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THE CHARGED CURRENT COUPLINGS AND CP VIOLATION
IN THE B MESON SYSTEM

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A B S T R A C T

We consider a lifetime range for the B meson from the observed upper limit down to $0.3 \cdot 10^{-12}$ sec. Under this constraint all elements of the flavour mixing matrix are strongly restricted. By fitting the ϵ parameter in K^0 decay, a relatively large CP violating phase angle is required.

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The mixing of the quark fields as they enter weak interactions is decisive for all flavour changing weak interaction processes. Some progress in determining or restricting the elements of the Kobayashi-Maskawa matrix¹ has already been reported². In the present paper we consider a particularly interesting range of values for the lifetime of the B-meson, namely a region around 10^{-12} sec which is presently studied using vertex detectors³. In case the B-meson lifetime turns out to be not too different from this value, very interesting consequences arise: We find that the magnitudes of all matrix elements are determined with small uncertainties. Even $\sin \delta$, the CP-violating quantity, is limited to be relatively large. These results allow us to discuss particle-anti-particle mixings and CP-violating effects, in particular of the B-meson system.

Our experimental input is the following:

- 1) $|V_{ud}|$ (the ud matrix element of the mixing matrix) = 0.973 ± 0.0024 as obtained from μ -meson and nuclear β -decay.
- 2) $|V_{us}| = 0.227 \pm 0.016$ from a Cabibbo-analysis of K_{13} and Hyperon-decays.
- 3) $|V_{cd}| = 0.24 \pm 0.03$ from opposite sign dimuon production by neutrinos.
- 4) Results from the study of semileptonic decays of B-mesons⁴, where the analysis⁵ of the lepton momentum spectra gives

$$\Gamma(B \rightarrow \mu \nu x_u) / \Gamma(B \rightarrow \mu \nu x_c) \lesssim .1$$

(90% confidence level). This corresponds to

$$|V_{ub}/V_{cb}| \lesssim 0.2.$$

The new input is a limited range for the B-meson lifetime:

- 5) $0.3 \times 10^{-12} \text{ sec} \lesssim \tau_B \lesssim 1.4 \times 10^{-12} \text{ sec}.$

The upper bound was set by the Jade group⁶. The lower bound which

we use here is of interest for two reasons: i) it is within the reach of present experiments, ii) a positive result within the above range would lead to an almost unique mixing matrix as will be demonstrated below.

We find it convenient to work with a parametrization of the charged current mixing matrix as suggested by Maiani⁷ ($S_\theta = \sin \theta$, $C_\theta = \cos \theta$ etc.)

	d	s	b
u	$C_\beta C_\theta$	$C_\beta S_\theta$	S_β
c	$-S_\beta S_\gamma C_\theta e^{i\delta'}$	$C_\gamma C_\theta - S_\beta S_\gamma S_\theta e^{i\delta'}$	$S_\gamma C_\beta e^{i\delta'}$
t	$-S_\beta C_\gamma C_\theta + S_\gamma S_\theta e^{-i\delta'}$	$-S_\beta S_\theta C_\gamma - S_\gamma C_\theta e^{-i\delta'}$	$C_\beta C_\gamma$

The total width of the B-meson is obtained from a quark decay graph analyses to be⁸

$$\Gamma^B = \Gamma_0^B \left[2.75 S_\gamma^2 + 7.7 S_\beta^2 + \mathcal{O}(S_\gamma^4, S_\beta^2 S_\gamma^2) \right] \quad (1)$$

$$\Gamma_0^B \equiv \frac{G^2 m_b^5}{192 \pi^3} = 1.08 \times 10^{14} \left(\frac{m_b}{5 \text{ GeV}} \right)^5 \text{ sec}^{-1}$$

