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ON THE LIGHT MESON MASS FORMULAE FROM QUANTUM CHROMODYNAMICS

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A B S T R A C T

We reconsider the Gell-Mann-Okubo light meson mass formulae using the moment sum rules ratios and taking into account non-leading quark masses and vacuum condensate effects. We also extract the strange quark mass value from the ϕ - ρ mass difference. The poorly known masses of the ($I = 0$, $J^{PC} = 0^{++}$) scalar mesons are discussed.

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1. - INTRODUCTION

One of the areas where outstanding progress has been made in extending the applicability domain of quantum chromodynamics (QCD) to the understanding of the meson properties, is the use of the Laplace (Borel) transform sum rules¹⁾ in the analysis of the two (three) point function of currents and current divergences. The main interesting features of the method are on the one hand the amplification of the lowest meson contribution to the sum rule at a moderate value of the sum rule scale of about 1 GeV due to the exponential factor, and on the other hand, the inclusion of non-perturbative terms via the operator product expansion (OPE) of the two (three) point Green function correlation functions. Recently, the moment sum rule ratio introduced in Ref. 1) in the analysis of the meson masses, has been formulated in the context of potential models²⁾ and has given encouraging results in the analysis of bottomium and charmonium systems. In this paper, we shall reconsider the well-known Gell-Mann-Okubo (GMO) meson mass formulae within such a sum rule. These mass formulae concern the mesons belonging to the same irreducible representation of $U(3)_F$ global symmetry according to the usual quark model scheme. We shall see in this paper how the quark and gluon dynamics can break such mass formulae in the real world where the $U(3)_F$ chiral symmetry is broken by quark masses and quark and gluon vacuum condensates. Our prototype method for such an analysis is the sum rule:

$$R\left(\frac{1}{M^2}\right) = \frac{d}{d(-1/M^2)} \log \int_0^\infty dt e^{-t/M^2} \frac{1}{\pi} \text{Im} \Pi(t) . \quad (1.1)$$

Here $\Pi(Q^2)$ denotes the two point correlation functions:

$$\Pi(Q^2) \equiv \int d^4x e^{iqx} \langle 0 | T J_H(x) (J_H(0))^\dagger | 0 \rangle , \quad (1.2)$$

associated with the hadronic current J_H , and the variable M^2 emerges applying, to both sides of the dispersion relation obeyed by $\Pi(Q^2)$, the Laplace transform operator:

$$\hat{L} \equiv \frac{(-1)^N}{(N-1)!} (Q^2)^N \cdot \frac{\partial^N}{(\partial Q^2)^N} \quad \cdot \quad (1.3)$$

$\lim_{\substack{Q^2 \rightarrow \infty \\ N \rightarrow \infty}} \frac{Q^2}{N} \equiv M^2$

As we can see, the interesting feature of the sum rule in Eq. (1.1) is its direct sensitivity to the meson mass, as one can easily verify when working in a zero width approximation. The optimization of the resonance contribution to the sum rule can be obtained according to the potential model criterion by requiring:

$$\frac{d}{d(-1/M^2)} \mathcal{R}(M^2) \Big|_{M^2 = M_0^2} = 0 \quad (1.4a)$$

and

$$-\frac{d^2 \mathcal{R}}{d(-1/M^2)^2} \Big|_{M^2 = M_0^2} \geq 0 \quad (1.4b)$$

The actual value of M_0 for light quark systems is not fixed with a good accuracy both from theory and the data. However, various applications of the SVZ idea which match the lowest meson pole dominance contribution to the spectral function and the quasi-perturbative or quasi-non-perturbative evaluation of the two point function indicate a value

$$M_0 \approx (0.8 \sim 1) \text{ GeV} \quad (1.5)$$

quasi-independent of the types of meson channels analyzed. So, it becomes reasonably quite good to equate various sum rules at the same M_0^2 and to take the lowest ground state contribution to the sum rule. In such a case, the difference of two sum rules suitably normalized is less dependent on the continuum effect as one can see if a QCD model for the continuum is used. Such continuum effect can become totally zero if one uses an $SU(3)_F$ symmetry for the continuum threshold and for the meson coupling constants. The former can be realized in the region where the continuum effect starts to show up while the latter is well verified experimentally.

