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POLARIZED BEAMS IN HIGH-ENERGY e^+e^- STORAGE RINGS

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INTRODUCTION

The polarization of electron/positron beams in storage rings is fundamentally governed by the quantized nature of synchrotron radiation. On the one hand, beams can become polarized by the Sokolov-Ternov spin-flip asymmetry of the photon emission, whereas on the other hand, the abrupt energy loss excites orbit oscillations which perturb the spin motion and give rise to depolarizing effects. It is the balance between these two aspects of the same phenomenon that determines the degree of beam polarization that can be built up and maintained in a machine.

In this paper we shall first recall the main features of the radiative polarization mechanism, followed by some important properties of solutions of the Thomas-BMT equation. The motion of the spin-precession axis in the presence of orbit oscillations will then be discussed, leading to the concept of the spin-orbit coupling vector. The influence of radiation damping and the cumulative effects of many photon emissions leads to spin diffusion and thus to depolarization in the vicinity of spin resonances. The origin of the driving terms for these resonances will be examined, in particular for high-energy electron storage rings, and means of reducing their depolarizing influence will be outlined. Finally, expectations for polarized beams in LEP will be presented.

Before proceeding on these topics it is instructive to consider the time scales of the various physical processes involved; these are shown against a logarithmic scale in Fig. 1 for a typical (but non-existent) electron storage ring of about 25 GeV energy. It is seen that characteristic times span more than fourteen orders of

Fig. 1 Characteristic times of processes at 25 GeV energy;
 γ = Lorentz factor, ρ = bending radius, r_0 = classical radius,
 Q_β = betatron tune, α = fine structure constant, λ_c = Compton wave-
length, a = gyromagnetic anomaly.

magnitude, ranging from less than 10^{-10} s for the duration of the quantum emission process to over 10^4 s for a depolarizing time desirably exceeding the polarization time by a factor of ten.

An important feature of Fig. 1 is the clustering together of betatron-oscillation, orbit-harmonic and spin-precession periods, a factor which is of some significance in the spectrum of depolarizing resonances. By contrast, the large separations between times for polarization, radiation damping and oscillation modes helps to simplify calculations by the use of averaging methods. The separation of time scales between the oscillation modes, the interval between quanta emitted by a single electron and the duration of the quantum emission process enable the latter to be considered as an abrupt random process with no correlation between successive photons.

RADIATIVE POLARIZATION

Spontaneous polarization of an electron or positron moving in a transverse magnetic field $\vec{B}$ results from an asymmetry in the probability of photon emission according to the initial spin state, parallel (+) or anti-parallel (-), to $\vec{B}$. This process, predicted by Ternov, Loskutov and Korovina,¹ and calculated in detail by Sokolov and Ternov² for a uniform field, gives the probability with spin-flip as:

$$w_{\uparrow\downarrow}(\pm) = \frac{1}{2\tau_p} \left(1 \pm \frac{8}{5\sqrt{3}} \right) = \frac{\xi^2}{6} w_0 \left(1 \pm \frac{8}{5\sqrt{3}} \right) \quad (1)$$

where

$$\frac{1}{\tau_p} = \frac{5\sqrt{3}}{8} \frac{cr_0\lambda_c\gamma^5}{\rho^3}$$

is the polarization rate

$$w_0 = \frac{5}{2\sqrt{3}} \frac{cr_0\gamma}{\lambda_c\rho}$$

is the photon emission rate and

$$\xi = \frac{3}{2} \frac{\lambda_c\gamma^2}{\rho} = \frac{\hbar\omega_{cr}}{mc^2\gamma}$$

is the ratio of the critical photon energy and the particle energy, and is typically of the order of 10^{-6} . The probability of photon emission with spin flip is evidently very small, which accounts for the long polarization time, but the asymmetry with respect to initial spin state is large, which leads to a high degree of final polarization in a beam.

There is also an asymmetry in the probability of photon emission without spin flip, given by

$$w_{\uparrow\uparrow}(\pm) = w_0 \left[1 - \xi \left(\frac{16}{15\sqrt{3}} \pm 1 \right) \right]. \quad (2)$$

Here the asymmetry is small but its probability relatively large, being proportional to ξ rather than ξ^2 . Under some conditions this could be used to enhance the polarization, as mentioned below.

