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GRAVITATION AND SYMMETRY BREAKING

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A B S T R A C T

Among the possible vacua, quantum gravity within the stationary phase approximation selects the vacuum with vanishing cosmological constant. Classical background gravity coupled to matter renders the highest Higgs potential minimum stable, allowing for spontaneous supersymmetry breaking. The influence of the non-renormalizability of supergravity on the symmetry breaking pattern is studied.

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1. The fundamental interactions in particle physics are described by spontaneously broken gauge theories. In such theories, the vacuum energy and thus the cosmological constant are generally non-zero. However, all observations indicate that the cosmological constant is zero. It is natural that this apparent dilemma can only be solved once gravity is coupled to matter.

Supergravity¹⁾ (or local supersymmetry) is likely to be the candidate for a quantum gravity. For example, it is finite up to two loops on-mass shell²⁾. The close connection of gravity and matter interactions makes supergravity a natural framework for discussing the problem of the cosmological constant.

Weinberg³⁾ recently pointed out how gravity might affect the other interactions. The supergravity scalar potential has several supersymmetric (SUSY) vacua. If one adjusts one of them to have zero energy (zero cosmological constant), the others have negative energy density. Although one would expect the former to decay, Weinberg argues that gravity can make it stable.

In this note we sketch an alternative approach. Some of our results are not new; but in light of the recent discussion about the question of which minimum is realized in a theory with gravity, we feel it is important to emphasize them. We first show that if gravity is quantized in a canonical path integral formalism, it selects the vacuum with zero cosmological constant in the stationary phase approximation. This result holds modulo quantum renormalizations.

We also consider the above issue in a more phenomenological way, where gravity is considered as a classical background field interacting with matter^{4),5)}. This approach leads to the surprising result that the vacuum with the largest matter energy is in fact stable against quantum decay. It leads also to Weinberg's result when applied in the same context. In the case of global SUSY, where the SUSY-breaking vacuum is always higher than the SUSY-preserving one, it leads to spontaneous SUSY breaking. Similar spontaneous SUSY breaking can be realized in supergravity.

In supergravity, the Planck scale $M_P \sim 10^{19}$ GeV enters the scalar potential. This can lead to unwanted vacua, where the symmetries are already broken at M_P , instead of their relevant scale (say, $M_X \sim 10^{15}$ GeV, for grand unification). We investigate some examples of such minima and their rôle in symmetry breaking.

2. Consider for brevity a scalar particle coupled to gravity, with action

$$S = \int d^4x \left\{ -\frac{1}{16\pi G} e R + e V(\phi) + \frac{1}{2} e g_{\mu\nu} (\partial^\mu \phi)(\partial^\nu \phi) \right\}, \quad (1)$$

where $g_{\mu\nu}$ is the metric tensor, ϕ the scalar field, $e = \sqrt{\det g}$, R is the Riemann scalar and G is the Newton constant. $V(\phi)$ is an arbitrary scalar potential which may include a cosmological constant. In Eq. (1), we have already rotated time to imaginary time. From the action S one can define quantum gravity by the canonical path integral formalism through the partition function

$$Z = \int \mathcal{D}g_{\mu\nu} \mathcal{D}\phi e^{-S}, \quad (2)$$

modulo ghosts.

The quantum theory defined by (2) has serious ultra-violet divergences in the gravity sector which cannot be renormalized. Overlooking this, we proceed to make the semi-classical approximation. We determine a stationary minimum of the action for fields $\phi, g_{\mu\nu}$ which are uniform in space-time:

$$\left. \frac{\delta S}{\delta \phi} \right|_{\phi = \text{const.}} = e \left. \frac{\delta V}{\delta \phi} \right|_{\phi = \text{const.}} = 0; \quad (3)$$

$$\left. \frac{\delta S}{\delta g_{\mu\nu}} \right|_{g_{\mu\nu} = \text{const.}} = \frac{1}{2} g^{\mu\nu} e V_{\min} = 0. \quad (4)$$

$V_{\min}$ is the value of the scalar potential at its minimum [obtained from (3)] and is identified with the cosmological constant. As long as $\sqrt{\det g} \neq 0$, $V_{\min} = 0$. Thus the cosmological constant vanishes in the stationary phase approximation. The extrema defined by (4) are classically stable with respect to gravitational fluctuations.

Condition (4) cannot always be realized; in any case, it implies fine tuning of parameters. If (4) does not hold, then the stationary phase approximation does not exist.