The angles β and γ give directly the amplitudes for the decay into u and c-quarks, respectively. Eq. (1) contains QCD and phase space

corrections. The main uncertainty is the effective mass of the b-quark. We use $m_b = 5.0 \pm 0.2$ GeV. We allow β , γ , and θ to lie in the first quadrant ($0 < \beta, \gamma, \theta < \pi/2$); in this case the phase δ' must be allowed to lie in its full range ($-\pi < \delta' < \pi$). From the constraints quoted above, and with the aid of the unitarity of the mixing matrix, one finds

$$\begin{aligned} 0.04 &\lesssim \sin \gamma \lesssim 0.12 \\ 0. &\lesssim \sin \beta \lesssim 0.02 \\ 0.222 &\lesssim \sin \theta \lesssim 0.243 \end{aligned} \quad (2)$$

and $|v_{cs}| \gtrsim 0.97$. It is evident from these values that the small angle approximation $s_\theta \approx \theta$, $s_\beta \approx \beta$, $s_\gamma \approx \gamma$ is justified. The mixing matrix takes the simplified form

$$\begin{pmatrix} 1 & \theta & \beta \\ -\beta \gamma e^{i\delta'} - \theta & 1 & \gamma e^{i\delta'} \\ -\beta + \gamma \theta e^{-i\delta'} & -\beta \theta - \gamma e^{-i\delta'} & 1 \end{pmatrix} \quad (3)$$

The next topic concerns the estimate of β and the CP-violating phase δ' . For this purpose we calculate the CP-violating parameter ϵ_K , observed in the $K^0 - \bar{K}^0$ system. Generally one has

$$\epsilon = \frac{M_{21} - M_{12}}{M_{21} + M_{12} + 2\sqrt{M_{12} M_{21}}} \quad (4)$$

where $M_{12} = m_{12} - i \gamma_{12}/2$, $M_{21} = m_{12}^* - i \gamma_{12}^*/2$ and m_{12} and γ_{12} are the dispersive and absorptive parts of the off-diagonal element M_{12} in the particle - antiparticle mass matrix. For the $K^0 - \bar{K}^0$ system one can use

the approximate relation

$$2\sqrt{2} |\epsilon_{K^0}| \approx \frac{\text{Im } m_{12}}{\text{Re } m_{12}} \quad (5)$$

Eq. (5) holds because experimentally

$$\text{Im } \eta_{12} \ll \text{Im } m_{12} \quad \text{and} \quad \Delta M \approx -\frac{\Delta\Gamma}{2} \quad (6)$$

where ΔM and $\Delta\Gamma$ are the differences in mass and widths of K_L and K_S . For the evaluation of ϵ_{K^0} we use the well-known box diagrams, i.e. we replace M_{12} by $M_{12}^{(\text{Box})}$. When the mass of the t-quark does not exceed 60 GeV, the real part of $m_{12}^{(\text{Box})}$ is proportional to the square of the charmed quark mass. Additional contributions to $\text{Re } m_{12}$ from the t-quark are negligible because of the smallness of β and γ . An approximate representation for ϵ_K is⁹

$$2\sqrt{2} |\epsilon_K| = \frac{2\beta\eta \sin \delta'}{\theta} \left\{ -1 + \frac{\eta_3}{\eta_1} \ln \frac{m_t^2}{m_c^2} + \frac{\eta_2}{\eta_1} \frac{m_t^2}{m_c^2} \left[\eta^2 - \frac{\beta\eta}{\theta} \zeta \right] \right\} \quad (7)$$

For values of the top quark mass as high as 60 GeV, formula (7) together with (2) gives the bounds

$$2 \times 10^{-2} \geq \beta \geq 4 \times 10^{-4}$$

and

$$\begin{aligned} \sin \delta' &\geq 0.01 \quad \text{for } \cos \delta' < 0 \\ \text{or } &\geq 0.05 \quad \text{for } \cos \delta' > 0 \end{aligned} \quad (8)$$

For values of the top quark mass around 30 GeV and a lifetime

close to the upper bound, β and δ' are still more restricted

$$1.2 \times 10^{-2} \geq \beta \geq 0.6 \times 10^{-2}$$

$$\sin \delta' \geq 0.50 \quad (9)$$

Because of the smallness of γ , which was the starting point of this study, β and δ' are much restricted. Very small values for the CP-violating parameter δ' are excluded for the range of the B-meson lifetime considered.