Generalization of the spin-flip probability to the case of arbitrary magnetic fields, by Baier and Katkov,³ gives

$$w_{\uparrow\downarrow}(\vec{\zeta}) = \frac{1}{2\tau_p} \left[1 - \frac{2}{9} (\vec{\zeta} \cdot \vec{v}) + \frac{8}{5\sqrt{3}} \vec{\zeta} \cdot (\vec{v} \times \vec{v}) \right], \quad (3)$$

where $\vec{\zeta}$ is the initial spin vector and $\vec{v}$, $\dot{\vec{v}}$ the velocity and acceleration vectors, respectively, of the particle.

POLARIZATION WITH MACHINE IMPERFECTIONS

Nearly all depolarizing phenomena may be expressed in terms of the spin-orbit coupling vector $\vec{\Gamma} = \gamma \partial \vec{n} / \partial \gamma$, sometimes designated $\vec{d}$. This quantity describes the energy dependence of the spin precession axis, and a description of its physical origin will be discussed in a later section.

It has been shown by Derbenov and Kondratenko⁴ that the polarization time τ_p and the asymptotic level of polarization $P(\infty)$ in a storage ring with alignment and field errors are given quite generally by

$$\frac{1}{\tau_p} = \frac{5\sqrt{3}}{8} c r_0 \lambda_c \gamma^5 \left\langle \frac{1}{|\rho^3|} \left[1 - \frac{2}{9} (\vec{n} \cdot \vec{v})^2 + \frac{11}{18} |\vec{\Gamma}|^2 \right] \right\rangle \quad (4)$$

and

$$P(\infty) = - \frac{8}{5\sqrt{3}} \frac{\left\langle \frac{1}{|\rho^3|} \hat{b} \cdot (\vec{n} - \vec{\Gamma}) \right\rangle}{\left\langle \frac{1}{|\rho^3|} \left[1 - \frac{2}{9} (\vec{n} \cdot \vec{v})^2 + \frac{11}{18} |\vec{\Gamma}|^2 \right] \right\rangle}, \quad (5)$$

where $\hat{b}$ is the unit vector parallel to the magnetic field $\vec{B}$, $\vec{n}$ is the unperturbed precession axis, and the averages are taken over the machine circumference and the particle distribution. For a perfect planar machine both $\vec{\Gamma}$ and $(\vec{n} \cdot \vec{v})$ vanish, leading to an asymptotic polarization of $8/5\sqrt{3} = 92.4\%$. A value of $|\vec{\Gamma}|^2$ around unity or more leads to a substantial depolarization.

The term $\vec{\Gamma}$ in the numerator of Eq. (5) together with the non-spin-flip asymmetry in Eq. (2) can in principle be used to increase the level of polarization to about 95%,⁴ but the practical application has not yet been sufficiently studied so we shall not discuss this in detail.

THOMAS-BMT EQUATION

The relativistic spin-precession equation, first derived by Thomas⁵ and later reformulated in covariant four-vector notation by Bargmann, Michel and Telegdi,⁶ may be written in the form

$$\frac{d\vec{p}}{dt} = \vec{\Omega} \times \vec{p},$$

with

$$\vec{\Omega} = -\frac{e}{m\gamma} \left[(\gamma a) \vec{B} - (\gamma - 1) a \frac{(\vec{v} \cdot \vec{B}) \vec{v}}{v^2} + \left(\gamma a - \frac{\gamma}{\gamma^2 - 1} \right) \frac{\vec{E} \times \vec{v}}{c^2} \right], \quad (6)$$

where $a = (g - 2)/2$ is the gyromagnetic anomaly, about 1.16×10^{-3} for electrons and positrons, $\vec{B}$ and $\vec{E}$ are the magnetic and electric field vectors, and $\vec{v}$ is the particle velocity vector. In eq. (6), $\vec{\Omega}$ is referred to a coordinate system $(\hat{x}, \hat{y}, \hat{z})$ rotating with the particle trajectory, $\hat{y}$ being the tangent vector to the orbit and $\hat{z}$ the normal to the orbit plane. In the case of a perfect planar orbit $\vec{B} = \vec{B}_z$, and with $\vec{E} = 0$ the precession axis $\vec{\Omega}$ reduces to

$$\vec{\Omega} = -\frac{e \vec{B}_z}{m\gamma} (\gamma a).$$

The precession wave number or spin tune $\nu = \gamma a$ is the number of spin precession turns per orbital period; it has a value of about 104 at 46.5 GeV.