2. - THE GMO MASS FORMULAE FOR VECTOR MESONS

In deriving the GMO mass formulae we shall assign the following quark content of vector mesons assuming an ideal mixing between the ϕ and ω mesons*):

*) -----
The smallness of the observed ϕ - ω mixing has been discussed in Ref. 1) within the QCD sum rules approach. The main contribution to such mixing comes from the four-fermion vacuum condensates which is the α_s^3 graph shown in Fig. 1. The contribution to the ϕ - ω two-point function is about $10^{-4} (0.6 \text{ GeV}^2/M^2)^3$ which is irrelevant in the following analysis.

$$(K^*)^- \equiv \bar{u}s, \quad \rho^+ \equiv \bar{u}d, \quad \phi \equiv \bar{s}s \quad (2.1)$$

We get the theoretical part of the sum rule which comes from the evaluation of the diagrams in Figs. 2 to 4^{*)}

$$R_{th}^\rho \approx M^2 \left\{ 1 - \frac{1}{9 \log^2 M/\Lambda} - \frac{2\pi}{3} \frac{\alpha_s \langle F^2 \rangle}{M^4} + \left(\frac{B_\rho}{M} \right)^6 \right\},$$

$$R_{th}^{K^*} \approx M^2 \left\{ 1 - \frac{1}{9 \log^2 M/\Lambda} + \frac{\bar{m}_s^2}{M^2} \rho_{K^*} \left(\frac{\bar{m}_s^2}{M^2} \right) - \right. \quad (2.2a)$$

$$\left. \frac{2\pi}{3} \frac{\alpha_s \langle F^2 \rangle}{M^4} \left(1 + 12\pi \frac{m_s \langle \bar{s}s \rangle}{\alpha_s \langle F^2 \rangle} \right) + \left(\frac{B_{K^*}}{M} \right)^6 \right\} \quad (2.2b)$$

and

$$R_{th}^\phi \approx M^2 \left\{ 1 - \frac{1}{9 \log^2 M/\Lambda} + \frac{\bar{m}_s^2}{M^2} \rho_\phi \left(\frac{\bar{m}_s^2}{M^2} \right) - \right. \quad (2.2c)$$

$$\left. - \frac{2\pi}{3} \frac{\alpha_s \langle F^2 \rangle}{M^4} \left(1 + 24\pi \frac{m_s \langle \bar{s}s \rangle}{\alpha_s \langle F^2 \rangle} \right) + \left(\frac{B_\phi}{M} \right)^6 \right\},$$

where we have worked in the chiral limit $m_u = m_d \sim m_\pi^2 \approx 0$; $\bar{m}_s$ is the running mass of the s quark which is defined to one-loop as ($\hat{m}_s$ is the invariant mass):

$$\bar{m}_s(M^2) = \frac{\hat{m}_s}{(\log M/\Lambda)^{4/9}} \quad (2.3)$$

for $SU(3)_C \times SU(3)_F$ and in the $\overline{MS}$ scheme^{**)} ; $\alpha_s \langle F^2 \rangle$ is the gluon condensate contribution, $\langle \bar{s}s \rangle$ is the quark vacuum condensate and B_i is the contribution of the dimension six-vacuum condensate which could be known if the factorization

^{*)} One has to notice that radiative corrections in the unit operator are smaller in the moment ratios than in the original Laplace transform sum rule. The non-perturbative effects, however, are more important in the former than in the latter.

^{**)} For a review, see, e.g., Ref. 3).

hypothesis¹⁾ has been used. The function ρ_i ($i = \phi, K^*$) is known to lowest order as^{*)}:

$$\rho_\phi\left(\frac{\bar{m}_s^2}{M^2}\right) \equiv \frac{\int_0^1 dx e^{-\frac{\bar{m}_s^2}{M^2 x(1-x)}}}{\int_0^1 dx x(1-x) e^{-\frac{\bar{m}_s^2}{M^2 x(1-x)}}} \quad (2.4a)$$

and

$$\rho_{K^*}\left(\frac{\bar{m}_s^2}{M^2}\right) \equiv \frac{\int_0^1 dx x e^{-\frac{\bar{m}_s^2}{M^2(1-x)}}}{\int_0^1 dx x(1-x) e^{-\frac{\bar{m}_s^2}{M^2(1-x)}}} \quad (2.4b)$$