3. Quantum gravity so far remains ill-defined. However, one expects that at energy scales lower than the Planck mass $1/\sqrt{G}$, one can neglect quantum fluctuations of gravity. One may then include gravitational effects through the classical background gravity formalism^{4),5)}. In this description the action is as in (1) and the partition function is defined with only the matter fields integrated. Implicitly we assume G to be small and $g_{\mu\nu}$ is now a space-time dependent background metric satisfying Einstein's equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu}. \quad (5)$$

$R_{\mu\nu}$ is the Ricci tensor and $T_{\mu\nu}$ is the stress-energy tensor of the quantized matter fields. The quantized stress-energy tensor can be well defined only for a renormalizable theory^{5),6),7)}. For a non-renormalizable theory, we shall take $T_{\mu\nu}$ to be the classical stress-energy tensor.

Again, we find a stationary phase for the space-time uniform field ϕ yielding the minima constraint (3). Using $T_{\mu\nu}|_{\min} = g_{\mu\nu} V_{\min}$ in (5) and contracting with $g^{\mu\nu}$, we obtain

$$R|_{\min} = 32\pi G V_{\min}. \quad (6)$$

Hence, the total action of the matter and gravitational field is

$$S|_{\min} = \int d^4x (-e V_{\min}). \quad (7)$$

Typically there is more than one minimum. The minimum with the lowest action (per unit four volume), that is, with the highest $V_{\min}$, is stable against quantum decay. This is a surprising result in contradistinction to pure matter considerations alone.

In the case of supergravity, this result implies that, restricting ourselves to supersymmetric vacua, the one with zero cosmological constant is stable, confirming the conclusion of Weinberg^{3)*)}.

4. Consider a global supersymmetric theory. A general Higgs potential has the form

$$V = \left| \frac{\partial f(z)}{\partial \bar{z}} \right|^2 - \Lambda, \quad (8)$$

where we explicitly include a constant Λ , and f is a function of the complex scalar field. If, in addition to the SUSY (S) minima with vanishing $\partial f/\partial z$, the potential also allows for SUSY breaking (S.B.), then

$$V_{\min}^{\text{S.B.}} > V_{\min}^{\text{S.}} \quad (9)$$

We can also adjust Λ so that

$$V_{\min}^{\text{S.B.}} = 0. \quad (10)$$

If we couple this to background gravity^{**)}, the previous analysis tells us that the most stable vacuum (at zero temperature) has spontaneous SUSY breaking with zero cosmological constant.

In supergravity, one cannot have an explicit cosmological constant as in (8). It is possible to break SUSY spontaneously. A well-known example is the Polony⁸⁾ potential

$$V = e^{zz^*} \left(\left| \frac{\partial f}{\partial \bar{z}} + z^* f \right|^2 - 3|f|^2 \right),$$

with $f(Z) = Z + \beta$. For $\beta \geq 2.6$, (9) is satisfied⁹⁾, but not (10). However, in more complicated potentials, both (9) and (10) could be realized.

*) Whereas in Ref. 3), the condition $|V_{\min}| \leq M_x^6 / M_p^2$ with $M_x \sim 10^{15}$, $M_p \sim 10^{19}$ must be met, our result holds also, if $|V_{\min}| \leq M_x^4$.

**) This theory can be understood as the limit of supergravity when the metric becomes a classical field and the gravitino is eliminated. It is invariant under metric dependent global supersymmetric transformations.

5. In supergravity, the potential contains non-renormalizable terms, suppressed by inverse powers of M_p . These new terms allow new minima. In light of the previous analysis, the new minima could be relevant. To understand the effects of these terms, we consider the potential of a scalar ϕ in the adjoint representation of $SU(N)$. It is well known that ϕ can be taken to be a real diagonal traceless $N \times N$ matrix with elements ϕ_i , $i = 1, \dots, N$, $\sum \phi_i = 0$. The potential is then a function of the invariants $I^n = \text{tr}(\phi^n)$, $n = 2, \dots, N$:

$$V = f(I_2, \dots, I_N) + \lambda I^1$$

$$= \alpha m^2 I^2 + \beta m I^3 + \gamma (I^2)^2 + \delta I^4 + V^{n.r.} + \lambda I^1, \quad (11)$$

where m is the scale associated with ϕ and the Lagrange multiplier λ is added to enforce the tracelessness condition. The first terms in (11) are the renormalizable ones, and $V^{n.r.}$ contains the inverse powers of M_p . In the absence of $V^{n.r.}$, at an extremum of (11), there are at most three different values for the ϕ_i ¹⁰⁾ ($dV/d\phi_i = 0$ is a cubic equation). At a minimum, only two are different, leading to a breaking of $SU(N)$ into $SU(M) \times SU(N-M) \times U(1)$ ^{10), 11)}, $M < N$, in agreement with Michel's conjecture ¹²⁾.

