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SUPERSYMMETRY AND FUNDAMENTAL INTERACTIONS
IN THE REGION OF THE FERMI SCALE

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SUMMARY

1. Why supersymmetry ?
2. N=1 matter-supergravity couplings, the super-Higgs effect and the global limit.
3. Supergravity-induced supersymmetry breaking and low-energy effective Lagrangians.
4. Towards the construction of realistic models.

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1. WHY SUPERSYMMETRY?

Non-gravitational interactions are nowadays described, in the framework of relativistic quantum field theory, by gauge forces.

In this context one of the fundamental problems is the origin of the several physical energy scales. In the case of the strong forces, the typical scale, i.e. the inverse hadronic size or interaction range, is satisfactorily accounted for by the introduction of the Λ parameter in Quantum Chromodynamics (QCD)¹. This parameter arises from a scale-invariant Lagrangian through the phenomenon of dimensional transmutation, which relates it to a basic dimensionless coupling constant with the help of the renormalization group.

In the case of the electroweak phenomena, the relevant energy scale is given by

$$G_F^{-\frac{1}{2}} \simeq 293 \text{ GeV} , \quad (1.1)$$

where G_F is the Fermi coupling constant.

A possible way of explaining this new physical scale, in analogy to QCD, would be to invoke a new strong force, based on a non-Abelian gauge theory, with a related Λ parameter of

order $G_F^{-\frac{1}{2}}$. This force might give rise to the binding of a composite Higgs field, responsible for a dynamical breaking of the electroweak symmetry², and/or even perhaps of composite quarks, leptons and weak gauge vector bosons³. The problem of this viewpoint is that, in order to make it more concrete, one has to rely on strong assumptions on the essentially unknown dynamics of the hypothetical new force. These assumptions can at best be justified on the basis of naive analogies with QCD, and this only in some cases.

The alternative possibility in describing the physical origin of the Fermi scale relates it to the vacuum expectation value of a fundamental scalar field ϕ , such that

$$\langle \phi \rangle = (\sqrt{2} G_F)^{-\frac{1}{2}} = 246 \text{ GeV} . \quad (1.2)$$

This is the case of the standard $SU(2)_L \times U(1)$ model⁴ with the Higgs doublet -- whose electrically neutral component is ϕ -- taken as a basic degree of freedom rather than an effective field.

The main advantage of this viewpoint is that it can be realized in the context of a weak-coupling field theory. This in turn makes possible the application of the usual perturbative techniques to give physical predictions.

However, in the framework of perturbative field theory, scalar masses usually receive huge (quadratically divergent) radiative corrections⁵. This is in sharp contrast to the case of fermion masses, which control suitable chiral symmetries and are for this reason protected from acquiring big radiative corrections. More specifically, the typical expressions for the one-loop contributions to these masses are

$$\delta m_S^2 = A g^2 \Lambda^2 \quad (1.3)$$

$$\delta m_F = B g^2 m_F \log \frac{\Lambda}{m_F} \quad (1.4)$$

where A, B are numerical constants of order unity, g is a dimensionless gauge or Yukawa coupling, m_F is a tree-level fermion mass, and Λ is the momentum cut-off.

As a consequence of Eq. (1.3), barring accurate cancellations in A or even between the correction δm_S^2 and a bare mass term, the scalar mass itself will be at least of order $g\Lambda$ and therefore

$$\langle \phi \rangle = \frac{m_S}{\sqrt{2\lambda}} = O\left(\frac{g\Lambda}{\sqrt{2\lambda}}\right), \quad (1.5)$$

where λ is a dimensionless scalar self-coupling constant. This equation, together with Eq. (1.2) implies that in a weak coupling theory ($\lambda < 1$), the cut-off Λ cannot exceed by orders of magnitude $G_F^{-1/2}$. In other words, the validity of the weakly coupled fundamental Higgs scalar model cannot be pushed much further up than the Fermi scale itself⁵ *).

An important fact to notice at this point is that the numerical parameter A defined in Eq. (1.3) is not positive definite but it receives contributions opposite in sign from virtual boson and fermion exchanges, as is the case for the total vacuum energy of a given matter system of bosons and fermions. This observation makes the vanishing of A possible -- thus avoiding the scale problem -- which would, however, be accidental unless related to a symmetry principle. This symmetry, which has also to keep under control the potential quadratic divergences introduced in higher orders of perturbation theory, must organize bosons and fermions in suitable multiplets with related masses and coupling constants⁷.

*) In theories with a scale M much bigger than $G_F^{-1/2}$, the scalar mass requires additional contributions $O(gM)$. In the context of Grand Unified Theories (GUTs) this is called the hierarchy problem⁶.

The only candidate for such an invariance is an extension of the Fermi-Bose symmetry⁸ displayed by the free kinetic Lagrangian of an equal number of bosons and fermions, i.e.

$$\mathcal{L} = -(\frac{1}{2}|\partial_\mu z_i|^2 + \bar{\chi}_{Li} \not{\partial} \chi_R^i) \quad (1.6)$$

$$\begin{aligned} \delta z_i &= \bar{\epsilon}_R \chi_{Li} , \\ \delta \chi_{Li} &= + \frac{1}{2}(\not{\partial} z_i) \epsilon_R , \end{aligned} \quad (1.7)$$

where ϵ_R is a constant anticommuting Weyl spinor. Under Eqs. (1.7) $\mathcal{L}$ changes by a total derivative. The symmetry given by Eqs. (1.7) has an extension to interacting Lagrangians and it gives rise to an algebraic structure called supersymmetry⁹.

Unbroken supersymmetry implies relations amongst coupling constants and, most importantly, degeneracy between bosons and fermions belonging to the same supermultiplet. A realistic particle spectrum therefore demands the breaking of supersymmetry characterized by intramultiplet mass splittings Δm_i .

As we have said, supersymmetry may indeed require that the coefficient A in Eq. (1.3) vanishes identically with at most a logarithmic divergence analogous to the fermionic case [Eq. (1.4)]¹⁰. In the presence of supersymmetry breaking, however, the finite corrections to the scalar mass may be sizeable, being of order $\delta m_S^2 = O(g^2 \Delta m_i^2)$. One can say that the quadratic divergence is replaced by the typical square mass splitting, which is how the supersymmetry breaking enters the radiative correction diagrams.