The above functions are plotted in Fig. 5. In the region of M about 0.7 to 1 GeV, we can approximate them by:

$$\rho_\phi \approx \frac{5.40}{4.93} \quad \text{and} \quad \rho_{K^*} \approx \frac{2.26}{2.61} \quad \text{for} \quad \hat{m}_s \approx \frac{0.4}{0.2} \text{ GeV}. \quad (2.5)$$

We have used the value of $\hat{m}_s$ obtained recently^{4),5)}. The asymptotic behaviour of these functions is:

$$\rho_\phi \approx 2\rho_{K^*} \approx 6 \left\{ 1 - 2 \frac{\bar{m}_s^2}{M^2} \log \frac{M^2}{\bar{m}_s^2} + O\left(\frac{\bar{m}_s^2}{M^2}\right) \right\}. \quad (2.6)$$

Keeping the lowest meson contribution into the sum rule and equating various sum rules at the same M of the value about Eq. (1.5), we get**)

*) These expressions come from the expression of the two-point function $\pi(Q^2)$ which corresponds to the parametrization $\pi^{\mu\nu} \equiv -(g^{\mu\nu}q^2 - q^\mu q^\nu)\pi(Q^2) + g^{\mu\nu}D(q^2)$. The other parametrization $\pi^{\mu\nu} \equiv -(g^{\mu\nu}q^2 - q^\mu q^\nu)\pi(1) + q^\mu q^\nu \pi(0)$ leads to an expression of $\pi(1)$ different from $\pi(q^2)$ for unequal quark mass surrounding the internal loop [see, e.g., ref. 3)].

**) If we assume that the continuum threshold is $SU(3)_F$ symmetry, one can see using the QCD model for the continuum that their effects cancel when taking the difference of two sum rules.

$$M_\phi^2 - M_\rho^2 \approx \bar{m}_s^2 f_\phi - \frac{16\pi^2}{3} \frac{m_s \langle \bar{s}s \rangle}{M_0^2} + \frac{B_\phi^6 - B_\rho^6}{M_0^4} \quad (2.7)$$

and

$$M_{K^*}^2 - M_\rho^2 \approx \bar{m}_s^2 f_{K^*} - \frac{8\pi^2}{3} \frac{m_s \langle \bar{s}s \rangle}{M_0^2} + \frac{B_{K^*}^6 - B_\rho^6}{M_0^4} \quad (2.8)$$

i.e., the remaining pieces in Eqs. (2.7) and (2.8) are the flavour dependent ones^{*)}. From the above equations, it is easy to get the quadratic GMO mass formulae for vector mesons:

$$M_\phi^2 - M_\rho^2 \approx 2(M_{K^*}^2 - M_\rho^2) + \bar{m}_s^2 (f_\phi - 2f_{K^*}) + \frac{B_\phi^6 + B_\rho^6 - 2B_{K^*}^6}{M_0^4} \quad (2.9)$$

We see that in the $SU(2)_F$ chiral limit, the dimension four vacuum condensate preserves the GMO mass formulae. The quark mass breaks the sum rule to order equal or higher than $\bar{m}_s^4/M_0^2$ as can be seen from Eqs. (2.5) and (2.6). If we assume an $SU(3)_F$ symmetry for the dimension six flavour dependent vacuum condensate, then we can ignore the effect of the term B_i^6/M_0^4 of Eq. (2.9). Using Eq. (2.5), we deduce:

$$M_\phi^2 - M_\rho^2 \approx 2(M_{K^*}^2 - M_\rho^2) + (1 \sim 4) 10^{-2} \text{ GeV}^2. \quad (2.10)$$

The data for the meson masses lead to:

$$0.44 \approx 0.39 + (1 \sim 4) 10^{-2} \quad (2.11)$$

in units of GeV^2 , i.e., the QCD effects tend to restore the small discrepancy between the two sides of the sum rule. A better improvement of our result in Eq. (2.10) needs a more careful analysis of the possible $1/M_0^4$ contributions to the sum rule and the evaluation of various sources of radiative corrections,

^{*)} Notice that from Eqs. (2.7) and (2.8) it is easy to see that the $SU(3)_F$ breaking terms explain why M_ϕ is much larger than M_{K^*} while the latter is much larger than M_ρ .

