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A SUPERSYMMETRIC VARIANT OF DASHEN'S FORMULA

G. Veneziano
CERN -- Geneva

ABSTRACT

In SUSY-QCD Dashen's formula is shown to be accompanied, in a SUSY vacuum, by a more useful relation giving the behaviour of $F_{\pi_i}^a M_{ij} F_{\pi_j}^b$ in the chiral limit. As a consequence, the previously noted properties of SUSY vacua at small mass become even more puzzling.

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1. Increasing attention has been recently given to the question of how chiral symmetries are realized in supersymmetric (SUSY) gauge theories. Since this is a strong coupling problem and no reliable non-perturbative techniques are available, a clear cut answer does not look within reach for the time being.

A more heuristic approach, which has been followed by a number of authors¹⁻⁵⁾, consists in constructing effective Lagrangians which, in terms of a small number of low energy degrees of freedom, incorporate the Ward identities of the true effective action. The attitude here is that such effective Lagrangians, when making sense, represent *possible* realizations of the underlying theory. At the same time, results obtained from it can suggest properties of more general validity as well as the way to prove them.

Unfortunately, the situation that has emerged, despite considerable effort, is still very intriguing. While in the absence of (or for heavy) matter a nice realization of super Yang-Mills theory was constructed¹⁾ and its features agreed with more model independent results⁶⁾, the limit of small matter mass ($m/\Lambda \ll 1$) lead to surprising behaviours for various physical quantities in SUSY-QCD. We now briefly summarize those results.

Assuming the relevant degrees of freedom of SUSY-QCD to be the SUSY completion of those of QCD⁷⁾, the authors of Ref. 2 found an effective Lagrangian which, at $m \neq 0$, exhibited as many SUSY vacua as required by the value of Witten's index⁶⁾. It was also found, however, that the physics around such vacua, while perfectly reasonable at $m/\Lambda \sim O(1)$, became very unusual at $m/\Lambda \ll 1$. The reason for this was traced back to the runaway (to infinity) of the SUSY vacua as m went to zero. It was also argued that this funny behaviour could signal either that chirality was not broken as in ordinary QCD, or that SUSY itself was broken as $m \rightarrow 0$. No nice looking realization of either possibility was however given.

It was subsequently proven⁸⁾ that the runaway of the vacuum was a general feature of SUSY-QCD, as long as chirality was spontaneously broken and previous arguments²⁾ for chirality protection by SUSY were reinforced.

Later on it was pointed out by Peskin³⁾ that, at least by giving up the implementation of anomalous Ward identities in $\mathcal{L}_{\text{eff}}$, it was possible to construct models which possessed, besides the zero energy vacua, an extra higher energy relative minimum which became the true vacuum just at $m = 0$ *) with consequent breaking of supersymmetry. Such a possibility had been considered earlier for phenomenological purposes by a number of authors⁹⁾. Furthermore, current algebra at small, non-zero m/Λ was found to be of the conventional (QCD) type around the (false) SUSY-breaking vacuum but still pathological around the (true) SUSY-vacua ($F_\pi \rightarrow 0$, $m_\pi \rightarrow \infty$, in the model of Ref. 3).

Improvements and extensions of Peskin's work were given in Ref. 4 where it was also noticed that Peskin's variables are related to those of Ref. 2 by a nonlinear transformation^{**)}. In all these models the chiral properties of SUSY vacua agree with the general results of Ref. 8.

Finally, Kitazawa⁵⁾ argued that the usual chirality breaking QCD condensate should appear in the planar (large N_c) limit of SUSY-QCD, implying again that some bosonic bilinears should get divergent expectation values for $m \rightarrow 0$.

2. In this note we shall prove another model independent property of SUSY vacua in SUSY-QCD, whose implications will make its chiral limit even more peculiar, unless chirality is unbroken by those vacua. The general validity of this new result was suggested by its occurrence in all SUSY-QCD effective Lagrangians studied so far. It says that the well known Dashen formula¹⁰⁾ of chiral perturbation theory:

$$\sum_{i,j} F_{\pi_i}^a M_{ij}^2 F_{\pi_j}^b = \langle 0 | [Q_S^a, [Q_S^b, \mathcal{L}_m]] | 0 \rangle \quad (1)$$

*) Insisting on their way of implementing the supercurrent anomaly the authors of Ref. 2 found it impossible to construct a model with these features (unpublished).