The same line of the previous arguments now leads to the conclusion that the individual Δm_i cannot exceed, as an order of magnitude, the Fermi scale itself. With broken supersymmetry the absence of fine tuning in δm_S^2 requires that

all the known particles find their superpartners in the energy range of the Fermi scale.

The detailed prediction of the mass spectrum of all these new particles requires an understanding of the mechanism of supersymmetry breaking. Most predictive, in particular, are the theories where the breaking is spontaneous⁹. In such a case, and restricting ourselves to a weak coupling model, the following "low-energy theorem" holds¹¹:

$$\Delta m_i^2 = g_i M_S^2, \quad (1.8)$$

where g_i is the coupling to the i^{th} multiplet of the Goldstone fermion (goldstino) ψ_g , and M_S is the supersymmetry-breaking scale defined by the goldstino term in the spinorial supersymmetry current¹¹,

$$J_{\mu A} = M_S^2 (\gamma_\mu \psi_g)_A. \quad (1.9)$$

Two schemes of spontaneous supersymmetry breaking can be considered, depending on the size of the primordial supersymmetry-breaking scale M_S . In the first case¹¹

$$M_S^2 = O(\Delta m_i^2) = O(G_F^{-1}), \quad (1.10)$$

and g_i is a coupling of the same order of the electroweak gauge or Yukawa couplings. In the second case¹²

$$M_S^2 = O(M G_F^{-\frac{1}{2}}), \quad (1.11)$$

where M is a mass scale much bigger than $G_F^{-\frac{1}{2}}$. Here g_i must consequently be very small, $g_i = O(G_F^{-\frac{1}{2}}/M)$. In suitable supersymmetric theories the coupling g_i may indeed be given as a ratio of different mass scales, to all orders of perturbation theory (decoupling theorem)¹².

A close inspection of models for particle interactions based on a "low" supersymmetry-breaking scale, $M_S = O(G_F^{-\frac{1}{2}})$,

leads to a series of difficulties, both theoretical and phenomenological¹³. In many respects, the main problem is that in models based on the minimal gauge group $SU(3) \times SU(2) \times U(1)$ and with $M_S = O(G_F^{-1/2})$, the tree-level mass spectrum is unrealistic because of at least one unacceptably light scalar partner of the quarks^{11,14,15}. Possible ways of avoiding this difficulty seem to introduce a host of new problems.

In the alternative point of view, $M_S^2 = O(M G_F^{-1/2})$, $M \gg G_F^{-1/2}$, the most promising situation is met by identifying M with the Planck mass M_P . This may happen in theories where there are scalar fields acquiring vacuum expectation values of order M_P . The real interest of this situation lies in the fact that supergravity is brought in, since the effects of couplings vanishing as inverse powers of M_P are in such a case no longer negligible.

In order to explore the physical consequences of a theory with a supersymmetry-breaking intermediate between the Fermi and the Planck scale, the proper framework is set by a supersymmetric Yang-Mills Lagrangian coupled to supergravity¹⁶⁻¹⁸.

2. $N = 1$ MATTER-SUPERGRAVITY COUPLINGS, THE SUPER-HIGGS EFFECT, AND THE GLOBAL LIMIT

Supersymmetry is a relativistic symmetry which enlarges the Poincaré algebra to an algebra containing, in addition to the Poincaré generators, spinorial charges obeying anti-commutation relations.

In the case of a single spinorial charge, the basic anti-commutation relation is⁸

$$\{Q_A, Q_B\} = -\frac{1}{2}(\gamma^\mu C)_{AB} P_\mu, \quad (2.1)$$

with the additional commutation relations

$$[Q_A, P_\mu] = 0, \quad [Q_A, M_{\mu\nu}] = (\sigma_{\mu\nu})_A^B Q_B, \quad (2.2)$$

with $\sigma_{\mu\nu} = \frac{1}{4}[\gamma_\mu, \gamma_\nu]$, C_{AB} being the charge conjugation matrix. The four-component spinor Q_A is a self-conjugate operator. For the free Lagrangian given in Eq. (1.6), the explicit form of Q_A is

$$Q_A = \int d^3x J_{0A}(x) \quad (2.3)$$

in terms of the Noether current

$$J_{\mu L} = \not{\partial} z^{*i} \gamma_\mu \chi_{Li} \quad (2.4)$$

The free Lagrangian (1.6) and the transformation laws (1.7) give the simplest realization of the supersymmetry algebra defined by Eqs. (2.1) and (2.2). However, supersymmetric Lagrangians exist for interacting field theories as well. Most importantly, it is possible to construct, in a highly constrained way, supersymmetric generalizations of ordinary Yang-Mills theories¹⁹ and of the theory of gravitation²⁰.

In the case of a single spinorial charge ($N = 1$ theories), these Lagrangians involve three kinds of basic supermultiplets, all made of a boson-fermion doublet with adjacent helicities $(|\lambda|, |\lambda| + \frac{1}{2})$, with the following particle content:

i) chiral supermultiplets

$$S_i = (z_i, \chi_i), \quad (2.5)$$

where z_i denote complex scalar fields and χ_i are left-handed Weyl spinors;

ii) vector supermultiplets

$$V^\alpha = (\lambda^\alpha, B_\mu^\alpha) , \quad (2.6)$$

where λ^α are left-handed Weyl spinors and B_μ^α are spin 1 vector fields, with the index α running over the adjoint representation of a gauge group G ;

iii) gravitational supermultiplet

$$E = (\psi_{\mu A}, e_{\mu m}) \quad \begin{array}{l} \mu = \text{world vector index} \\ m = \text{flat vector index} \\ A = \text{spinor index} \end{array} \quad (2.7)$$

where $e_{\mu m}$ is the (spin 2) vierbein field and $\psi_{\mu A}$ is a Rarita-Schwinger spin 3/2 field.

In $N = 1$ supersymmetry the chiral supermultiplets S_i can be assigned to arbitrary -- in particular, complex -- representations of the gauge group G . It is important to observe that this is a unique property of $N = 1$ supersymmetry if one considers supersymmetry charges commuting with the gauge group G . Higher N supersymmetry algebras (Q_A^i , $i = 1, \dots, N$) require an extension of the supermultiplets such that spin 1/2 chiral fermions are always assigned to real representations of G . This is the case for the $N = 2$ and $N = 4$ Yang-Mills theories. Unless one is willing to introduce a major change in the present description of weak interactions (heavy mirror fermions?), this is a temporarily sufficient reason for sticking to the $N = 1$ case as the interesting one.