Solutions of Eq. (6) correspond, in general, to rotations about some axis $\vec{n}$. An important property of any two such solutions $\vec{P}_1, \vec{P}_2$ for a given $\vec{\Omega}$ is

$$\frac{d}{dt} (\vec{P}_1 \cdot \vec{P}_2) = 0, \quad (7)$$

which implies a constant angle between $\vec{P}_1$ and $\vec{P}_2$ and a constant length $|\vec{P}|$, by putting $\vec{P}_1 = \vec{P}_2$. The precession axis $\vec{n}$ is evidently also a solution and, locally, $\vec{n} = \vec{\Omega}$. The properties of $\vec{n}$ under general conditions determine the polarization state of the beam.

PERIODIC ORBIT

We first consider the spin motion of a particle moving on a periodic (closed) orbit at constant energy and without oscillations. It is then convenient to replace the independent variable t by the azimuthal variable $\theta = ct/R$, where R is the average radius of the machine, and the precession equation becomes:

$$\frac{d\vec{P}}{d\theta} = \vec{\Omega}(\theta) \times \vec{P} \quad (8)$$

with the property

$$\vec{\Omega}(\theta) = \vec{\Omega}(\theta + 2\pi)$$

on a periodic orbit. Now since all solutions of Eq. (8) correspond to rotations about some axis $\vec{n}_0$, this axis is also a solution with the particular periodic property

$$\vec{n}_0(\theta) = \vec{n}_0(\theta + 2\pi) .$$

It is also evident that, the orbit being periodic in θ , $\vec{n}_0(\theta)$ repeats itself from one revolution to the next indefinitely.

More generally, close orbits exist for arbitrary (constant) energies deviating from a nominal energy γ_0 (within practical aperture limits). Except for a perfect planar orbit, the periodic spin solution is then energy dependent and we write, to first order:

$$n(\theta+2\pi, \gamma_0+\delta\gamma) = \vec{n}(\theta, \gamma_0+\delta\gamma) = \vec{n}_0(\theta) + \gamma \frac{\partial \vec{n}}{\partial \gamma}(\theta) \frac{\delta\gamma}{\gamma_0} \quad (9)$$

for an energy deviation $\delta\gamma$.

Particles in general execute betatron and synchrotron (energy) oscillations around a closed orbit; their trajectories cannot be periodic since this would lead to instability of the orbital motion. We are therefore led to seek a formulation in which the precession axis $\vec{n}$ can conveniently be defined in the presence of aperiodic orbit oscillations.

ORBIT OSCILLATIONS

Recalling from Fig. 1 the large separation of time scales between radiation damping and quantum processes on the one hand, and oscillation periods on the other, we examine the transformation properties of the orbital motion and of the spin motion over one revolution of the machine. During one revolution period the effects of radiation damping and quantum diffusion have a very small influence and we can for the moment neglect them in discussing the orbital and spin motions.

We let $X = (x, x', y, y', z, z')$ represent an orbit coordinate vector, where x, z are, respectively, the radial and vertical positions, y the position along the direction of motion, and the primes denote the corresponding conjugate variables; all are taken with respect to a reference particle moving on the ideal closed orbit with azimuthal variable θ . Then from Eq. (6), the statement

$$\vec{\Omega}(X, \theta) = \vec{\Omega}(X, \theta+2\pi)$$

simply specifies that the structure is periodic. We may define a general mapping of the orbit X and the spin $\vec{P}$ between two arbitrary positions θ and θ' by

$$M(\theta, \theta') : (X, \vec{P}) \rightarrow (MX, \vec{P}') ,$$

which implies that the spin motion depends on the orbital motion but that the orbital motion does not depend on the spin motion. (The magnetic moment of a particle has a negligible influence on its trajectory.) In general $\vec{P}' \neq \vec{P}$; also, the mapping need not necessarily be linear.