The extremization of (11) leads to

$$0 = \left\langle \frac{\partial V}{\partial \phi_i} \right\rangle = \left\langle \sum_{n=2}^N n \frac{dV}{dI^n} \cdot \phi_i^{n-1} \right\rangle + \lambda; \quad i = 1, \dots, N. \quad (12)$$

These equations have at most $N-1$ different solutions $\langle \phi_i \rangle$, leading to at least the conserved symmetry group $SU(2) \times [U(1)]^{N-2}$. Furthermore, if $\partial V / \partial I^n = 0$ for all n , then all ϕ_i can be different, leading even to the breaking $SU(N) \rightarrow [U(1)]^{N-1}$ ^{*)}. Michel's conjecture does not apply here ¹³⁾. We conclude that non-renormalizable terms may induce new symmetry breaking patterns.

In practice, $M_p \gg m$. Then it is easy to see that there are "old" solutions, i.e., solutions with $\langle \phi_i \rangle \sim m$ and "new" solutions, with $\langle \phi_i \rangle \sim M_p$. (This is analogous to the quadratic equation $c + bx + a^{-1}x^2$, for $a \rightarrow \infty$. One solution tends smoothly towards the solution of $c + bx = 0$, the other grows with a .)

*) A potential which satisfies this criterion is, for instance, $V = \sum C_n (I^n - x_n)^2$, with global minimum $I^n = x_n$ if $C_n \geq 0$.

The old solutions lead to the same symmetry breaking pattern as the renormalizable potential. To see this, we write $\vec{\phi} = \vec{a} m + m/M_p \vec{b} m + \dots$ ($\vec{\phi} = \{\phi_1 \dots\}$) where the dots denote higher powers in m/M_p , and the a_i are of order unity (if the coupling constants of the potential are not ridiculously small). At zero order in M_p , the a_i can take only two different values. But can $\vec{b}$ point in a different direction from $\vec{a}$? If this were the case, one could have the interesting situation where there is an automatic breaking hierarchy $\sim m/M_p$ (*). But this cannot be so. The extremal conditions for two ϕ_i with the same a_i but, for the moment, different b_i (say, $i=1,2$) are of the form:

$$\begin{aligned} m^2 p \phi_1 + m q \phi_1^2 + r \phi_1^3 + \frac{\phi_1^4}{M_p} + \dots &= 0, \\ m^2 p \phi_2 + m q \phi_2^2 + r \phi_2^3 + \frac{\phi_2^4}{M_p} + \dots &= 0. \end{aligned} \quad (13)$$

We form the difference of these equations and expand in $1/M_p$:

$$(b_1 - b_2) \frac{m^4}{M_p} (p + 2q a_1 + 3r a_1^2) = 0. \quad (14)$$

Either $b_1 = b_2$, or

$$p + 2q a_1 + 3r a_1^2 = 0$$

must be satisfied, along with

$$p + q a_1 + r a_1^2 = 0.$$

These two equations are compatible only if $4pr = q^2$. Barring unwanted fine tuning, the b_i are equal. The above reasoning can be generalized to arbitrary potentials; only the new solutions can generate new breaking patterns.

These possible new solutions, which are clearly unwanted, have a vacuum energy density $\sim M_p^4$. For such a large energy density, the quantum nature of gravity must be taken into account, and one should use the results of Section 2, where we find that the vacuum energy must vanish. Thus, if we fine tune an old solution to have zero energy density, the new ones are not really minima in the sense of Eqs. (3) and (4), and the fine tuning can be sustained.

*) For example, $SU(3)$ could be broken to $SU(2) \times U(1)$ at m , and to $U(1) \times U(1)$ at m^2/M_p .

6. To summarize the results:

- 1) In quantum gravity with matter, within the stationary phase approximation, it is natural to have $V_{\min} = 0$.
- 2) Classical background gravity coupled to matter, surprisingly, makes the vacuum with highest matter energy density stable.
- 3) As an application of the above, we find that when background gravity is coupled to globally supersymmetric matter, spontaneous symmetry breaking can be realized. This can also be implemented in supergravity.
- 4) Non-renormalizable terms in the potential usually induce undesirable effects such as yielding unwanted symmetry breaking patterns. The vacuum energy density is then typically $\sim M_p^4$. This problem can only be dealt with inside a quantum gravity. The arguments in Section 2 [point 1) above] suggest that when a vacuum with a correct symmetry breaking pattern is fine tuned to have zero cosmological constant, the unwanted breaking patterns do not occur.

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