For $N > 4$ the gauge group G does not commute with the supersymmetry generators Q_A^i , leaving open the possibility of describing the observed interactions in the framework of extended supergravity theories²¹. We will not consider this possibility in the present article.

The basic building block of the $N = 1$ locally supersymmetric Lagrangians is the $N = 1$ pure supergravity Lagrangian^{*}) of the gravitational supermultiplet²⁰

$$\mathcal{L}_{SG} = -\frac{1}{2} e R - \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma. \quad (2.8)$$

(D_ρ denotes the spinor covariant derivative with spin 3/2 contortion contribution to the spin 1/2 connection). This supermultiplet must be coupled to an arbitrary Yang-Mills system, which is specified by the gauge group G and by the transformation properties under G of the chiral multiplets S_i .

The arbitrariness of the $N = 1$ coupled supergravity Yang-Mills Lagrangians consists, in general^{**}), of a non-canonical modification of the chiral and Yang-Mills kinetic parts¹⁷. They involve respectively a real and G -invariant function $\mathcal{G}(z, z^*)$ and an analytic function $f_{\alpha\beta}(z)$ which transforms as the symmetric product of the adjoint representation of G . These arbitrary functions, because of supersymmetry, proliferate in all interaction terms as well.

With a view to later reference, it is useful to split the complete Lagrangian, in the presence of the gravitational couplings, as follows:

- i) the bosonic part containing derivatives of the boson fields:

^{*}) Hereafter, unless otherwise stated, we set the gravitational coupling $k \equiv 1/M_P = 1$.

^{**}) Further possible generalizations of the $N = 1$ supergravity Yang-Mills Lagrangians are excluded by demanding that there be no higher derivative couplings and no supersymmetric invariants more than are quadratic in the Yang-Mills field strength multiplet.

$$\begin{aligned}
 e^{-1} \mathcal{L}_{BK} = & -\frac{1}{2} R + \mathcal{G}_j^i D_\mu z_i D^\mu z^{*j} \\
 & - \frac{1}{4} \operatorname{Re} f_{\alpha\beta} F_{\mu\nu}^\alpha F^{\mu\nu\beta} + \frac{i}{4} \operatorname{Im} f_{\alpha\beta} F_{\mu\nu}^\alpha \tilde{F}^{\mu\nu\beta}, \quad (2.9) \\
 \mathcal{G}^i = & \frac{\partial \mathcal{G}}{\partial z_i}, \quad \mathcal{G}_i = \frac{\partial \mathcal{G}}{\partial z^{*i}}, \quad \mathcal{G}_j^i = \frac{\partial^2 \mathcal{G}}{\partial z_i \partial z^{*j}};
 \end{aligned}$$

ii) the scalar potential:

$$\begin{aligned}
 e^{-1} \mathcal{L}_P = & -V(z, z^*) = e^{-\mathcal{G}} (\mathcal{G}^i \mathcal{G}_i^{-1j} \mathcal{G}_j + 3) \\
 & - \frac{1}{2} \tilde{g}^2 \operatorname{Re} f_{\alpha\beta}^{-1} (\mathcal{G}^i T_i^{\alpha j} z_j) (\mathcal{G}^k T_k^{\beta \ell} z_\ell); \quad (2.10)
 \end{aligned}$$

iii) the kinetic fermionic terms:

$$\begin{aligned}
 e^{-1} \mathcal{L}_{FK} = & -\frac{1}{4} \operatorname{Re} f_{\alpha\beta} \bar{\lambda}^\alpha \not{D} \lambda^\beta - \frac{1}{4} e^{-1} \bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma \epsilon^{\mu\nu\rho\sigma} \\
 & + \mathcal{G}_j^i \bar{\chi}_{Li} \not{D} \chi_R^j - \frac{i}{8} \operatorname{Im} f_{\alpha\beta} e^{-1} D_\mu (e \bar{\lambda}^\alpha \gamma_5 \gamma^\mu \lambda^\beta) \\
 & + \text{h.c.} \quad (2.11)
 \end{aligned}$$

iv) the quadratic part in the fermion fields without derivative terms:

$$\begin{aligned}
 e^{-1} \mathcal{L}_F^{(2)} = & e^{-\mathcal{G}/2} \bar{\psi}_{\mu R} \sigma^{\mu\nu} \psi_{\nu R} - \bar{\psi}_R \gamma (e^{-\mathcal{G}/2} \mathcal{G}^i \chi_{Li} \\
 & - \frac{i}{2} \tilde{g} \mathcal{G}^i T_i^{\alpha j} z_j \lambda_L^\alpha) + e^{-\mathcal{G}/2} [\mathcal{G}^{ij} - \mathcal{G}^i \mathcal{G}^j \\
 & - \mathcal{G}^\ell \mathcal{G}_\ell^{-1k} \mathcal{G}_k^{ij}] \times \bar{\chi}_{Li} \chi_{Lj} \\
 & - 2i \tilde{g} \mathcal{G}_i^k z^{*j} T_j^{\alpha i} \bar{\lambda}_L^\alpha \chi_{Lk} \\
 & + \frac{1}{2} f_{\alpha\beta}^k \left(\frac{1}{2} e^{-\mathcal{G}/2} \mathcal{G}_\ell \mathcal{G}_k^{-1\ell} \bar{\lambda}_L^\alpha \right. \\
 & \left. + i \tilde{g} \operatorname{Re} f_{\alpha\gamma}^{-1} \mathcal{G}^i T_i^{\gamma j} z_j \bar{\chi}_{Lk} \right) \lambda_L^\beta + \text{h.c.} \quad (2.12)
 \end{aligned}$$

v) four-fermion interactions terms, or other interactions of dimension greater than four, containing covariant derivatives of boson fields:

