} = 14$ eV results for the electron neutrino mass and assume the same relation to hold for the other generations, too, we predict $m_{\nu_\mu} \approx 0.6$ MeV and $m_{\nu_\tau} \approx 170$ MeV.

The very heavy state $(\nu_4, m_4 \approx O(1) \text{ TeV})$ is mostly an equal mixture of ν_L^C and N_L . The two other massive states $(\nu_2$ and $\nu_3)$ couple to e_L only very weakly (with a suppression factor μ_1/μ_2), while they couple to E_R with nearly the full weak interaction strength. Therefore, if the mass m_E of the charged mirror lepton is considerably less than m_W , $W^\pm$ will decay into $E\nu_2$ or $E\nu_3$ pair. Also some mirror quarks may be lighter than $W^\pm$, so that there may be also new quark decay channels. Therefore, we expect $W^\pm$ decay width to be larger than in the GWS model.

The mixing of neutrinos and mirror neutrinos in (16) will induce mirror mixing in the charged lepton sector. This happens radiatively through $\phi_f^+ \phi_F^+$ mixing, as depicted in Fig. 1. The mixed scalar propagator can be read off from the matrix (14). For a small external momentum we find for the mixed mass term of the charged lepton and its mirror partner

$$\delta_{fF} \approx \frac{\lambda_3}{16\pi^2} \frac{\mu_1 \mu_2}{M} \ln \left(\frac{M^2}{\lambda_3 v_F^2} \right), \quad (18)$$

where we have used the fact that $\mu_1 = g_Y' v_f$ and $\mu_2 = \tilde{g}_Y v_F$ ^{*}). If we take $\mu_1 \approx O(m_f)$, $\mu_2 \approx O(m_F)$ and $\lambda_3 v_F^2 \approx O(m_W^2)$, we obtain the following upper limit for the mixing angle θ_{fF}

$$\theta_{fF} \approx \frac{\delta_{fF}}{m_F} \lesssim 3 \times 10^{-5} m_f \text{ (GeV)}. \quad (19)$$

The number in (19) would be two orders of magnitude larger, if we assume the relation $g_Y' \approx 10g_Y$, and the correspondingly $\tilde{g}_Y \approx 10g_Y$, as suggested by the neutrino mass considerations above. In any case the mixing angle is small enough to suppress mirror contributions to the anomalous magnetic moments of ordinary leptons beyond all observable limits¹⁵⁾. It is obvious that if the mixing mass parameter m_1 [Eq. (4)] goes to zero δ_{fF} vanishes.

Quarks and mirror quarks do not mix in our model at tree level. Nor are such mixings induced radiatively since the quark sector of the Lagrangian possesses an unbroken global "mirroriness" symmetry. This means that mirror quarks cannot decay into ordinary ones, in contrast to charged leptons which will decay very rapidly (with a rate $\sim m_F^5$) to ordinary fermions, even though the mixing angles are small. One interesting consequence is that there will be stable mirror matter containing some stable mirror quarks. The lack of quark/mirror-quark transitions also means that there will be no heavy mirror quark contributions to the flavour changing neutral currents, e.g., to the box diagrams of $K^0 - \bar{K}^0$ mass difference [for a discussion, see Ref. 15)].

^{*\phi_f^0 and ϕ_F^0 . As being very small compared with μ_1 , this contribution is, however, not included in the mass matrix (16).}