***) This was independently noticed by the author (private communication to M. Peskin).

relating pseudogoldstone decay constants and masses to the vacuum expectation value (v.e.v.) of a commutator, is accompanied, is a SUSY vacuum, by an amusing variant reading^{*)}:

$$\sum_{i,j} F_{\pi_i}^a M_{ij} F_{\pi_j}^b = \langle 0 | [Q_5^a, [Q_5^b, V_{\theta=0}]] | 0 \rangle \quad (2)$$

In Eq. (2) δV is the chirality breaking part of the SUSY-QCD superpotential which is given by:

$$V(x, \theta) = - \sum_{i=1}^{N_f} m_i q_i \tilde{q}_i \quad ; \quad \mathcal{L}_m^{(x)} = \int d^2\theta V(x, \theta) + h.c. \quad (3)$$

where $q_i = (\phi_i, \psi_i, F_i)$ are the (chiral) left-handed quark superfields (for various flavours) and $\tilde{q}_i$ are the (chiral) antiquark superfields.

Before turning to the derivation of Eq. (2), let us point out why it can be useful. The point is that the double commutator appearing in Eq. (1) is not the component of a supermultiplet (see, for example, Refs. 2 and 3) and hence its estimation is difficult. On the contrary, the right-hand side of Eq. (2) is the lowest component of a chiral superfield and can be evaluated for $m \rightarrow 0$ following previous results⁸⁾. Indeed, it was already realized²⁾ that the quantity $F_{\pi}^2 M_{\pi}$ was much more stable than $F_{\pi}^2 M_{\pi}^2$ ^{**) against modifications of $\mathcal{L}_{\text{eff}}$.}

3. Now to the proof of Eq. (2)^{***)}. The occurrence of M_{ij} (rather than M^2) suggests considering a fermionic two-point function. Denoting by $V_{\theta\alpha}$ the θ -component of the composite chiral superfield in Eq. (3) we write, for a SUSY vacuum, the obvious identity^{†)},

*) For the anomaly free R-current of SUSY-QCD there is a simple correction factor [see Eq. (11) below].

**) This does not mean, as suggested in Ref. 5, that Dashen's formula is not satisfied. It can be proven (G. Veneziano, unpublished) that Eq. (1) always holds as a consequence of chiral Ward identities, provided that the sum over i, j runs over all the states (see also Ref. 2).

***) An elegant rederivation of Eq. (2), using manifestly supersymmetric superfield techniques, has been given meanwhile by G. Shore (private communication and to appear).

†) Factors of 2, i , and signs are convention-dependent and neglected here for simplicity. The result [Eq. (2)] can be easily normalized at the end.

$$\begin{aligned} \int d^4x \, \varepsilon_{\alpha\beta} T \langle 0 | (\delta_s^a V_{\theta_\alpha}(0), \delta_s^b V_{\theta_\beta}(x)) | 0 \rangle = \\ = \int d^4x \, \varepsilon_{\alpha\beta} T \langle 0 | (\delta_s^a V_{\theta=0}, \{Q_\alpha, \delta_s^b V_{\theta_\beta}\}) | 0 \rangle \end{aligned} \quad (4)$$

where we have indicated by δ_s^a the variation under a generic axial transformation which commutes with SUSY (for the case of the R-symmetry, see below). Since $\bar{Q}_\alpha$ commutes with $V_{\theta=0}$ we can add to the anticommutator in Eq. (4) $\{\bar{Q}_\alpha, \delta_s^b V_{\theta_\beta}\} \varepsilon_{\alpha\beta}$. It then turns out that the following identity holds in SUSY-QCD:

$$\begin{aligned} \{Q_\alpha, \delta_s^b V_{\theta_\beta}\} \varepsilon_{\alpha\beta} + \{\bar{Q}_\alpha, \delta_s^b V_{\bar{\theta}_\beta}\} \varepsilon_{\alpha\beta} = \\ = \delta_s^b [\int d^2\theta V + h.c.] = \partial_\mu J_\mu^b \end{aligned} \quad (5)$$

This equation may look trivial and valid separately for the θ and $\bar{\theta}$ parts. In fact it is not, due to the fact that the elimination of the auxiliary fields of q and $\tilde{q}$ involves chirality breaking equations of motions making Q_α and δ_s non-commuting. Equation (5) however still holds due to the identity

$$[\lambda^b, m^2] = \{\lambda^b, m\}m - m\{\lambda^b, m\} \quad (6)$$

where m is the quark mass matrix. Combining Eqs. (4) and (5) we obtain

$$\begin{aligned} \varepsilon_{\alpha\beta} \int d^4x \, T \langle 0 | (\delta_s^a V_{\theta_\alpha}(0), \delta_s^b V_{\theta_\beta}(x)) | 0 \rangle = \\ = \langle 0 | \delta_s^b \delta_s^a V_{\theta=0} | 0 \rangle = \langle 0 | [Q_s^b, [Q_s^a, V_{\theta=0}]] | 0 \rangle \end{aligned} \quad (7)$$