as well as possible effects of the meson width corrections. For the M_0^{-4} terms from QCD, one can have a correction of the type $\bar{m}_S^2/M_0^4 \alpha_S \langle F^2 \rangle$ due to two mass insertions in the quark loop where the gluon is condensed, which may lead to an effect less than $7 \cdot 10^{-3} \text{ GeV}^2$ for $M_0 \geq 0.8 \text{ GeV}$. We can estimate the four-fermion operator contribution using the order of magnitude estimate due to the factorization hypothesis of Ref. 1). Then:

$$B_i^6 \approx 3 \cdot \frac{448}{81} \pi^3 \alpha_S \langle \bar{\psi}_i \psi_i \rangle^2. \quad (2.12)$$

Assuming $B_{K^*}^6 \approx B_\phi^6$ which is less than B_0^6 if the $SU(3)_F$ breaking of the condensate is taken into account^{6),7)}, i.e., using $\langle \bar{s}s \rangle / \langle \bar{d}d \rangle \approx 0.6$, we get a contribution less than $2 \cdot 10^{-2} \text{ GeV}^2$ for $M_0 \geq 0.8 \text{ GeV}$ and using $\langle \bar{d}d \rangle^{1/3} \approx -0.178 \text{ GeV}$ consistent with $\bar{m}_u + \bar{m}_d \approx 30 \text{ MeV}$ ⁴⁾, $\bar{\alpha}_S(M_0) \approx 0.4$ for $\Lambda \approx 150 \text{ MeV}$. Such a contribution goes in the good direction in confronting the two sides of the sum rule and may compete with the leading mass contribution. So, we also need a more accurate analysis of non-perturbative effects if we want to improve the GMO mass formulae further, but as we have seen, the contributions of such terms depend strongly on the value of M_0 used and may be totally negligible for $M_0 \approx 1 \text{ GeV}$.

3. - THE STRANGE QUARK MASS VALUE FROM THE VECTOR MESON MASS DIFFERENCES

Analyzing the ϕ or K^* meson sum rules, we have seen that the quark mass plays an important role due to the anomalous large values of the function ρ_{ϕ, K^*} defined in Eqs. (2.4). Then, we believe that an accurate determination of the strange quark mass can be obtained from the ϕ - ρ or K^* - ρ mass differences. Taking the leading contribution in Eq. (2.5) and using the corrected kaon PCAC relation⁶⁾:

$$(m_u + m_s) \langle \bar{u}u + \bar{s}s \rangle \approx -(0.6) 2 M_K^2 f_K^2 \quad (3.1)$$

together with

$$\langle \bar{s}s \rangle / \langle \bar{d}d \rangle \approx 0.6 \quad (3.2)$$

and $m_s/m_d \approx 25$, we deduce

$$m_s \langle \bar{s}s \rangle \approx -0.5 M_K^2 f_K^2, \quad (3.3)$$

while the $SU(3)_F$ symmetry relation would get a value of about twice Eq. (3.3). Then, we deduce for $M_0 \approx 0.8 \sim 1 \text{ GeV}^*$:

$$\bar{m}_s(M_0) \approx 250 \text{ MeV} \quad (3.4)$$

to which corresponds the value of the invariant mass:

$$\hat{m}_s \approx 314 \text{ MeV} \quad (3.5)$$

for $\Lambda \approx 150 \text{ MeV}$. If instead we had used the PCAC estimate of the condensate $m_s \langle \bar{s}s \rangle \approx -M_K^2 f_K^2$, we would have got: $\bar{m}_s(M_0) \approx 231 \text{ MeV}$ which is not too different from the one in Eq. (3.4) where the $SU(3)_F$ breaking effects to the quark condensate have been taken into account. One should notice that previous analysis⁵⁾ which keeps only the linear quark mass terms in the mass expansion of the meson mass difference is not justified due to the fact that in the case of Eq. (2.5) the quadratic mass term contribution is more important than the former.

Our result in Eq. (3.4) is comparable to recent determinations^{4),5)} and is significantly higher than the orthodox value of about 150 MeV of the strange quark mass.