4. - NEUTRAL CURRENTS

Unification of the electroweak forces in our model is two-fold. The electromagnetic interactions are associated with the $SU(2)_W \times U(1)_Y$ part of the gauge group, whereas $U(1)_U$ is involved only in the weak interactions. There are two mixing angles, θ_W and ω , which we call electroweak and weak mixing angles, respectively. The photon field is given in terms of the former as $A = \sin\theta_W W_3 + \cos\theta_W B$, as in the GWS model, where W_3 and B are the neutral gauge fields of $SU(2)_W$ and $U(1)_Y$, respectively. The electroweak mixing angle can be expressed in terms of the corresponding coupling constants, g_W and g_Y , as $\sin\theta_W = g_Y/(g_W^2 + g_Y^2)^{1/2}$. The two neutral weak bosons with definite mass are

$$\begin{aligned} Z &= \sin\omega (\cos\theta_W W_3 - \sin\theta_W B) - \cos\omega D, \\ Z' &= \cos\omega (\cos\theta_W W_3 - \sin\theta_W B) + \sin\omega D, \end{aligned} \quad (20)$$

where D is the gauge field of $U(1)_U$. The masses and the value of the weak mixing angle ω can be obtained by diagonalizing the (W_3, B, D) mass matrix which for the VEVs (11) takes the form

$$\mathcal{M} = \frac{v_F^2 + v_f^2}{8} \begin{pmatrix} g_W^2 & -g_W g_Y & -g_W g_U t \\ -g_W g_Y & g_Y^2 & -g_Y g_U t \\ -g_W g_U t & -g_Y g_U t & g_U^2(1+r) \end{pmatrix}, \quad (21)$$

where $t = (v_F^2 - v_f^2)/(v_F^2 + v_f^2)$ and $r = 1 + 2v_S^2/(v_F^2 + v_f^2)$. Defining $p = g_U \cos\theta_W/g_W \equiv \eta \cos\theta_W$, we find

$$\begin{aligned} m_{Z(Z')}^2 &= \frac{1}{2} \frac{m_W^2}{\cos^2\theta_W} \left[1 + p^2 r \mp \sqrt{[1 - p^2 r]^2 + p^2 t^2} \right], \\ \tan 2\omega &= \frac{pt}{1 - p^2 r}, \end{aligned} \quad (22)$$

where $m_W = \frac{1}{2} g_W (v_F^2 + v_f^2)^{1/2}$ is the mass of the charged weak gauge boson. One eigenvalue of the matrix (21) is of course zero, corresponding to the photon field. It is important to remark that the hierarchy $v_f \ll v_F$ makes $t \approx 1$

and hence the connection between the $SU(2)_L \times U(1)_Y$ and $U(1)_U$ interactions depends directly on the unknown coupling constant g_u . Unless we put g_u negligibly small, there is a mixing with non-trivial consequences in the neutral current phenomenology.

The effective neutral current Lagrangian at low momentum transfer is given by

$$\begin{aligned} \mathcal{L}_{eff}^{NC} = & - \frac{g_w^2}{\cos^2 \theta_w} \left[\left(\frac{\cos^2 \omega}{m_{Z'}^2} + \frac{\sin^2 \omega}{m_Z^2} \right) (J_3 - \sin^2 \theta_w J_Q)^2 \right. \\ & + \rho^2 \left(\frac{\sin^2 \omega}{m_{Z'}^2} + \frac{\cos^2 \omega}{m_Z^2} \right) J_U^2 \\ & \left. + \rho \sin 2\omega \left(\frac{1}{m_{Z'}^2} - \frac{1}{m_Z^2} \right) (J_3 - \sin^2 \theta_w J_Q) \cdot J_U \right], \end{aligned} \quad (23)$$

where $J_3 = \bar{\psi} \gamma T_3 \psi$, $J_Q = \bar{\psi} \gamma Q \psi$ and $J_U = \bar{\psi} \gamma \frac{U}{2} \psi$ (ψ represents all fermions). In order to compare the model with data, we have determined theoretical expressions for the standard phenomenological parameters¹⁶⁾ of different low-energy processes. They are listed in the Table, where the following notations are used:

$$\begin{aligned} x &= \sin^2 \theta_w, \\ a &= \sin^2 \omega + \cos^2 \omega \left(\frac{m_Z}{m_{Z'}} \right)^2, \\ b &= \eta^2 \left[\cos^2 \omega + \sin^2 \omega \left(\frac{m_Z}{m_{Z'}} \right)^2 \right], \\ c &= \frac{1}{2} \eta \sin 2\omega \left[1 - \left(\frac{m_Z}{m_{Z'}} \right)^2 \right], \\ \rho &= \frac{m_W^2}{m_Z^2 \cos^2 \theta_w}. \end{aligned} \quad (24)$$