We now consider a one-turn mapping from θ to $\theta + 2\pi$ which we designate M_θ . Then for all X , θ there exists $\vec{n}(X, \theta)$ such that

$$M_\theta\{X, \vec{n}(X, \theta)\} = \{M_\theta X, \vec{n}(X, \theta)\}.$$

This corresponds to the "hairy ball" theorem which states that a vector field on the surface of a sphere vanishes at at least one point. Physically the meaning is that whilst the orbit vector X transforms to $M_\theta X$ ($\neq X$ in general), arbitrary solutions $\vec{P}$ of the Thomas-BMT equation corresponding to the orbit X do not transform into themselves; however, there is one solution $\vec{n}(X, \theta)$ which does, and this is the precession axis of the other spin solutions.

It should be noted that $\vec{n}$ is thus defined for one turn, starting at θ , and must be redefined for successive turns, consequently

$$\vec{n}(X, \theta) \neq \vec{n}(M_\theta X, \theta + 2\pi).$$

In the original formulation of Derbenev and Kondratenko⁷ the orbit vector X was expressed in action-angle variables I_j, ψ_j ($j = x, y, z$), with the equality:

$$\vec{\Omega}(I_j, \psi_j, \theta + 2\pi) = \vec{\Omega}(I_j, \psi_j, \theta) = \vec{\Omega}(I_j, \psi_j + 2\pi, \theta).$$

The first part of the equality expresses the periodicity of the structure and the second part the periodicity with oscillation phase. The precession axis $\vec{n}$ then satisfies the same periodicity conditions

$$\vec{n}(I_j, \psi_j, \theta + 2\pi) = \vec{n}(I_j, \psi_j, \theta) = \vec{n}(I_j, \psi_j + 2\pi, \theta).$$

In this formulation it is important to note that the I_j, ψ_j represent points in the phase space and not the coordinates of a particular orbit. However, both representations are equivalent and in the limit of vanishing oscillation amplitudes $X \rightarrow 0$ or $I_j \rightarrow 0$, the precession axis $\vec{n}(\theta) \rightarrow \vec{n}_0(\theta)$, the unperturbed axis, as would be expected from continuity.

Now, since $\vec{n}(\theta)$ is the axis around which all other spin solutions precess with spin tune ν , a Fourier analysis of $\vec{n}$ does not contain the frequency ν but only a spectrum of orbit-oscillation frequencies. This is a useful property, because it enables us to calculate most depolarization processes in terms of the perturbation of $\vec{n}$, without considering individual spins, as long as the spin tune is not too close to a resonance.

SYNCHROTRON OSCILLATIONS

To illustrate the behaviour of the precession axis in the presence of a perturbation we consider a simple example involving only synchrotron (energy) oscillations excited by an energy jump $\delta\gamma$ (< 0) resulting from the emission of a single photon. We neglect betatron oscillations, a good approximation away from the betatron spin resonances, and for the moment we ignore the radiation damping, which is small in one turn.

Before the photon emission the spin $\vec{P}$ of a particle is precessing around the axis $\vec{n}_0(\gamma_0)$ with no orbit oscillation, as in Fig. 2a. After photon emission the energy loss $\delta\gamma$ results in a new precession axis $\vec{n}(\gamma_0 + \delta\gamma)$ around which the spin $\vec{P}$ starts to precess, with $\vec{n}(\gamma_0 + \delta\gamma) = \vec{n}_0(\gamma_0) + \vec{\Gamma}(\delta\gamma/\gamma_0)$ to first order. The spin-orbit coupling vector $\vec{\Gamma} = \gamma \partial \vec{n} / \partial \gamma$ characterizes the sensitivity of $\vec{n}$ to energy fluctuations and oscillations.

In Fig. 2b we follow the evolution of $\vec{n}$ projected on a plane tangent to the unit sphere at $\vec{n}_0$, but considering successive one-turn transformations from θ to $\theta + 2\pi$, $\theta + 4\pi$, and so on. The corresponding precession axes are denoted $\vec{n}_1$, $\vec{n}_2$, etc. Now since we are neglecting radiation damping and considering only a single

Fig. 2 a) Perturbation of precession axis $\vec{n}$ from emission of a photon.
b) Evolution of one-turn precession axis with synchrotron oscillation.

quantum emission, the perturbation $\vec{\Gamma}(\delta\gamma/\gamma_0)$ has a constant magnitude and the locus of $\vec{n}_1, \vec{n}_2$, etc., must lie on a circle centered on $\vec{n}_0$. Furthermore, the phase advance on this circle between successive one-turn axes $\vec{n}_1, \vec{n}_2, \dots$ is evidently the synchrotron-oscillation phase advance per turn $\psi_s = 2\pi Q_s$, where Q_s is the synchrotron tune which has typically a value of around 0.1 in high-energy e^+e^- storage rings.