4. - THE GMO MASS FORMULAE FOR THE O^{++} SCALAR MESONS

In the scalar mesons case, the quark model assigns the δ meson as the scalar partner of the ρ and the κ as the scalar partner of the K^* meson. They can be represented by the following current divergence:

$$J_\delta \equiv i(m_u - m_d) : \bar{u}d : , \quad (4.1a)$$

$$J_\kappa \equiv i(m_u - m_s) : \bar{u}s : . \quad (4.1b)$$

The currents in Eq. (4.1) are renormalization group invariant and their normalization is known by analogy with the pion current:

$$\langle 0 | J_H | H \rangle = \sqrt{2} M_H^2 f_H , \quad H \equiv \delta, \kappa , \quad (4.2)$$

*) One can see that at this low value of M_0 the dimension six operators contribution to Eq. (2.7) is less than 5%.

with f_H the scalar meson decay amplitude analogous to $f_\pi \approx 93$ MeV. For the isoscalar channel, the representation like in Eq. (4.1) becomes impossible due to the fact that the neutral current has a zero divergence. Then, an alternative representation is needed to describe the isoscalar $I = 0$ scalar particles. The octet particle can be represented by the current:

$$J_8 = \frac{i}{\sqrt{6}} : \bar{u}u + \bar{d}d - 2\bar{s}s : \quad (4.3a)$$

while the singlet one is:

$$J_1 = \frac{i}{\sqrt{3}} : \bar{u}u + \bar{d}d + \bar{s}s : \quad (4.3b)$$

In fact J_1 and J_8 are no longer renormalization group invariant and the normalization of the matrix element $\langle 0 | J_{i=1,8} | s \rangle$ is not well defined. However, as far as we shall work with moment ratios, these problems will not appear to leading order.

Using the meson current in Eq. (4.1), we deduce from the result in Ref. 4b) to two loops: ($m_u = m_d = 0$)

$$R^\delta \approx 2M^2 \left\{ 1 - \frac{2}{9 \log M/\Lambda} + 0.35 \frac{\log \log M^2/\Lambda^2}{4 \log^2 M/\Lambda} - \frac{\pi}{3} \frac{\alpha_s \langle F^2 \rangle}{M^4} \left(1 - \frac{12\pi m_\pi^2 g_\pi^2}{\alpha_s \langle F^2 \rangle} \right) + \left(\frac{B_\delta}{M} \right)^6 \right\} \quad (4.4)$$

and ($m_u \ll m_s$):

$$R^\kappa \approx 2M^2 \left\{ 1 - \frac{2}{9 \log M/\Lambda} + 0.35 \frac{\log \log M^2/\Lambda^2}{4 \log^2 M/\Lambda} + \frac{\bar{m}_s^2}{M^2} \left(1 - \frac{4}{27 \log M/\Lambda} \right) - \frac{\pi}{3} \frac{\alpha_s \langle F^2 \rangle}{M^4} \left[1 + \frac{4\pi m_s \langle \bar{s}s \rangle}{\alpha_s \langle F^2 \rangle} \left(1 + 2 \frac{\langle \bar{u}u \rangle}{\langle \bar{s}s \rangle} \right) \right] + O\left(\frac{1}{M^6}\right) \right\} \quad (4.5)$$

where the radiative corrections in the $\bar{m}_s^2$ term have been obtained from the general result in Ref. 8). Saturating the δ and κ sum rule by the δ and κ meson and equating these sum rules at the same M_0^2 , we get:

$$M_K^2 - M_\delta^2 \approx 2 \bar{m}_s^2 - \frac{8\pi^2}{3} \frac{m_s \langle \bar{s}s \rangle}{M_0^2} \left(1 + 2 \frac{\langle \bar{u}u \rangle}{\langle \bar{s}s \rangle} \right) + \mathcal{O}\left(\frac{1}{M_0^4}\right) \quad (4.6)$$