We can see that in the limit $v_S \gg v_F, v_f$, where $a \approx 1$, $b \approx c \approx 0$ and $\rho \approx 1$, the GWS model values of the parameters are recovered. From (22) it is obvious that ρ is always larger than one, that is, m_Z is lighter than in the GWS model. The second possible situation is, as we mentioned earlier, that Z becomes very light and Z' takes the rôle of the standard model Z boson. The expressions of the Table are still applicable in this case, provided a , b , c and ρ in (24) are redefined by changing the rôles of m_Z and $m_{Z'}^1$: a , b , and c should be multiplied by $(m_{Z'}/m_Z)^2$ and ρ replaced by $\rho' = m_W/(\cos^2 \theta_w m_Z^2)$.

The table is written by assuming that mirror mixing can be neglected and that mirror particles are heavy enough not to enter the processes considered. The mixing, possible only for leptons in our model, can be taken into account by replacing the parameter g_A by $g_A \cos 2\theta_e$, where θ_e is the eE mixing angle. Also, instead of κ we would have two neutrino-neutrino interaction parameters, $\kappa_V = \kappa$ and $\kappa_A = \kappa \cos 2\theta_e$, where θ_e is the $\nu_e N_e$ mixing angle. Note that the vector parameters g_V and κ_V are not affected by the mirror mixing, because the fermion and its mirror partner have identical vector couplings.

The most stringent bound resulting from the NC data concerns the value of ρ ; experimentally $|1 - \rho| \lesssim 0.03$ ¹⁷⁾. We use this number to determine a lower limit for the VEV v_S of the isosinglet Higgs field. In Fig. 2, ρ as well as ρ' , are plotted as functions of v_S for two reasonable values of $p = \frac{g_u \cos \theta_W}{g_W} = 1$ and $p = 0.5$. We have taken $v_F = 250$ GeV and neglected terms of the order of v_F/v_S . For $p = 1$ we can read from the figure the limits $v_S \geq 0.52$ TeV and $m_Z \geq 3.2 m_W / \cos \theta_W \approx 0.29$ TeV, and for $p = 0.5$, $v_S \geq 0.62$ TeV and $m_Z \geq 1.8 m_W / \cos \theta_W \approx 0.16$ TeV. We have assumed that $m_W / \cos \theta_W \approx 91$ GeV as in the standard model. With $v_S = 0.52(0.62)$ TeV and $p = 1(0.5)$ we obtain for the parameters a, b and c given in (24) $a = 0.996(0.995)$, $b = 0.12(0.12)$ and $c = 0.06(0.04)$. The values of the NC parameters listed in the Table will in these cases hence deviate at most 10-15% from their GWS values, which is of about the same size as is the present experimental inaccuracy for them¹⁷⁾. It could therefore be possible that the mass of the second Z boson is as low as about 200 GeV. In order to make a more explicit prediction, a full analysis of all NC data should, of course, be carried out. We will leave this for a further study.

5. - FINAL COMMENTS

In the case of a light Z boson, the parameters a, b and c each have a term proportional to the inverse square of the light mass m_Z which would make them too large. For b and c this increase is partly compensated by a decrease of g_u , which makes the overall factor η diminish. The situation changes and might become acceptable, however, when m_Z is much smaller than the typical momentum transfer q of the experiments considered, since instead of m_Z^{-2} we then have $q^{-2} + m_Z^{-2} \approx q^{-2}$. This fact was in another connection emphasized by Fayet¹⁰⁾ (in his model $m_Z \lesssim 300$ MeV would be allowed).