Meanwhile, the spin is precessing around $\vec{n}_1, \vec{n}_2, \dots$ with a phase advance of $\psi_p = 2\pi\nu$ per turn, also shown in Fig. 2b. A resonant situation occurs if $\psi_p = 2\pi k_0 + k_s \psi_s$, where k_0, k_s are integers, i.e. if:

$$\nu = k_0 + k_s Q_s .$$

Such resonances are excited by errors in the closed orbit with a non-vanishing Fourier component k_0 near to the spin tune ν ; the synchrotron oscillations modulate the perturbation, giving rise to spin satellite resonances of order k_s . These can cause depolarization even if the spin tune is not very close to the integer k_0 , especially in high-energy e^+e^- rings which have a relatively large Q_s and large-amplitude synchrotron oscillations.

NUMERICAL INTERLUDE

The scale of the diagrams in Fig. 2 has been exaggerated for clarity and it is instructive to examine typical numerical values. The emission of a single photon of critical energy $\hbar\omega_c$ causes a relative energy loss $\delta\gamma/\gamma_0$ of about 10^{-6} ; this is the parameter ξ of Eq. (1). We also saw in Eq. (5) that a magnitude $|\vec{\Gamma}| \approx 1$ gives a substantial degree of depolarization, and corresponds to an angular perturbation $\vec{n} - \vec{n}_0 = \vec{\Gamma}\delta\gamma/\gamma_0$ of 10^{-6} radian. Thus, first-order perturbation theory is quite adequate if $|\vec{\Gamma}| \gg 1$, i.e. not too close to a resonance.

We can compare this perturbation of 10^{-6} from a single photon with a simple classical representation of a beam 92.4% polarized, in which all the spins are imagined to lie on the surface of a cone of half angle $\arccos(0.924) = 0.39$ radian. This number, which characterises the average spread in spin orientation of an almost fully polarized beam, is large compared with the perturbation from a single photon emission. However, many photons are emitted randomly, about $5\pi\alpha\gamma/\sqrt{3}$ per turn, causing a random walk of $\vec{n}$ reaching 0.39 radian in a time which can be shown to be approximately equal to the polarization time. We thus have a simple semi-quantitative physical model of the spin-diffusion mechanism arising from quantum fluctuations of the precession axis, showing that $|\vec{\Gamma}| \sim 1$ corresponds to approximately equal polarization and depolarization times.

RADIATION DAMPING

In the presence of radiation damping of orbit oscillations the locus of $\vec{n}$ is no longer a circle as in Fig. 2b but a spiral converging on $\vec{n}_0$. For $|\vec{\Gamma}| \ll 1$ this spiral is very fine, deviating only slightly from a circle over one oscillation period. The spin vector $\vec{P}$ can then follow $\vec{n}$ adiabatically during the damping process; furthermore, the quantum fluctuations of $\vec{n}$ are also small so there is a high degree of coherence in the spin motion and therefore little depolarization.

In contrast near a resonance $|\vec{\Gamma}| \gtrsim 1$, and a loss of coherence arises both from the enhanced effects of the quantum fluctuations and from the coarser nature of the spiral, the latter inducing a non-adiabatic component of spin precession around $\vec{n}$.

It is important to draw a distinction between the effects of quantized energy loss on the orbital motion and on the spin motion. In the case of orbital motion, the average energy loss is recovered from the RF acceleration system, a classical process which leads to damping of oscillations and an equilibrium with the quantum excitation. Spin motion has, in the normal sense, no kinetic damping mechanism associated with it. Fortunately, spin-flip polarization fulfils an equivalent function but it can hardly be considered as analogous to orbit damping in view of the much longer time constant and the subtle quantum-mechanical asymmetry involved.