$$\begin{aligned}
e^{-1} \mathcal{L}'_F = & \frac{1}{2} \text{Re } f_{\alpha\beta} (\frac{1}{2} \bar{\lambda}^\alpha \gamma^\mu \sigma^{\rho\sigma} \psi_\mu F_{\rho\sigma}^\beta \\
& - \frac{1}{2} \bar{\lambda}_L^\alpha \gamma_\mu \lambda_R^\beta \mathcal{G}_{\mu z_i}^i) - \frac{1}{2} f_{\alpha\beta}^i \bar{\chi}_{Li} \sigma F^\alpha \lambda_L^\beta \\
& + \frac{1}{8} e^{-1} \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_\nu \psi_\rho \mathcal{G}_{\sigma z_i}^i - \mathcal{G}_i^j \psi_{\mu L} \not{z}^{*i} \gamma^\mu \chi_{Lj} \\
& - \bar{\chi}_{Li} \not{z}_j \chi_R^k (\mathcal{G}_k^{ij} + \frac{1}{2} \mathcal{G}_k^i \mathcal{G}^j) + \frac{1}{32} \mathcal{G}_\ell^{-1k} f_{\alpha\beta}^\ell f_{\gamma\delta}^* \\
& \times \bar{\lambda}_L^\alpha \lambda_L^\beta \gamma_\gamma \lambda_R^\delta + \frac{3}{32} (\text{Re } f_{\alpha\beta} \bar{\lambda}_L^\alpha \lambda_R^\beta)^2 \\
& + \frac{1}{8} \text{Re } f_{\alpha\beta} \bar{\lambda}^\alpha \gamma^\mu \sigma^{\rho\sigma} \psi_\mu \bar{\psi}_\rho \gamma_\sigma \lambda^\beta + \frac{1}{2} f_{\alpha\beta}^i (\bar{\chi}_{Li} \sigma^{\mu\nu} \lambda_L^\alpha \\
& \times \bar{\psi}_{\nu L} \gamma_\mu \lambda_R^\beta + \frac{1}{4} \bar{\psi}_R \gamma \chi_{Li} \bar{\lambda}_L^\alpha \lambda_L^\beta) + \frac{1}{8} \mathcal{G}_i^j \bar{\chi}_R^i \not{d} \chi_{Lj} \\
& \times (\epsilon^{abcd} \bar{\psi}_a \gamma_b \psi_c - \bar{\psi}_a \gamma_5 \gamma^d \psi_a) + \frac{1}{16} \bar{\chi}_{Li} \gamma^\mu \chi_R^j \bar{\lambda}_R^\delta \gamma_\mu \lambda_L^\gamma \\
& \times (-2 \mathcal{G}_j^i \text{Re } f_{\gamma\delta} + \text{Re } f_{\alpha\beta}^{-1} f_{\alpha\gamma}^i f_{\beta\delta}^*) \\
& - \frac{1}{16} \bar{\chi}_{Li} \sigma_{\mu\nu} \chi_{Lj} \bar{\lambda}_L^\gamma \sigma^{\mu\nu} \lambda_L^\delta \text{Re } f_{\alpha\beta}^{-1} f_{\alpha\gamma}^i f_{\beta\delta}^j \\
& + \frac{1}{16} \bar{\chi}_{Li} \chi_{Lj} \bar{\lambda}_L^\gamma \lambda_L^\delta (-4 \mathcal{G}_k^{ij} \mathcal{G}_\ell^{-1k} f_{\gamma\delta}^\ell + 4 f_{\gamma\delta}^{ij} \\
& - \text{Re } f_{\alpha\beta}^{-1} f_{\alpha\gamma}^i f_{\beta\delta}^j) + (-\frac{1}{2} \mathcal{G}_{k\ell}^{ij} + \frac{1}{2} \mathcal{G}_m^{ij} \mathcal{G}_n^{-1m} \mathcal{G}_{k\ell}^n \\
& - \frac{1}{4} \mathcal{G}_k^i \mathcal{G}_\ell^j) \bar{\chi}_{Li} \chi_{Lj} \bar{\chi}_R^k \chi_R^\ell + \text{h.c.} \tag{2.13}
\end{aligned}$$

In all previous equations, covariant derivatives D_μ are meant to be covariant both with respect to the gauge group G and to gravitation.

Spinorial covariant derivatives contain the spin-connection with spin 3/2 contortion contribution²⁰. The symbol $\tilde{g}$ denotes the gauge coupling constant, and the gravitational coupling $k \equiv 1/M_P$ has been set equal to unity.

It is important to observe that the above Lagrangian can be defined provided the function $\mathcal{G}(z, z^*)$ is of the form

$$\mathcal{G}(z, z^*) = J(z, z^*) - \log |g(z)|^2, \tag{2.14}$$

where $g(z)$ is a non-vanishing analytic function of z_i . In fact the original superspace Lagrangian contains two different supersymmetric-invariant pieces related to $J(z, z^*)$ and $g(z)$, respectively¹⁷. These two terms can be cast together in the final Lagrangian given above in terms of $\mathcal{G}(z, z^*)$ only if $g(z) \neq 0$. In other words, the singularity of Eq. (2.14) for $g(z) = 0$ tells us that the final Lagrangian is not simply obtained in this case by the replacement $\mathcal{G} \rightarrow J$. The correct substitution rules in this case are²²:

$$\begin{aligned} e^{-\mathcal{G}}, e^{-\mathcal{G}/2} &\rightarrow 0 \\ \mathcal{G}^i D_\mu z_i &\rightarrow J^i D_\mu z_i + \frac{3}{2} i q_\alpha B_\mu^\alpha \\ \tilde{g}_\alpha \mathcal{G}^{iT\alpha j} z_j &\rightarrow \tilde{g}_\alpha J^{iT\alpha j} z_j - \frac{3}{2} q_\alpha \end{aligned} \quad (2.15)$$

For other derivatives of $\mathcal{G}$ not in the form given by formulas (2.15), replace $\mathcal{G} \rightarrow J$.

In formulas (2.15), q_α are arbitrary dimensionless constants which might not vanish only for Abelian $U(1)$ factors of G . They are related, as seen by the third formula in Eq. (2.15), to the Fayet-Iliopoulos term^{23,24}.

The tree-level vacuum state defined by the minimization of the potential in Eq. (2.10),

$$V_i = \frac{\partial V}{\partial z_i} = 0, \quad (2.16)$$

may give rise to spontaneous supersymmetry breaking. This is characterized in a model-independent way by some of the quantities^{16,17}

$$e^{-\mathcal{G}/2} \mathcal{G}^i, \quad \tilde{g} \mathcal{G}^{iT\alpha j} z_j \quad (2.17)$$

being different from zero at the extremum satisfying Eq. (2.16). In fact, the would-be goldstino field is, in general, given by

$$\eta_L = e^{-\mathcal{G}/2} \mathcal{G}^i \chi_{Li} - \frac{i}{2} \tilde{g} \mathcal{G}^{i\alpha j} z_j \lambda_L^\alpha \quad (2.18)$$

which appears in Eq. (2.12) with the minimal universal coupling

$$- \bar{\psi}_R \cdot \gamma \eta_L + \text{h.c.} \quad (2.19)$$

to the gravitino field. Through this coupling, local supersymmetry invariance allows the goldstino to be rotated away by a special choice of the supersymmetry gauge. This is the super-Higgs effect²⁵.