One can notice from Eq. (4.6) that the $SU(3)_F$ chiral symmetry leading breaking terms contribution is positive. So why the κ mass is much heavier than the δ one. In the same way and using the quark representation in Eqs. (4.3), we get:

$$M_1^2 - M_\delta^2 \approx 2 \bar{m}_s^2 - \frac{16\pi^2}{3} \frac{m_s \langle \bar{s}s \rangle}{M_0^2} + \mathcal{O}\left(\frac{1}{M_0^4}\right), \quad (4.7a)$$

$$M_\delta^2 - M_\delta^2 \approx 4 \bar{m}_s^2 - \frac{32\pi^2}{3} \frac{m_s \langle \bar{s}s \rangle}{M_0^2} + \mathcal{O}\left(\frac{1}{M_0^4}\right), \quad (4.7b)$$

which, after the elimination of the quark mass effect, lead to:

$$M_1^2 \approx M_K^2 - \frac{8\pi^2}{3} \frac{m_s \langle \bar{s}s \rangle}{M_0^2} \left(1 - 2 \frac{\langle \bar{u}u \rangle}{\langle \bar{s}s \rangle} \right) + \mathcal{O}\left(\frac{1}{M_0^4}\right), \quad (4.8)$$

$$M_\delta^2 \approx 2M_K^2 - M_\delta^2 - \frac{16\pi^2}{3} \frac{m_s \langle \bar{s}s \rangle}{M_0^2} \left(1 - \frac{2\langle \bar{u}u \rangle}{\langle \bar{s}s \rangle} \right) + \mathcal{O}\left(\frac{1}{M_0^4}\right). \quad (4.9)$$

Using the effective mass $M_K \approx 1.2 \text{ GeV}^*$ and $M_\pi \approx 980 \text{ MeV}$, we get:

$$M_1 \approx 1.1 \text{ GeV} \quad (4.10a)$$

$$M_\delta \approx 1.3 \text{ GeV} \quad (4.10b)$$

Confronting our result to the known values of the $S^*(975)$ and of the $\epsilon(1.3)$ masses from the data⁹⁾, we are tempted to conclude that the S^* is mainly composed of the singlet part of the isoscalar current while the ϵ is more composed of the nonet part of the current. Such a result differs from the one previously stated¹⁰⁾ and a clarification of the situation from the data is still needed. Then, we also examine the assumption of an ideal mixing between the $I = 0, 0^{++}$ scalar mesons. So one can adopt the following quark representation:

*⁹⁾ This mass is much lower than the κ mass given in the particle data tables. It would correspond to the mass of an effective resonance with which one can reproduce (forgetting background and inelastic effects) the $\kappa\pi$ phase shift data⁹⁾.

$$J_X \equiv i: \bar{s}s: \quad , \quad (4.11a)$$

$$J_Y \equiv \frac{i}{\sqrt{2}} : \bar{u}u + \bar{d}d: \quad (4.11b)$$

of the scalar mesons X and Y . Then, one can easily find that, to leading order of $SU(2)_F$ chiral symmetry ($m_u = m_d$), the Y meson is degenerated to the δ mesons as is the case of the ω and ρ mesons in the vector channel. The Y particle could be identified as the $S^*(975)$ or even as the old $\epsilon(800)$ which actually disappears in the particle data listings as wished by some authors¹¹⁾. In fact, the data on the S^* decay do not, a priori, exclude the identification of the S^* with the Y due to the observed copious decays of the S^* into $\pi^+\pi^-$ rather than into a $K\bar{K}$ pair. However, various analyses⁹⁾ on the $\pi\pi$ and $K\bar{K}$ phase shift data seem to favour an S^* coupled strongly to the $K\bar{K}$ systems but cannot copiously decay into a $K\bar{K}$ pair because of the too small phase space. So, if one takes seriously the result of such analyses, then one has to identify the S^* with the X particle which has a large $\bar{s}s$ component in its amplitude; the Y particle can be identified in this case with the $\epsilon(800)$. The mass of the X particle is predicted from the GMO like mass formula to be:

$$M_X^2 \approx M_f^2 + 6 \bar{m}_s^2 - \frac{48\pi^2}{3} \frac{m_s \langle \bar{s}s \rangle}{M_0^2} \quad . \quad (4.12)$$

Equation (4.12) shows that due to $SU(3)_F$ breaking, the mass of an isoscalar particle with a large $\bar{s}s$ component in its current must be higher than the δ meson mass. Equation (4.12) leads to:

$$M_X^2 \approx 3M_K^2 - 2M_f^2 - \frac{24\pi^2}{3} \frac{m_s \langle \bar{s}s \rangle}{M_0^2} \left(1 - \frac{2 \langle \bar{u}u \rangle}{\langle \bar{s}s \rangle} \right) , \quad (4.13)$$

after the elimination of the s quark mass term from Eq. (4.6). Using the input in Section 3, we deduce:

$$M_X \approx 1.45 \text{ GeV} . \quad (4.14)$$

The prediction of the S^* mass in such a scheme differs notably from the expectation of having an S^* mass, with a large $\bar{s}s$ component, around 975 GeV^{9),12)}. The result is so because, as we can see from Eq. (4.12), the chiral symmetry breaking effects tend to increase the X meson mass from the δ mass. The only way how the ideal mixing could survive is to assign the $S^*(975)$ or the