Our approach of simply using the box diagram formulas can be questioned. Only the short range contributions are described correctly after the QCD corrections¹⁰ have been employed. In principle, long range QCD corrections on the box diagram (considered as the most general flavor flow diagram for the $K^0 - \bar{K}^0$ matrix element) should also be taken into account. However, long range effects can not be computed reliably. To some extent the significance of the box diagram formulas can be tested by the $K_L - K_S$ mass difference:

$$\frac{1}{2} \Delta M_{K^0} = \text{Re } m_{12}^{(\text{Box})} \quad (10)$$

The value of the reduced matrix element

$$X = \left\langle K^0 \left| \left[\bar{d} \gamma_\mu (1 - \gamma_5) s \right]^2 \right| \bar{K}^0 \right\rangle \quad (11)$$

is then a measure of the unknown corrections. According to ref. (11)

X is bounded by hadronic sum rules

$$X \leq (1.5 \pm 0.2) \frac{4}{3} f_K^2 M_{K^0} \quad (12)$$

Using (10) we find

$$X = (1.2 \pm 0.1) \frac{4}{3} f_K^2 M_{K^0} \quad (13)$$

for the range of angles and the top quark masses considered here.

Consequently the long-range contribution to $\text{Re } m_{12}$ is small.

Thus, X is in agreement with the bound and very close to the value obtained by vacuum insertion. Because of the smallness of β and γ

the formula

$$\Delta M_K = \frac{G^2 m_c^2}{8\pi^2} \eta_1 s_\theta^2 c_\theta^2 X \quad (14)$$

is a good approximation of the box diagram calculation. Using again the box diagram formula for γ_{12} the first of the formulas in (6) is verified.¹² With regard to the process $K_L^0 \rightarrow \mu^+ \mu^-$ we find that the box diagram contribution¹³ for this reaction is much smaller than the first order weak process via 2 photons. Even though the experimental information on the $K_L^0 \rightarrow \mu^+ \mu^-$ decay in general restricts β and γ , the values determined in this paper are well within the allowed region.

The above discussion gives us some confidence for the use of the box diagrams. We use therefore the exact box diagram formulas given in reference 14, in which the effects of a finite M_W and QCD corrections are included. These accurate numerical studies satisfy all constraints quoted above and lead to

$$\begin{aligned} \sin \delta' & \begin{cases} \geq 0.03 & \text{For } \cos \delta' < 0 \\ \geq 0.07 & \text{For } \cos \delta' > 0 \end{cases} \\ 1.5 \times 10^{-2} & \geq \beta \geq 7 \times 10^{-4} \\ 0.11 & \geq \gamma \geq 0.05 \end{aligned} \quad (15)$$

in agreement with the rough estimates given above. In figures (1 a-c) we present the allowed values for β and δ for $\gamma = 0.06$, which corresponds to B lifetimes, $0.77 \times 10^{-12} \text{ sec} \leq \tau_B \leq 0.95 \times 10^{-12} \text{ sec}$. Note that the CP phase angle $\sin \delta' \geq 0.1$. A smaller value of the lifetime would of course extend the β, δ' ranges to smaller values. Because of the interest in a large phase angle δ' , we give in table 1 the mixing and CP-violation in the $B-\bar{B}$ and $D-\bar{D}$ systems for $\sin \delta' = 1$ and $\gamma = 0.06$ and 0.09 .