In general the tree-level vacuum state defined by Eq. (2.14) has a non-vanishing energy density

$$V \neq 0 \quad \text{at} \quad V_i = 0. \quad (2.20)$$

The physical requirement of a vanishing cosmological constant ($V = 0$ at $V_i = 0$) may be obtained, in some situations, by an adjustment of the parameters of the theory. Under this circumstance, the gravitino mass has a well-defined meaning and it is given, from Eq. (2.12), by the model-independent formula

$$m_{3/2} = e^{-\mathcal{G}/2}. \quad (2.21)$$

The $N = 1$ supergravity Lagrangians coupled to a Yang-Mills matter system are not renormalizable, in general, not even in the limit where the gravitational coupling is set to zero, or $M_P \rightarrow \infty$, because of the arbitrary functions $\mathcal{G}(z, z^*)$ and $f_{\alpha\beta}(z)$. In this limit, where local supersymmetry reduces to global supersymmetry, renormalizability

is achieved if (the gravitational coupling is explicitly re-introduced)

$$f_{\alpha\beta}(z) = (1 + i \theta_{\alpha}) \delta_{\alpha\beta} \quad (2.22)$$

$$\mathcal{G}(z, z^*) = -\frac{1}{2} |z_i|^2 - M_P^2 \log \left| \frac{g(z)}{M_P^3} \right|^2, \quad (2.23)$$

where the so-called superpotential $g(z)$ is a gauge-invariant polynomial, at most of degree three, in the scalar variables z_i (and not of their conjugate z_i^*). The θ_{α} are real constants respecting the gauge transformation properties of $f_{\alpha\beta}$. More generally, complex constants in $f_{\alpha\beta}$ or in the first term of $\mathcal{G}(z, z^*)$ may always be absorbed in the limiting Lagrangian by a suitable redefinition of the fields and of the gauge coupling constants.

With the limiting forms, Eqs. (2.22) and (2.23), we obtain*) the most general renormalizable globally supersymmetric Yang-Mills Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{Y.M.}} = & -\frac{1}{4} \left(F_{\mu\nu}^{\alpha} \right)^2 - \frac{1}{2} \bar{\lambda}^{\alpha} \not{D} \lambda^{\alpha} \\ & - \frac{1}{2} \bar{\chi}_{Li}^{\dagger} \not{D} \chi_R^i - \frac{1}{2} |D_{\mu} z_i|^2 - 2 |g^i|^2 \\ & - \frac{1}{2} \left(-\frac{1}{2} \tilde{g} z^{*i} T_i^{\alpha j} z_j + \xi^{\alpha} \right)^2 \\ & + (g^{ij} \chi_{Li} \chi_{Lj} - i \tilde{g} T_j^{\alpha k} z_k \bar{\lambda}_R^{\alpha} \chi_R^j + \text{h.c.}) \\ & + \frac{i}{4} \theta_{\alpha} \left[F_{\mu\nu}^{\alpha} \tilde{F}^{\mu\nu\alpha} - \frac{1}{2} \partial_{\mu} (\bar{\lambda}^{\alpha} \gamma_5 \gamma_{\mu} \lambda^{\alpha}) \right]. \end{aligned} \quad (2.24)$$

Notice the constants ξ^{α} which are possibly present in association with the generators T^{α} of isolated $U(1)$ factors of the gauge group G and correspond to the so-called

) In taking the global limit, we have to use the original formulation of the Lagrangian in terms of the functions $J(z, z^)$ and $g(z)$ as defined in Eqs. (2.14).

Fayet-Iliopoulos terms²³. They arise in the global limit as follows. If the superpotential $g(z)$ vanishes, from Eqs. (2.15) one simply has

$$\xi^\alpha = q^\alpha M_P^2. \quad (2.25)$$

In association with gauge transformations of Abelian factors of G , a non-vanishing $g(z)$ may transform²⁶ as

$$g(z_i) e^{ig_\alpha \omega_\alpha T_i^\alpha} = e^{i\beta_\alpha \omega_\alpha} g(z_i) \quad (2.26)$$

since the function $\mathcal{G}(z, z^*)$ does not feel the phase of $g(z_i)$. In this case, from Eqs. (2.10) and (2.26) we have

$$\xi^\alpha = \beta^\alpha M_P^2. \quad (2.27)$$

In the flat limit, $M_P \rightarrow \infty$ with ξ^α finite, the superpotential is strictly gauge-invariant and the Lagrangian acquires an additional global $U(1)$ R-symmetry.

3. SUPERGRAVITY-INDUCED BREAKING AND LOW-ENERGY EFFECTIVE LAGRANGIANS

In attempts to construct realistic models of particle interactions, the locally supersymmetric Lagrangians described in the previous section will contain, other than the Planck scale, some lower energy scales. Amongst them we do not distinguish at this point between $G_F^{-1/2}$ and any other possible new fundamental scale, e.g. the unification mass. What is important is that in the formal limit $M_P \rightarrow \infty$, a well-defined renormalizable Lagrangian exists describing the non-gravitational interactions of the "low-energy" fields.

This limit becomes non-trivial in the presence of some scalar fields x_i acquiring vacuum expectation values of order M_P which break supersymmetry^{16,27}. For definiteness we shall assume that these fields are singlets under the gauge group G . The full set of scalar fields z_i , belonging to the chiral supermultiplets, splits therefore into $z_i = (x_i, y_i)$. The remaining fields y_i are called "low-energy" fields and generally transform non-trivially²⁸ under G .