In Fayet's supersymmetric model¹⁰⁾ the extra $U(1)$ factor is introduced in order to make spin-0 sleptons and squarks heavier than their spin- $\frac{1}{2}$ partners. In the original version of the model the U charge assignment is $U(\psi_L) = -U(\psi_R)$ universally for all fermions within a family, which is essentially different from our assignment. Especially, no U charge can be zero since it would mean that the corresponding fermion and its supersymmetric counterpart were degenerate in mass. There is an interesting connection between this model and our model. We introduced the new $U(1)$ factor because of mirror particles, whereas in the supersymmetric model it is done because of the supersymmetric partners of the conventional particles. In the latter case, however, the U assignment leads to a triangle anomaly. To cure this, one has to add mirror particles also to this model^{10),18)}.

ACKNOWLEDGEMENTS

We thank D.V. Nanopoulos for reading the manuscript. Financial support from the Emil Aaltonen Foundation (K.E.) and the Particle Physics Committee, Finland (J.M.) is gratefully acknowledged.

Theoretical expressions for the Hung-Sakurai parameters

Parameter	$SU(2)_W \times U(1)_Y \times U(1)_U$	Standard GWS
κ	$\rho[a + b(1-x) + 2c\sqrt{1-x}]$	1
$2g_V$	$\rho[a(-1+4x) + 3b(1-x) + 2c(1+2x)\sqrt{1-x}]$	$-1 + 4x$
$2g_A$	$\rho[-a - b(1-x) - 2c\sqrt{1-x}]$	-1
α	$\rho[a(1-2x) - \frac{1}{4}b(1-x) - 2cx\sqrt{1-x}]$	$1 - 2x$
β	$\rho[a + b(1-x) + 2c\sqrt{1-x}]$	1
γ	$\rho[-\frac{2}{3}ax - \frac{1}{6}b(1-x) - \frac{2}{3}c(1+x)\sqrt{1-x}]$	$-\frac{2}{3}x$
δ	0	0
h_{VV}	$\rho[a(-\frac{1}{2}+2x)^2 - \frac{9}{4}b(1-x) + 3c(\frac{1}{4}-x)\sqrt{1-x}]$	$(-\frac{1}{2}+2x)^2$
h_{VA}	$\rho[a(-\frac{1}{4}+x) - \frac{3}{4}b(1-x) + c(-\frac{1}{4}+x)\sqrt{1-x}]$	$\frac{1}{4} - x$
h_{AA}	$\rho[\frac{1}{4}a + \frac{1}{4}b(1-x) + \frac{1}{4}c\sqrt{1-x}]$	$\frac{1}{4}$
$\tilde{\alpha}$	$\rho[a(-1+2x) + \frac{4}{3}b(1-x) + c(\frac{1}{3}+2x)\sqrt{1-x}]$	$-1 + 2x$
$\tilde{\beta}$	$\rho[a(-1+4x) + 3b(1-x) + 2c(1+2x)\sqrt{1-x}]$	$-1 + 4x$
$\tilde{\gamma}$	$\rho[\frac{2}{3}ax + \frac{1}{3}b(1-x) + \frac{1}{3}c(1+2x)\sqrt{1-x}]$	$\frac{2}{3}x$
$\tilde{\delta}$	0	0