BETATRON OSCILLATIONS

The energy loss from photon emission gives rise to excitation of betatron oscillations because of the non-vanishing energy dispersion in a storage ring. As in the case of synchrotron oscillations a one-turn precession axis can be defined whose locus is a spiral together with a random walk. The spin resonance condition is then

$$\nu = k_0 + k_z Q_z$$

for vertical betatron oscillations, which are usually the more important. For $k_0 = 0$, pure betatron resonances are driven only by the normal oscillation amplitude; since $Q_z \gg 1$ these resonances are widely spaced. The presence of orbit-error harmonics introduces driving terms for $k_0 \neq 0$, which are modulated by the betatron oscillations, producing families of spin satellites analogous to those of synchrotron oscillations but with wider spacing in general.

WHAT MAKES $\vec{\Gamma}$ LARGE?

In a perfect storage ring without errors the orbit lies everywhere in the median plane and particles experience essentially only a vertical component B_z of the magnetic field, since the vertical oscillation amplitudes are exceedingly small. The precession axis $\vec{n}_0$ of the spin motion is then everywhere vertical, independent of energy and of horizontal betatron amplitude, and $\vec{\Gamma} = 0$ as a consequence.

Practical storage rings have errors due to magnet misalignments and field imperfections, causing the orbit to deviate from the ideal median plane. The focusing forces from quadrupoles and the end fields of bending magnets then subject particles both to radial field components B_x and to longitudinal fields B_y , which cause the precession axis $\vec{n}$ to vary with azimuth, energy and oscillation amplitudes. In high-energy e^+e^- storage rings, longitudinal field components from magnet ends are relatively weak and their effects on spin motion are further weakened by the factor (γa) in the Thomas-BMT equation. However, detector solenoids are normally strong enough that their influence on the spin motion must be compensated locally, either by "anti-solenoids" or by special orbit bumps.

The major contribution to large values of $\vec{\Gamma}$ remains the radial field components B_x associated with orbit deviations and energy dispersion in the vertical plane, though vertical betatron oscillations enhanced by coupling from the horizontal motion also play an important role. In addition, a storage ring equipped with spin rotators, to obtain longitudinal polarization at the interaction point, necessarily has some intentional vertical bending; the resulting $\vec{\Gamma}$ induced locally must be compensated nearby to prevent it propagating around the machine and generating a large contribution to $|\vec{\Gamma}|^2$ in Eq. (5).

VERTICAL ORBIT-ERROR HARMONIC

Vertical orbit deviations are of particular importance since they can combine with betatron and synchrotron oscillations to excite a wide spectrum of resonances given by the general form

$$\nu = \gamma a = k_0 + k_x Q_x + k_z Q_z + k_s Q_s .$$

The integer k_0 identifies the Fourier harmonics of orbit errors which can drive integer spin resonances; these are spaced throughout the spectrum at intervals of $mc^2/a \approx 440$ MeV. Considering these integer resonances alone we can expand the orbit deviation in a Fourier series:

$$z(\theta) = \sum_{k=1}^{\infty} z_k \cos(k\theta + \phi_k)$$

and take only the harmonic $k \sim \nu$ nearest to the spin tune which makes the dominant contribution. It can then be shown⁸ that the corresponding component of the spin-orbit coupling vector is given by

$$|\vec{\Gamma}|_k = \frac{2\nu^2 k^2}{(\nu^2 - k^2)} \frac{z_k}{R}, \quad (10)$$

where R is the average radius of the machine.

The resonance denominator demonstrates how $\vec{\Gamma}$ can become large if ν is close to an integer k for which there is an appreciable amplitude z_k of orbit harmonic. Equation (10) also shows how the sensitivity to orbit errors increases at high energies, being proportional to γ^3 for $k \approx \nu = \gamma a$.

More generally one must take account of the spectrum of perturbations including also those arising from betatron and synchrotron oscillations, and sum the perturbations over all significant terms.

Synchrotron-oscillation satellite resonances

A further complication arises at high energies because of the large energy spread of the beam, given by

$$\sigma_E = \left[\frac{55}{64\sqrt{3}} \hbar c \, mc^2 \right]^{\frac{1}{2}} \frac{\gamma^2}{\sqrt{\rho}},$$

leading to a correspondingly large spread in spin tune $\sigma_\nu = a\sigma_E$. When this spread occupies a large fraction of the space between integer resonances (1 in spin tune, 440 MeV in energy), particles in the Gaussian tails of the beam are unavoidably close to the integer resonances, as shown schematically in Fig. 3a. The situation is aggravated in the presence of synchrotron-oscillation spin satellites, which further reduce the space available for the spread in spin tune, as in Fig. 3b.