The requirement that there remain indeed "light" fields with renormalizable gauge interactions can be fulfilled under the condition that a decoupling occurs between the x -type and y -type supermultiplets, and furthermore that the gravitino mass is finite for $M_P \rightarrow \infty$ (Ref. 28). This can be made more precise by restoring the dimensional gravitational constant in the functions $f_{\alpha\beta}(z)$ and $\mathcal{G}(z, z^*)$. The dimensionless function $f_{\alpha\beta}$ must tend, as $M_P \rightarrow \infty$, to the constant limit (2.22) for finite x_i and y_i . Defining

$$\mathcal{G}(z, z^*) = J(z, z^*) - M_P^2 \log \left| \frac{g(z)}{M_P^3} \right|^2, \quad (3.1)$$

the y -dependent part of $J(z, z^*) = J(x, x^*; y, y^*)$ must behave as²⁹

$$\begin{aligned} J(M_P \zeta, M_P \zeta^*; y, y^*) \approx & -\frac{1}{2} y_i A_j^i(\zeta, \zeta^*) y^{*j} + B_i(\zeta, \zeta^*) y_i \\ & + B_i^*(\zeta, \zeta^*) y^{*i}. \end{aligned} \quad (3.2)$$

Analogously, the y -dependent part of the superpotential $g(z) = g(x, y)$, after rescaling of the x -variables $x_i = M_P \zeta_i$, must be a finite polynomial of at most degree three in the variable y_i with possibly ζ -dependent coefficients.

These requirements correspond to those in Eqs. (2.22) and (2.23). Equation (3.1) defines the superpotential $g(z)$ and the Kahler potential $J(z, z^*)$ up to a Kahler transformation,

$$\begin{aligned} J(z, z^*) &\rightarrow J(z, z^*) + f(z) + f^*(z^*) \\ g(z) &\rightarrow g(z) \exp f(z)/M_P^2 \end{aligned} \quad (3.3)$$

This in turn means that the behaviour as $M_P \rightarrow \infty$ of the y -independent part of $g(z)$ consistent with (3.2) is

$$g(M_P \zeta, 0) = O(M_P^2) . \quad (3.4)$$

Let us now consider the general explicit form of the limiting Lagrangian as $M_P \rightarrow \infty$ in the fields y_i , after the x_i having acquired v.e.v.'s of order M_P which break supersymmetry. Equation (3.4) implies a finite gravitino mass given by

$$m_{3/2} = \lim_{M_P \rightarrow \infty} \frac{\langle g \rangle}{M_P^2} \exp -(\langle J \rangle / 2M_P^2) . \quad (3.5)$$

This mass provides the scale of all the explicit supersymmetry-breaking terms appearing in the limiting Lagrangian for the low-energy fields y_i (Ref. 28)*), which

*) The low-energy fields also include, out of the x -type superfields, the goldstino, which provides the missing degrees of freedom to form the massive spin 3/2 gravitino and two G singlet scalar fields. These fields are not included in the "low-energy" Lagrangian because in the limit $M_P \rightarrow \infty$ they do not have interactions with the y -type superfields. Their lightness may be of cosmological relevance³⁰.

has the general form*)

$$\begin{aligned} \mathcal{L} = & \mathcal{L}(\text{supersymm}; f) + m_{3/2} (c_\alpha \bar{\lambda}_{\alpha_L} \lambda_{\alpha_L} + \text{h.c.}) \\ & - m_{3/2} [h(y) + h^*(y^*)] - \frac{1}{2} m_{3/2}^2 y^{*j} \Lambda_j^i y_i . \end{aligned} \quad (3.6)$$

Here the first term is the globally supersymmetric Lagrangian given by Eq. (2.24) with a superpotential

$$f(y) = \lim_{M_P \rightarrow \infty} g(\langle x \rangle, y) \exp -(\langle J \rangle / 2M_P^2) . \quad (3.7)$$

The function $h(y)$ is another analytic polynomial, invariant under G , at most of degree 3; the c_α 's in the gaugino mass terms are complex coefficients of order unity; and, finally, Λ_j^i are the elements of an arbitrary numerical Hermitian matrix.

The result of Eq. (3.6) can be summarized by saying that the low-energy effects of the supergravity-induced supersymmetry breaking give rise to the general breaking terms which are soft in the supersymmetric sense³⁴, namely they do not alter the ultraviolet behaviour of the purely supersymmetric part of the Lagrangian at the level of the leading quadratic divergences. For this reason it can be shown that the Lagrangian (3.6) with arbitrary f and h polynomials is strictly renormalizable³⁴.

The origin of the various supersymmetry-breaking terms in Eq. (3.6) can be better understood by considering some special cases for the functions $\mathcal{G}(z, z^*)$ and $f_{\alpha\beta}(z)$ in the original Lagrangian. If these functions have the form given in Eqs. (2.22) and (2.23) even before the limit

*) Particular cases of Eq. (3.6) have been considered in Refs. 28 and 29, and also in Refs. 31 to 33.

$M_P \rightarrow \infty$ is taken, we get in the low-energy effective Lagrangian^{28,31}

$$h(y) = (A - 3) f(y) + \sum_j y_j \frac{\partial f}{\partial y_j}, \quad (3.8)$$

$$c_\alpha = 0, \quad (3.9)$$

$$\Lambda_j^i = \delta_j^i, \quad (3.10)$$

where A is a parameter depending on the particular way the x -fields enter in $g(z)$. For a linear dependence on x , we have $A = 3 - \sqrt{3}$ (Ref. 28). Keeping canonical kinetic terms for the matter multiplets,

$$J_i^j = -\frac{1}{2} \delta_i^j \quad (3.11)$$

and for the vector multiplets $f_{\alpha\beta}$ as in Eq. (2.22), an arbitrary function $h(y)$ not related to $f(y)$ may be obtained by introducing higher dimensional couplings of y to x through $g(z)$ in Eq. (3.1). Specifically, whereas $f(y)$ is still given by Eq. (3.7), $h(y)$ receives an additional contribution Δh other than that of Eq. (3.8):

$$\Delta h(y) = \lim_{M_P \rightarrow \infty} \sqrt{3} M_P \left\langle \frac{\partial g}{\partial x}(x, y) \right\rangle \exp -(\langle J \rangle / 2M_P^2), \quad (3.12)$$

where $\partial g / \partial x$ is the derivative of g in the direction of the would-be goldstino^{*)}.

