Ref.TH.3571-CERN

CLOSING THE WINDOW TO ASYMPTOTIC FREEDOM
WITH THE MIXED ACTION

K. H. Mütter ^{*)}
CERN — Geneva

and

T. Freimuth and K. Schilling
Physics Department, University of Wuppertal

ABSTRACT

We complement recent investigations of Bhanot and Dashen by presenting new Monte Carlo results for the mass gap in pure lattice gauge theory with the mixed fundamental adjoint SU(2) action on a 8^4 lattice. We find no signal for asymptotic freedom at $\beta_A = 1.21$.

*) On leave of absence from Physics Dept.,
University of Wuppertal, FRG.

A year ago, Bhanot and Dashen ¹⁾ investigated the universality of the string tension as obtained by blending into the usual Wilson action of pure SU(2) gauge theory, a piece due to the adjoint representation of the gauge group ²⁾ proportional to β_A :

$$S = \frac{\beta}{2} \sum_P \text{tr } U(P) + \frac{\beta_A}{4} \sum_P (\text{tr } U(P))^2 \quad (1)$$

[$U(P)$ is the usual plaquette variable].

They produced high statistics Monte Carlo data for the string tension σ at the particular point $\beta_A = 1.21$. Their data lack a convincing signal for asymptotic freedom; in particular, the envelope of the various measurements of Creutz ratios $\chi(I)$ appear to have a significantly steeper slope than produced by asymptotic freedom (AF). Nevertheless, these authors extracted a value for the renormalization group scale

$$\Lambda(\beta_A = 1.21) = (3 \pm 3) \cdot 10^{-3} \sqrt{\sigma} \quad (2)$$

by drawing a tangent with the AF slope to part of their data in the interval $1.5 \leq \beta \leq 1.6$. Assuming universality of the string tension σ , the Monte Carlo data for $\beta_A = 0$ and $\beta_A = 1.21$ yield a ratio of $r = \Lambda(1.21)/\Lambda(0)$ which disagrees by a factor of four with the perturbative prediction as obtained by background field methods ³⁾ within the one-loop approximation. This discrepancy has prompted a series of papers that try to improve the perturbative β_A dependence of r ⁴⁾. On the other hand, Gonzalez Arroyo et al. ⁵⁾ cast some doubt on the weak coupling interpretation of Bhanot's and Dashen's data, by demonstrating the string tension to follow largely a pattern predicted by a strong coupling expansion for $\beta_A > 0$.

It has been suggested that ratios of physical masses might be more sensible quantities for the study of universality ¹⁾. In that spirit, the authors of Ref. 6) determined the deconfinement temperature T_c in the mixed action case, and confirmed the value of r as quoted by Dashen and Bhanot ¹⁾. They conclude that $T_c/\sqrt{\sigma}$ is a universal quantity under change of β_A , $\beta_A > 0$.

In this somewhat confusing situation, it appears worth while to study a third physical quantity in order to clarify the situation. In this paper we therefore intend to present high statistics Monte Carlo data on the mass gap ξ^{-1} at the value $\beta_A = 1.21$.

We determine ξ by the long distance correlation method as advocated previously by two of us ^{7),8)}. With that method, the mass gap can be obtained by measuring the response of local operators such as

$$O_1(P) = \cos \Theta(P) = \frac{1}{4} \text{tr} \{U(P) + U^\dagger(P)\} \quad (3)$$

$$O_2(P) = \cos^2 \Theta(P) - \langle \cos \Theta(P) \rangle^2 \quad (4)$$

averaged over plaquettes "deep" inside the lattice to a change in boundary conditions. To be more precise, we define as in Ref. 7) the quantities $U_i(d)$ ($L_i(d)$) as the expectation values of the operators O_i on plaquettes P with distance $\geq d$ from the lattice surface with fixed (free) boundary conditions.

In Figs 1a and 1b, we plot the observed differences :

$$\Delta_i(d) = U_i(d) - L_i(d) \quad (5)$$

for various fixed values of d versus β . The Monte Carlo points exhibit a pronounced peak around $\beta \approx 1.4$ in both Δ_1 and Δ_2 . This peak is not hard to understand, its position is close to the end-point singularity of the phase transition line in the (β, β_A) plane, as Bhanot and Creutz ²⁾ determined it, sitting at :

$$(\beta, \beta_A) = (1.48 \pm .05, 1.2 \pm .04) \quad (6)$$

This peak is expected to entail a peak in the correlation length and thereby in the quantities Δ_i which were shown previously to be simply sums of correlation functions ⁷⁾.

It is illustrative to compare this peaking β dependence with the one obtained from the pure Wilson action, plotted as dotted lines in Figs. 1a,b as well. Obviously approaching the end point singularity amounts to a drastic narrowing of the broad structure observed with Wilson's action. Moreover, the height of the peak in $\Delta_i(d)$ appears to become less dependent on d than at $\beta_A = 0$. We would like to stress that this whole pattern is drastically different from the universality in Δ_i under change from Wilson's to Manton's action reported previously ⁸⁾.