These spin satellites of the integer resonances are of considerable importance in LEP since they impose tight limits on the permissible strengths of nearby orbit harmonics⁹. The "error-sensitivity enhancement factor" C is the ratio by which the relevant orbit harmonics must be reduced in the presence of synchrotron-oscillation satellites compared with the case of no satellites; the example shown in Fig. 4 indicates that values of C approaching 10 can occur even if the spin tune ν is mid-way between two satellites. The apparent poles at the satellites are unrealistic, since the first-order perturbation theory used in Ref. 9 is inaccurate for large C .

Fig. 3 a) Integer resonances $\nu = k_0$. b) Synchrotron-oscillation spin satellites overlapping energy distribution function.

Fig. 4 Error-sensitivity enhancement factor C .

Preliminary results of a second-order theory (C. Biscari, paper in preparation) indicate that the resonances are indeed narrower but that the minima of C between the resonances are not reduced by taking account of second-order terms.

The nominal parameters of LEP in the 50 GeV energy region are such as to make the value of C rather critically dependent on the energy spread and on the synchrotron tune Q_s . This is unfortunate for predictive purposes but leaves open the possibility of using small changes in some machine parameters to minimize experimentally the depolarizing effects of the satellites.

POSSIBLE CURES

We have seen that the major cause of depolarizing resonances is the ubiquitous spectrum of vertical closed-orbit harmonics, which not only drive the integer spin resonances and their synchrotron-oscillation satellites but also extend the otherwise sparse spectrum of betatron resonances. The first priority is therefore to reduce the more important harmonics to an acceptably low level, which can be done in principle by arrangements of dipole orbit correctors excited in a suitable pattern.

The main problem arises in measuring the amplitude and phase of orbit harmonics in order to know what corrections to apply. Normal methods of correction aim to reduce the maximum orbit excursions, which are associated mainly with harmonics near to the betatron tune, and will even tend to excite higher harmonics. Now high-energy storage rings have spin tunes typically a factor of two greater than the betatron tunes and the corresponding orbit harmonics contribute very little to the orbit displacement. It is therefore very difficult, perhaps impossible, to measure and analyse the orbit to the precision required for these higher harmonics.

Failing direct measurement of the errors, one can measure the polarization of the beam and apply systematic, predictable compensation of the appropriate harmonics by a searching procedure in order to increase the polarization. This requires a fast, high-resolution polarimeter, which is no great problem, but also some initial measurable polarization level, around 5% or more.

The effects of machine errors on the spin motion may be reduced by a procedure known as "spin matching", proposed by Chao et Yokoya¹⁰ and extended by Steffen¹¹. This involves choosing the orbit parameters in such a way that ten integrals can be made to vanish, or at least become small. The spin matching conditions are energy dependent and, even at selected energies, it may not be possible to satisfy them all because of machine design constraints. Nevertheless this method looks promising for reducing the sensitivity of the spin motion to orbit perturbations. Spin matching could also

be used to reduce depolarization from pure betatron resonances¹² and from the non-linear beam-beam effect¹³.

Depolarization from synchrotron-oscillation spin satellites is due to the extended tails of the normal Gaussian energy distribution. Non-linear wiggler magnets could be used to produce stronger damping for large-amplitude synchrotron oscillations⁹ and modify the energy distribution function to reduce the density of the tails overlapping the satellites. The same non-linear wigglers which may be necessary at low energies in LEP to reduce collective effects¹⁴ could be operated in a different mode at higher energies to reduce the density of the energy tails.