With canonical kinetic terms and an arbitrary $g(z)$, we are left with no gaugino mass terms and a universal scalar mass term. In general, $c_\alpha \neq 0$ is related to a non-minimal function $f_{\alpha\beta}(z)$, from Eq. (2.12), through the following relation:

*) This result is due to E. Cremmer, 1982 (unpublished).

$$m_{3/2}^2 c_{\alpha} \delta_{\alpha\beta} = \lim_{M_P \rightarrow \infty} M_P \left\langle f_{\alpha\beta}^i g_j [(-J/M_P^2)^{-1}]_i^j \exp(-G/2M_P^2) \right\rangle. \quad (3.13)$$

Finally, to obtain a non-universal mass matrix Λ_j^i for the scalar fields, a non-minimal function $J(z, z^*)$ has to be introduced. As an example, consider

$$J(z, z^*) = -\frac{1}{2} x^{i*} x_i - \frac{1}{2} y^{j*} y_j - \frac{x^{i*} x_i}{2M_P^2} y^{*l} A_{lm} y_m, \quad (3.14)$$

where A is a numerical Hermitian matrix. From the general potential given in Eq. (2.10), to first order in A , we obtain

$$\Lambda = \mathbb{1} + 3A. \quad (3.15)$$

The deviation from $\mathbb{1}$ of the Hermitian matrix Λ_j^i and the gaugino mass terms in Eq. (3.6) give a correction to the mass-sum rules obtained for minimal couplings¹⁷:

$$\sum_{J=0}^1 (2J+1) m_J^2 (-)^{2J} = 2N m_{3/2}^2 - 2g_{\alpha}^{\alpha} \text{Tr} T_{\alpha}. \quad (3.16)$$

This is connected to the modification of these sum rules for general functions $G(z, z^*)$, $f_{\alpha\beta}(z)$. For instance, a non-minimal $J(z, z^*)$ gives an additional contribution to Eq. (3.16) of the type³⁵

$$R_i^{\ell} g^i g_{\ell} e^{-G} \quad (3.17)$$

where R_i^{ℓ} is the contracted Riemann tensor constructed out of the Kahler metric g_i^j ³⁶. The term (3.17), for the example of Eq. (3.14), is actually proportional to $3m_{3/2}^2 \text{Tr} A$.

The Lagrangian (3.6) is a tree-level result. It is natural to wonder if loop corrections are negligible -- more precisely, if they commute with the $M_P \rightarrow \infty$ limit.

This is certainly the case for the Lagrangian (3.6) which is obtained from the most general locally supersymmetric original Lagrangian and, as such, it is stable under radiative corrections. Gravitational loops would give contributions at most of the same order as the ones already included in Eq. (3.6)^{37,38}. On the other hand, one cannot expect a strict stability of the low-energy Lagrangians obtained in the particular cases considered in Eqs. (3.8) to (3.10). In fact, even the radiative corrections of the same low-energy Lagrangian -- excluding therefore gravitational loops -- introduce in these cases new divergent as well as finite terms not contained in the tree-level Lagrangian^{38,39*)}. What may happen is that these contributions, using a cut-off $\Lambda \sim M_p$, may be numerically small. In some explicit cases this makes the particular Lagrangians worthy of consideration.

4. TOWARDS THE CONSTRUCTION OF REALISTIC MODELS

The Lagrangian given in Eq. (3.6) is suitable for describing a realistic theory for particle interactions. In principle, one may include in it any physical scale below the Planck mass. For example, the properties of supersymmetric GUTs could be investigated. In the following we shall concentrate on the properties of the low-energy physics in the region of the Fermi scale $G_F^{-1/2}$, restricting our attention to the minimal gauge group $G = SU(3) \times SU(2) \times U(1)$.

The chiral multiplets which define the theory include quark and lepton supermultiplets denoted by:

*) A particular example is obtained by integrating out the heavy degrees of freedom in GUTs⁴⁰, which in general modify the soft-breaking terms of the light fields³³.

$$\begin{aligned} Q(3, 2, \frac{1}{6}), \quad u^c(\bar{3}, 1, -\frac{2}{3}), \quad d^c(\bar{3}, 1, \frac{1}{3}) \\ L(1, 2, -\frac{1}{2}), \quad e^c(1, 1, 1) , \end{aligned} \quad (4.1)$$

the minimal Higgs structure of two SU(2) doublets

$$H(1, 2, -\frac{1}{2}), \quad H^c(1, 2, \frac{1}{2}) , \quad (4.2)$$

as well as a singlet supermultiplet Y. The numbers in parentheses denote the transformation properties under G of the various fields in a self-explanatory notation.

The chiral superfields in (4.2) are the minimal structure of Higgs fields which can describe the breaking of the electroweak $SU(2)_L \times U(1)$ group down to $U(1)_Q$ ⁴¹. It is also possible to choose the polynomials f(y) and h(y) in Eq. (3.6) such that the inclusion of the quark and lepton scalars in the over-all potential leaves the desired properties of the local minimum unaltered.

The constrained form of the supersymmetry-breaking terms in the Lagrangian (3.6) gives rise to rather general properties of the tree-level particle spectrum^{28,32,42}.

For illustrative purposes it is useful to consider a particularly simple case, where all details can be worked out explicitly. Let us take a Lagrangian with "minimal" functions $f_{\alpha\beta}(z)$ and $\mathcal{G}(z, z^*)$. In this case, the low-energy limit is specified by Eqs. (3.6) to (3.10). For simplicity we also take in Eq. (3.8) $A = 3$, as we get from a factorized form of the original superpotential³²,

$$g(x, y) = g_1(x) \left[1 + \frac{1}{\langle g_1(x) \rangle} g_2(y) \right] . \quad (4.3)$$

The function $g_2(y)$ is proportional to the low-energy superpotential f, which is chosen as follows:

$$f(y) = \beta Y(HH^c - \epsilon) + g_u QH^c u^c + g_d QHd^c + \dots , \quad (4.4)$$

where the multiplication of SU(2) doublets with the anti-symmetric tensor is understood and the dots indicate all the other super-Yukawa interactions. Perhaps a more elegant alternative to Eq. (4.4) would be to replace³¹ the term $-\beta\epsilon Y$ with $-\beta'Y^3$, which leads to a low-energy scale invariant superpotential, scale invariance being explicitly broken in the effective Lagrangian by the gravitino mass only. For our illustrative purposes, the two alternatives are equivalent, and we follow the simpler possibility, Eq. (4.4)*).

In order to study the particle spectrum, the over-all scalar potential must first be minimized. The minimum is obtained for

$$|\langle H_0 \rangle| = |\langle H_0^c \rangle| = \frac{m_{3/2}}{\beta} \sqrt{1 \pm \frac{\epsilon\beta^2}{m_{3/2}^2}} \quad (4.5)$$

$$|Y| = \frac{m_{3/2}}{|\beta|} \quad (4.6)$$

with the + [-] sign in (4.5) going together with $\epsilon > (m_{3/2}^2/\beta^2)$ [$\epsilon < (-m_{3/2}^2/\beta^2)$]. All other fields have vanishing expectation values.