The behaviour of $\Delta_i(d)$ implies a pronounced dip in the mass gap $\xi^{-1}(\beta)$ as shown in Figs. 2a and 2b. We observe that the rate of descent in $\xi^{-1}(\beta)$ is markedly larger (by a factor 2-3) than the prediction of AF, which is drawn for reference.

In Fig. 3 we finally present a combined plot of ξ^{-1} (as from Fig. 2) and the square root of the Creutz ratios $\sqrt{\chi(I)}^{1)}$ as functions of β . The latter data have been shifted up by a factor 2.2 ^{*)} in order to facilitate the comparison of the envelopes of ξ^{-1} and $\sqrt{\chi}$. It strikes the eye that both envelopes carry the same slope ! Note, however, that the size of the slope is off by a factor 2-3 from the AF value. Note, moreover, that the determination of $\Lambda(1.21)$, as quoted in Eq. (2), rests on the "tangent" with the AF slope drawn in Fig. 3 !

Our conclusion : we see no evidence whatsoever for AF behaviour in the Monte Carlo results, both for $\sqrt{\chi}$ and ξ^{-1} at $\beta_A = 1.21$. But there is no reason to be frightened about the status of lattice methods in gauge theories. The preceding exercise teaches us only that one has been going too far in the wrong direction when modifying the good old Wilson action in the above manner.

Any change of action by the addition of terms irrelevant with respect to the continuum limit will enlarge or diminish on a finite lattice the β window available for the study of continuum physics. Since we have obviously managed to close this β window at $\beta_A = 1.21$, we can neither determine properly a value for Λ nor must we worry about perturbation theory to explain any anomaly in that quantity. Instead, we should orient future research towards the construction of improved actions with larger β windows ⁹⁾.

Acknowledgements

We thank Dr. G. Bhanot for an interesting discussion.

*) Strangely enough, this value of 2.2 equals the universal ratio for $m_g/\sqrt{\sigma}$ obtained from Wilson's and Manton's action, which does imply AF ⁸⁾.

R E F E R E N C E S

- 1) G. Bhanot and R. Dashen, Phys.Letters 113B (1982) 299.
- 2) L. Caneschi, I.G. Halliday and A. Schwimmer, Nuclear Phys. B200 [FS4] (1982) 409;
G. Bhanot and M. Creutz, Phys.Rev. D24 (1981) 3212.
- 3) R. Dashen and D.J. Gross, Phys.Rev. D23 (1981) 2340.
- 4) H. Sharatchandra and P. Weisz, DESY preprint 81-083 (1981);
Yu.M. Makeenko and M.I. Polikarpov, ITEP preprint 152 (1981);
B. Grossman and S. Samuel, Phys.Letters 120B (1983) 383.
- 5) A. Gonzalez Arroyo, C.P. Korthals Altes, J. Peiro and M. Perrottet, Phys.Letters 116B (1982) 414.
- 6) R.V. Gavai, F. Karsch and H. Satz, Bielefeld preprint BI-TP 82/26 (October 1982).
- 7) K.H. Mütter and K. Schilling, Phys.Letters 117B (1982) 75.
- 8) K.H. Mütter and K. Schilling, Phys.Letters 121B (1983) 267.
- 9) K. Symanzik, in "New Developments in Gauge Theories", Eds. G. 't Hooft et al., Plenum Press, New York (1980) p.313;
P. Weisz, Nuclear Phys. B212 (1983) 1;
G. Martinelli, G. Parisi and R. Petronzio, Phys.Letters 114B (1982) 251;
B. Berg, S. Meyer, I. Montvay and K. Symanzik, "Improved continuum limit in the lattice $O(3)$ non-linear sigma model", DESY preprint (April 1983).

FIGURE CAPTIONS

Figure 1a : The difference $\Delta_1(d)$ computed according to Eq. (5) at $\beta_A = 1.21$, using the correlation of the operator O_1 from Eq. (3) to the boundary of an 8^4 lattice. The full lines connect the points in order to guide the eye. The dashed lines refer to the shape of the respective results drawn from use of the Wilson action ⁷⁾ (normalized and shifted in β to roughly match the full lines).

Figure 1b : Same as Fig. 1a, except that the operator O_2 [Eq. (4)] has been used.

Figures 2a,b : The mass gap ξ^{-1} vs β as obtained from the data in Figs. 1a,b, respectively.

Figure 3 : Combined view of ξ^{-1} (from Fig. 2a) and $\sqrt{X}$ [from Ref. 1)]. The values of $\sqrt{X}$ have been scaled up by a factor 2.2 in order to match the envelope to the ξ^{-1} data. The three lines represent the asymptotic freedom fit performed in Ref. 1).

Fig. 1a

Fig. 1b

Fig. 2a

Fig. 2b

Fig. 3