POLARIZATION RATE AT THE LOWER ENERGIES

The natural polarization rate of a beam from the Sokolov-Ternov effect is a very steep function of energy, varying as γ^5 in a given machine as seen in Eq. (1). Although the nominal design energy of LEP is about 85 GeV, it will operate for a considerable time in the 50 GeV range where the natural polarization time is about 3 hours. This can be reduced by the installation of asymmetric polarizing wigglers¹⁵ consisting of a sequence of three dipole magnets bending the orbit in the horizontal plane. The centre magnet is short and strong with a field B_+ and the outer magnets are long and weak, with field B_- , such that the total field integral over the three magnets vanishes leaving the orbit outside the wiggler unperturbed. Since the rate of polarization varies as B^3 , it is strongly enhanced by the centre magnet and only weakly reduced by the outer ones, leading to an overall increase in polarization rate by a factor

$$\left(\frac{\tau_P^W}{\tau_P}\right)^{-1} = 1 + \frac{B_+^3 \eta_+ + 2|B_-|^3 \eta_-}{\langle B_Z \rangle B_Z^2},$$

where $\eta_{\pm} = \ell_{\pm}/2\pi R$ are the normalised lengths of the bending magnets, B_Z is the normal bending field in the machine arcs and $\langle B_Z \rangle$ is this field averaged over the whole circumference.

The price to pay for the enhanced polarization rate is a reduction in the asymptotic polarization level to

$$P(\infty) = \frac{8}{5\sqrt{3}} \frac{B_+^2 - B_-^2}{B_+^2 + B_-^2}$$

and an increase in both the synchrotron radiation power and the beam energy spread. A large ratio $|B_+/B_-|$ minimizes these undesirable effects.

For the LEP machine a first step in speeding up the polarization will be made by a small modification to the emittance-control wigglers,¹⁶ which are required in any case to control the beam size for optimum luminosity. Making these somewhat asymmetric detracts in no way from their primary function, costs very little and decreases the polarization time by a factor of 2 or 3, to give about 90 minutes at 50 GeV with a polarization level of around 75%.

POLARIZED BEAMS IN LEP

There are encouraging prospects that transversely-polarized beams could be observed in LEP fairly early in Phase 1 at around 50 GeV. To make this possible certain modest provisions must be made during the construction period of the machine.

The most important requirement is a laser back-scattering polarimeter which would be necessary even if polarized beams were not required, in order to avoid confusion of physics results by unsuspected partially-polarized beams. It was concluded at the DESY Workshop¹⁷ that an argon-laser polarimeter, similar to those used at SPEAR and PETRA, would be quite suitable for LEP. However, a faster response and somewhat better resolution would be desirable, and a Nd-YAG laser, as recently tested at PETRA, might be preferred.

The normal LEP orbit-correction magnets will be largely adequate for compensating harmonics near the spin tune, though a few extra may have to be installed at strategic points for the best results. Special correction algorithms will be needed for minimizing these higher harmonics.

Simulation of machine errors and their influence on spin resonances will be used to determine the most favourable energies and beam-optics parameters for obtaining a measurable degree of polarization initially, and for working out correction strategies. The possibility of adjusting the optics parameters to satisfy the spin-matching constraints will also be examined.

In anticipation of success in obtaining transverse polarization, some effort will be devoted to studies of spin rotators for longitudinal polarization at the interaction points, and dedicated asymmetric wigglers should be studied for increasing further the rate and level of polarization.

During early operation in Phase 1, initial polarization studies will be made under the most favourable conditions possible. These include a low beam current to avoid instabilities, detuned low- β sections to reduce orbit errors and a suitable choice of energy as suggested by theory and computer simulations. In addition one would make full use of possibilities for varying critical machine parameters such as Q_s .

With these precautions we would expect to find a measurable level of polarization which could then be improved by application of the orbit-correction schemes, until finally we would hope to achieve a useful degree of polarization under normal operational conditions for physics.

The apparent optimism on which the above scenario is based stems from a number of causes. The earlier theoretical work of the Novosibirsk group has become more widely disseminated and better understood in recent years, and its extension to the higher range of energies has been considerably clarified. Experimental results from PETRA have demonstrated the feasibility of spin-resonance compensation¹⁷ and also shown that beam-beam depolarization is much less serious than had previously been believed¹⁷. The increasing interest in spin motion that has been developing in the last few years, particularly for electron storage rings, will stimulate new ideas and increase the chances of overcoming the extra difficulties at the higher energies. Finally, the physics motivation for polarized e^+ and e^- beams is now much stronger than had been the case for the smaller storage rings at lower energies¹⁸.

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