For the mass difference the formula

$$\Delta M_B = \frac{f_B}{f_K} \frac{M_B}{M_K} \Delta M_K \frac{m_t^2}{m_c^2} \frac{1}{\theta^2} \{ \beta^2 - 2\beta\gamma\theta \cos \delta' + \gamma^2 \theta^2 \cos(2\delta') \} \quad (16)$$

gives a good representation. It is quite interesting because of the quadratic dependence on the t-quark mass. The other parameters occurring in table 1 are defined as usual

$$\bar{\gamma} = \frac{\Gamma(\bar{B}^0 \rightarrow X^- l^+ \nu)}{\Gamma(\bar{B}^0 \rightarrow X^+ l^- \bar{\nu})} = \left| \frac{1(\mp) \epsilon_B}{1(\pm) \epsilon_B} \right| \frac{(\Delta M)^2 + (\Delta \Gamma/2)^2}{2\Gamma^2 + (\Delta M)^2 - (\Delta \Gamma/2)^2} \quad (17)$$

$$\gamma_2 \equiv \frac{l^{++} + l^{--}}{l^{++} + l^{+-} + l^{-+} + l^{--}} = \frac{\gamma + \bar{\gamma}}{1 + \gamma + \bar{\gamma} + \gamma \bar{\gamma}} \quad (18)$$

and

$$a \equiv \frac{l^{++} - l^{--}}{l^{++} + l^{--}} = \frac{\gamma - \bar{\gamma}}{\gamma + \bar{\gamma}} \quad (19)$$

with l^{++} the number of $(x^- l^+ \nu)(x^- l^+ \nu)$ coming from the production, subsequent mixing and decays of $B^0, \bar{B}^0$ pairs in $e^+ e^-$ annihilation¹⁵.

We conclude that a small mixing angle around $\gamma \approx 0.06$ implies an almost unique determination of the Kobayashi-Maskawa matrix. The $B^0-\bar{B}^0$ mixing is expected to be 10% and CP-violating effects in the B-system are of the order of 10^{-4} .

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<u>B - $\bar{B}$ System</u>					
m_t in GeV	20	30	40	50	60
γ	0.06	0.06	0.06	0.06	0.06
β	0.009	0.006	0.004	0.003	0.003
ΔM in GeV	$9.6 \cdot 10^{-14}$	$1.8 \cdot 10^{-13}$	$2.8 \cdot 10^{-13}$	$4.1 \cdot 10^{-13}$	$5.5 \cdot 10^{-13}$
$\Delta \Gamma$ in GeV	$3.7 \cdot 10^{-14}$	$3.2 \cdot 10^{-14}$	$2.9 \cdot 10^{-14}$	$2.8 \cdot 10^{-14}$	$2.8 \cdot 10^{-14}$
r	$2.8 \cdot 10^{-02}$	$8.5 \cdot 10^{-02}$	$1.9 \cdot 10^{-01}$	$3.3 \cdot 10^{-01}$	$4.7 \cdot 10^{-01}$
r_2	$5.2 \cdot 10^{-02}$	$1.4 \cdot 10^{-01}$	$2.7 \cdot 10^{-01}$	$3.7 \cdot 10^{-01}$	$4.4 \cdot 10^{-01}$
a	$-1.3 \cdot 10^{-03}$	$-6.0 \cdot 10^{-04}$	$-4.8 \cdot 10^{-04}$	$-1.6 \cdot 10^{-04}$	$-9.6 \cdot 10^{-05}$
<u>D - $\bar{D}$ System</u>					
ΔM in GeV	$1.9 \cdot 10^{-15}$	$1.9 \cdot 10^{-15}$	$1.9 \cdot 10^{-15}$	$1.9 \cdot 10^{-15}$	$1.9 \cdot 10^{-15}$
$\Delta \Gamma$ in GeV	$6.8 \cdot 10^{-19}$	$3.2 \cdot 10^{-19}$	$1.7 \cdot 10^{-19}$	$1.1 \cdot 10^{-19}$	$6.9 \cdot 10^{-20}$
r	$1.1 \cdot 10^{-05}$	$1.1 \cdot 10^{-05}$	$1.1 \cdot 10^{-05}$	$1.1 \cdot 10^{-05}$	$1.1 \cdot 10^{-05}$
r_2	$2.1 \cdot 10^{-05}$	$2.1 \cdot 10^{-05}$	$2.1 \cdot 10^{-05}$	$2.1 \cdot 10^{-05}$	$2.1 \cdot 10^{-05}$
a	$2.9 \cdot 10^{-02}$	$1.9 \cdot 10^{-02}$	$1.4 \cdot 10^{-02}$	$1.1 \cdot 10^{-02}$	$8.6 \cdot 10^{-03}$
Table 1(a)					

Table Caption

Table 1: Predictions for particle - antiparticle mixing and CP violation in neutral B and D systems for some allowed combinations of the mixing parameters and for different top quark masses.