REFERENCES

- 1) T.D. Lee and C.N. Yang, Phys. Rev. 104 (1956) 254.
- 2) F. Wilczek and A. Zee, Princeton preprint (1979);
M. Ida, Y. Kayama and T. Kitazoe, Prog. Theor. Phys. 64 (1980) 1745;
J. Maalampi and K. Enqvist, Phys. Lett. 97B (1980) 217;
R.N. Mohapatra and B. Sakita, Phys. Rev. D21 (1980) 1062;
K. Enqvist and J. Maalampi, Nucl. Phys. B191 (1981) 189;
H. Sato, Phys. Lett. 101B (1981) 233;
F. Wilczek and A. Zee, Phys. Rev. D25 (1982) 553.
- 3) N.S. Baaklini, Phys. Rev. D21 (1980) 343;
J. Chakrabarti, M. Popović and R.N. Mohapatra, Phys. Rev. D21 (1980) 3212;
C.W. Kim and C. Roisnel, Phys. Lett. 93B (1980) 343;
I. Bars and M. Günaydin, Phys. Rev. Lett. 45 (1980) 859;
Z.-q. Ma, T.-s. Tu, P.-y. Xue and X.-j. Zhou, Phys. Lett. 100B (1981) 399;
I. Umemura and K. Yamamoto, Phys. Lett. 100B (1981) 34;
M. Chaichian, Yu.N. Kolmakov and N.F. Nelipa, Phys. Rev. D25 (1982) 1377;
J.C. Pati, A. Salam and J. Strathdee, Phys. Lett. 108B (1982) 121.
- 4) I. Umemura and K. Yamamoto, Phys. Lett. 108B (1982) 37;
K. Yamamoto, Phys. Lett. 117B (1982) 297;
A. Masiero, M. Roncadelli and T. Yanagida, Phys. Lett. 117B (1982) 291;
K. Yamamoto, Kyoto University preprint RIFP-497 (1983).
- 5) S.L. Adler, Phys. Rev. 177 (1969) 2426;
J.S. Bell and R. Jackiw, Nuovo Cimento 60 (1969) 47.
- 6) V. Barger, E. Ma and K. Whisnant, Phys. Rev. D26 (1982) 2378.
- 7) K. Enqvist, K. Mursula, J. Maalampi and M. Roos, Helsinki University preprint HU-TFT-81-18 (1981);
J. Maalampi, K. Mursula and M. Roos, Nucl. Phys. B207 (1982) 233;
J. Maalampi and K. Mursula, Z. Phys. C - Particles and Fields 16 (1982) 83;
K. Enqvist, K. Mursula and M. Roos, Helsinki University preprint HU-TFT-82-51 (1982), to appear in Nucl. Phys. B.
- 8) S. Nandi, A. Stern and E.C.G. Sudarshan, Phys. Rev. D26 (1982) 2522.
- 9) T. Yanagida, in Workshop on the Unified Theory and the Baryon Number in the Universe, Proceedings, KEK-79-18 (1979);
M. Gell-Mann, P. Ramond and R. Slansky in Supergravity, eds. D.Z. Freedman and P. van Nieuwenhuizen (North-Holland, Amsterdam, 1980), p. 315.
- 10) P. Fayet, Phys. Lett. 69B (1977) 489; 70B (1977) 461 and talk given at the XVIIth Rencontre de Moriond (1982).
- 11) G.C. Branco, Phys. Rev. D22 (1980) 2901.
- 12) G. 't Hooft in Recent Developments in Gauge Theories, eds. G. 't Hooft et al., (Plenum Press, New York, 1980), p. 135.
- 13) H.E. Haber, G.L. Kane and T. Sterling, Nucl. Phys. B161 (1979) 493.
- 14) V.A. Lubimov et al., Phys. Lett. 94B (1980) 266.
- 15) See K. Enqvist, K. Mursula and M. Roos in Ref. 7).

- 16) P.Q. Hung and J.J. Sakurai, Phys. Lett. 63B (1976) 295.
- 17) I. Liede and M. Roos, Nucl. Phys. B167 (1980) 397;
J.E. Kim et al., Rev. Mod. Phys. 53 (1981) 211.
- 18) A.B. Lahanas and T.R. Taylor, Phys. Lett. 114B (1982) 132.

FIGURE CAPTIONS

- Fig. 1 : Radiative mirror mixing for the electron induced by the mixing of sterile neutrino ν_R and N_L .
- Fig. 2 : The ρ parameters for the two Z bosons, Z and Z', as the functions of the VEV v_S of the isosinglet Higgs field with two different values of $p = g_u \cos\theta_W / g_W$.

Fig. 1

Fig. 2