The last step in the proof consists in saturating the left-hand side of Eq. (7) with the fermionic partners χ_i of the pseudogoldstones π_i . It is easy to prove that, in a SUSY vacuum:

$$\langle 0 | \int_S^a V_{\theta_\alpha} | \chi_{i\beta} \rangle = \varepsilon_{\alpha\beta} F_{\pi_i}^a M_i, \quad M_i \equiv M_{\pi_i} \quad (8)$$

Our result Eq. (2) then follows in the basis of the eigenstates of M_{ij} from:

$$\begin{aligned} \varepsilon_{\alpha\beta} \int d^4x \, T \langle 0 | [\int_S^a V_{\theta_\alpha}^{(0)}, \int_S^b V_{\theta_\beta}^{(x)}] | 0 \rangle &= \\ &= F_{\pi_i}^a M_i \frac{1}{M_i} M_i F_{\pi_i}^b = F_{\pi_i}^a M_i F_{\pi_i}^b \end{aligned} \quad (9)$$

The case of the R-symmetry is a bit more complicated but, in principle, straightforward. Keeping track of the lack of commutativity of Q_α and Q_5^R :

$$[Q_5, [Q_\alpha, V_{\theta=0}]] = \frac{\chi_\psi}{\chi_\phi} [Q_\alpha, [Q_5, V_{\theta=0}]] \quad (10)$$

where $\chi_\psi = 1$, $\chi_\phi = 1 - N_f/N_c$ are the anomaly-free R-charges of ψ and ϕ , one arrives at:

$$\begin{aligned} F_{\pi_i}^R M_{ij} F_{\pi_j}^R &= \frac{\chi_\psi}{\chi_\phi} \langle 0 | [Q_5^R, [Q_5^R, V_{\theta=0}]] | 0 \rangle = \\ &= 4 \chi_\phi \chi_\psi \langle 0 | V_{\theta=0} | 0 \rangle \end{aligned} \quad (11)$$

4. Let us now discuss some examples of how Eq. (2) can be used.

a) For one flavour SUSY-QCD the right-hand side of Eq. (11) is given by:

$$(1 - 1/N_c) \langle m \phi \tilde{\phi} \rangle \underset{m/\Lambda \ll 1}{\sim} \left(m/\Lambda \right)^{1/N_c} \Lambda^3 \quad (12)$$

where our previous results⁸⁾ have been used for the chiral limit. Hence we get, for the model of Ref. 2, $F_{\pi\pi}^2 M_\pi + F_{\eta\eta}^2 M_\eta \sim (m/\Lambda)^{1/N_c} \Lambda^3$ which, although peculiar (at $N_c \rightarrow \infty$, for instance) is perhaps not completely unacceptable.

- b) In the case of Ref. 4 the same quantity [Eq. 12)] is related to $F_{\pi}^2 M_{\pi}$ (there is no " η " in that model) while in the case of Peskin³⁾ one obtains $F_{\pi}^2 M_{\pi} = 0(1)$ (fulfilled in his example by $M_{\pi} \sim 1/m^2$, $F_{\pi} \sim m$).
- c) In the 2-flavour case, as already realized within the effective Lagrangian²⁾, the situation is even worse. Taking a and b in Eq. (2) to be the $\pi^{\pm}$ currents one gets

$$\begin{aligned} F_{\pi^{\pm}}^2 M_{\pi^{\pm}} &= (m_1 + m_2) \langle \phi_1 \tilde{\phi}_1 + \phi_2 \tilde{\phi}_2 \rangle \sim_{m_i/\Lambda \rightarrow 0} \\ &\sim \frac{(m_1 + m_2)^2}{m_1 m_2} \left(\frac{m_1 m_2}{\Lambda^2} \right)^{1/N_c} \Lambda^3 \end{aligned} \quad (13)$$

where results from Ref. 8 have been used. One can easily check that Eq. (13) holds in the effective Lagrangian of Ref. 2 *). Accordingly, M_{π^+} and/or F_{π^+} must blow up when one of the two quark masses vanishes.

If we consider these results as unphysical, two ways out seem possible. One is that the true vacuum of SUSY-QCD is chiral invariant with $\langle \phi \tilde{\phi} \rangle \equiv 0$ and $F_{\pi} = 0$; the second is that the sick SUSY vacuum is somehow out of reach and the theory sits in a higher energy SUSY breaking state (like Peskin's, for instance) and is unable to tunnel to the lower vacuum below some critical value of m/Λ .

Deciding about these possibilities, or finding yet another solution, will be of crucial importance for the applications of SUSY gauge theories, in particular for the constructions of supersymmetric models of quarks and leptons¹¹⁾.

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*) Because of a misprint the left-hand side of Eq. (3.29) there should read $F_{\pi_{12}}^2$.

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