$\epsilon(800)$ to have the quark content in Eq. (4.11b) and to detect a strange particle with a mass around 1.4 GeV from the data. We hope that a further experimental effort will be made for a better clarification of the $I = 0, 0^{++}$ scalar mesons spectrum.

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REFERENCES

- 1) M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147 (1979) 385, 448.
- 2) J.S. Bell and R.A. Bertlmann, CERN preprint TH.3540 (1983) and references therein.
- 3) S. Narison, Phys. Report 84 (1982) 263.
- 4) a) S. Narison and E. de Rafael, Phys. Lett. 103B (1981) 57;
b) S. Narison, N. Paver, D. Treleani and E. de Rafael, Nucl. Phys. B212 (1983) 365;
c) T.N. Truong, CERN preprint TH.3277 (1982).
d) A.L. Kataev, N.V. Krasnikov and A.A. Pivovarov, Phys. Lett. 123B (1982) 93.
e) G. Penso and C. Verzegnassi, Trieste preprint 56/82/EP (1982).
- 5) For a review on various determinations of the quark masses, see, e.g., J. Gasser and H. Leutwyler, Phys. Reports C87 (1982) 77.
- 6) S. Narison, N. Paver and D. Treleani, Nuovo Cimento 74A (1983) 347.
- 7) a) P. Pascual and R. Tarrach, Phys. Lett. B116 (1982) 443;
b) S. Mallik, Nucl. Phys. B206 (1982) 90.
- 8) D. Broadhurst, Phys. Lett. 101B (1981) 423.
- 9) a) P. Estabrooks et al., Nucl. Phys. B133 (1978) 490;
b) D. Aston et al., Phys. Lett. 106B (1981) 235;
c) G. Mennessier, Z. für Phys. C16 (1983) 241 and private communication.
d) L. Montanet, CERN preprint EP 82-69 (1982).
e) N. Paver. private communication.
- 10) D. Morgan, Phys. Lett. 51B (1974) 71.
- 11) a) M. Aguilar-Benitez et al., Nucl. Phys. B40 (1978) 73.
b) Y. Fujii and M. Fukugita, Nucl. Phys. B85 (1979) 179;
c) M. Cerroda, A. Gonzalez-Arroyo, J.A. Rubio and F.J. Yndurain, Phys. Lett. 62B (1976) 353.
- 12) N.A. Törnqvist, University of Helsinki preprint HUTFT-82-1 (1982).

FIGURE CAPTIONS

- Fig. 1 : Leading QCD contribution to the ω - ϕ mixing.
- Fig. 2 : Leading contribution of the unit operator in the OPE analysis of the two-point function. $\otimes$ denotes the hadronic current insertion.
- Fig. 3 : a) Leading $\langle m\bar{\psi}\psi \rangle$ quark vacuum condensate contribution to the two-point function;
b) Leading $\alpha_s \langle F^2 \rangle$ gluon condensate contribution to the two-point function.
- Fig. 4 : a),b) $\langle \bar{\psi}\Gamma_1\psi\bar{\psi}\Gamma_2\psi \rangle$ four-fermion operator contribution to the two-point function;
c) $\langle m\bar{\psi}(\lambda a/2)\sigma^{\mu\nu}\psi F_{\mu\nu}^a \rangle$ contribution to the two-point function.
d) Example of $g f_{abc} \langle F_a^{\mu\nu} F_b^{\nu\rho} F_c^{\rho\mu} \rangle$, gluon condensate contribution to the two-point function.
- Fig. 5 : a) The function ρ_ϕ given in Eq. (2.4a) versus $1/M^2$ for two extremal values of $\hat{m}_s$;
b) The function f_{K^*} given in Eq. (2.4b) versus $1/M^2$ for two extremal values of $\hat{m}_s$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5a

Fig. 5b