(a) $\gamma = 0.06$, $\sin \delta' = 1$. ($\tau_B \approx 0.9 \cdot 10^{-12}$ sec)

(b) $\gamma = 0.09$, $\sin \delta' = 1$. ($\tau_B \approx 0.4 \cdot 10^{-12}$ sec)

m_t in GeV	<u>B - $\bar{B}$ System</u>				
	20	30	40	50	60
γ	0.09	0.09	0.09	0.09	0.09
β	0.004	0.003	0.002	0.001	0.001
ΔM in GeV	$1.6 \cdot 10^{-13}$	$3.4 \cdot 10^{-13}$	$5.8 \cdot 10^{-13}$	$8.6 \cdot 10^{-13}$	$1.2 \cdot 10^{-12}$
$\Delta \Gamma$ in GeV	$6.2 \cdot 10^{-14}$	$6.0 \cdot 10^{-14}$	$6.0 \cdot 10^{-14}$	$6.0 \cdot 10^{-14}$	$5.9 \cdot 10^{-14}$
r	$7.4 \cdot 10^{-02}$	$2.6 \cdot 10^{-01}$	$5.0 \cdot 10^{-01}$	$6.9 \cdot 10^{-01}$	$8.1 \cdot 10^{-01}$
r_2	$1.3 \cdot 10^{-01}$	$3.3 \cdot 10^{-01}$	$4.4 \cdot 10^{-01}$	$4.8 \cdot 10^{-01}$	$4.9 \cdot 10^{-01}$
a	$-9.0 \cdot 10^{-04}$	$-3.2 \cdot 10^{-04}$	$-1.4 \cdot 10^{-04}$	$-6.9 \cdot 10^{-05}$	$-3.8 \cdot 10^{-05}$
	<u>D - $\bar{D}$ System</u>				
	20	30	40	50	60
ΔM in GeV	$1.9 \cdot 10^{-15}$	$1.9 \cdot 10^{-15}$	$1.9 \cdot 10^{-15}$	$1.9 \cdot 10^{-15}$	$1.9 \cdot 10^{-15}$
$\Delta \Gamma$ in GeV	$3.9 \cdot 10^{-19}$	$1.5 \cdot 10^{-19}$	$7.2 \cdot 10^{-19}$	$4.1 \cdot 10^{-20}$	$2.6 \cdot 10^{-20}$
r	$1.1 \cdot 10^{-05}$	$1.1 \cdot 10^{-05}$	$1.1 \cdot 10^{-05}$	$1.1 \cdot 10^{-05}$	$1.1 \cdot 10^{-05}$
r_2	$2.1 \cdot 10^{-05}$	$2.1 \cdot 10^{-05}$	$2.1 \cdot 10^{-05}$	$2.1 \cdot 10^{-05}$	$2.1 \cdot 10^{-05}$
a	$2.2 \cdot 10^{-02}$	$1.3 \cdot 10^{-02}$	$8.8 \cdot 10^{-03}$	$6.4 \cdot 10^{-03}$	$4.9 \cdot 10^{-03}$
Table 1(b)					

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Figure Caption

Figure 1: Allowed regions of combinations of $\sin \delta'$ and β .

It was assumed $\gamma = 0.06$. (a) is for a top quark mass of 20 GeV, (b) for 40 GeV, and (c) for 60 GeV. The upper branches correspond to $\cos \delta' > 0$, the lower branches to $\cos \delta' < 0$.