To describe the particle spectrum it is useful to observe that the presence of two Higgs doublets introduces in the Lagrangian an approximate $SU(2)_R$ invariance which interchanges H and H^c and transforms the right-handed

*) The scale-invariant possibility may, in addition, suffer from the presence of unwanted global minima where electric charge and/or colour conservation are broken by the vacuum expectation values of s-leptons or s-quarks. (See J.M. Frère et al., as quoted in Ref. 39.)

quark fields as doublets⁴³. This symmetry, which becomes exact in the limit

$$\begin{aligned} g_u &= g_d , \\ g' &= 0 , \end{aligned} \tag{4.7}$$

[g' is the U(1) gauge coupling] is always preserved by the Higgs self-interactions introduced through the general functions f and h , since they depend on the invariant HH^c . Furthermore, the vacuum expectation values (4.5) break the over-all $SU(2)_L \times SU(2)_R$ symmetry down to the diagonal $SU(2)_{L+R}^*$, which becomes a useful approximate classification symmetry.

Defining for the scalar Higgs components

$$\begin{aligned} H &= \begin{bmatrix} D + \frac{1}{\sqrt{2}} (\sigma + iQ_3) \\ i\phi_- \end{bmatrix} & H_c &= \begin{bmatrix} -i\phi_{c+} \\ D_c + \frac{1}{\sqrt{2}} (\sigma_c + i\phi_{c3}) \end{bmatrix} \\ Y &= \frac{1}{\sqrt{2}} (y_1 + iy_2) , \end{aligned} \tag{4.8}$$

the physical scalar fields can be grouped into an $SU(2)_{L+R}$ isotriplet (a charged and a neutral field):

$$\begin{aligned} \frac{1}{\sqrt{2}} (\phi_- + \phi_{c+}^*); \frac{1}{\sqrt{2}} (\sigma - \sigma_c) , \quad I = 1 , \quad M_{\pm}^2 &= 4m_{3/2}^2 + M_W^2 \\ M_0^2 &= 4m_{3/2}^2 + M_Z^2 \end{aligned} \tag{4.9}$$

*) The survival of this group rests on $|\langle H^0 \rangle| = |\langle H_c^0 \rangle|$. It can be shown that this is a feature of the potentials arising from the Lagrangian (3.6) based on $SU(3) \times SU(2) \times U(1)$ in the "minimal" case and no ξ term associated with the hypercharge generator.

which becomes degenerate for $g' \rightarrow 0$ ($M_Z \rightarrow M_W$) and four (neutral) isosinglets

$$y_1, \quad y_2, \quad \frac{1}{\sqrt{2}} (\phi_3 + \phi_{3c}), \quad \frac{1}{\sqrt{2}} (\sigma + \sigma_3) \quad (4.10)$$

generally mixed. In the limit $m_{3/2} \ll M_W$, the isosinglets have the common mass $\sqrt{2} (\beta/g) M_W$, whereas in the opposite situation their masses become $m_{3/2}$, $m_{3/2}$, $2m_{3/2}$, $4m_{3/2}$, respectively.

Disregarding, for a moment, the problem of the gluino and photino masses, the gaugino fermionic sector is as follows. For $m_{3/2} \ll M_W$, we get one isotriplet of Dirac fermions,

$$\begin{aligned} \lambda_- + h_+^* , & \quad M = M_W , \\ \lambda_- + \lambda_+^* , & \quad M = M_W , \\ \frac{1}{\sqrt{2}} (h_0 - h_{c0}) + \lambda_z^* , & \quad M = M_Z , \end{aligned} \quad (4.11)$$

and a Dirac isosinglet

$$y + \frac{1}{\sqrt{2}} (h_0 + h_{c0})^* , \quad M = \frac{\sqrt{2}\beta}{g} M_W , \quad (4.12)$$

where $\lambda_\pm$, λ_Z are the $W^\pm$, Z gauginos and h, y are the fermionic components of the Higgs and Y supermultiplets.

Interestingly enough, in the opposite situation, $m_{3/2} \gg M_W$, we do get light fermions, in particular a charged w -ino with mass

$$M \approx M_W \left(\frac{M_W}{m_{3/2}} \right) \quad (4.13)$$

and mainly given by $(\lambda_- + \lambda_+^*)$, as well as a neutral fermion approximately given by $(y + \lambda_z^*)$, with mass:

$$M \simeq M_Z \left(\frac{M_Z}{m_{3/2}} \right). \quad (4.14)$$

A light charged w-ino may be the most distinctive feature of the new particle spectrum^{44,45}. It is interesting to know that the w-ino will predominantly decay into a gluino plus a quark-antiquark pair rather than into a photino plus a quark-antiquark (or a lepton-antilepton) pair with a relative branching ratio of order α_s/α . On the other hand, before it can be produced in e^+e^- , a light w-ino may be discovered in the W-boson decay⁴⁴

$$W^\pm \rightarrow \lambda_W^\pm + \text{photino}, \quad W^\pm \rightarrow \lambda_W^\pm + \text{z-ino}. \quad (4.15)$$

The counterparts of a w-ino substantially lighter than the W-boson ($m_{3/2} \gg M_W$) are relatively light quark and lepton scalar partners ($m_{3/2} \ll M_W$). In fact, the mass eigenstates of a quark q or a lepton ℓ superpartners are

$$Q_{1,2} = \frac{1}{\sqrt{2}} (Q \pm Q_c), \quad L_{1,2} = \frac{1}{\sqrt{2}} (L \pm L_c), \quad (4.16)$$

and the corresponding eigenvalues are³² expressed in terms of the fermion mass m by

$$m_{1,2} = |m \pm m_{3/2}|. \quad (4.17)$$

Notice that for the effective "minimal" Lagrangian with h related to f by Eq. (3.8), a simultaneous diagonalization in generation space of the fermion and scalar matter multiplets can be achieved since the tree-level fermion and scalar mass matrices are proportional to each other. A violation of this alignment would produce significant flavour-changing neutral-current effects as well as CP-violating effects⁴⁶.

At the tree-level, even after spontaneous symmetry breaking, the fermionic partners of the massless vector bosons, the photino (λ_γ) and the gluino (λ_g), are also massless. Needless to say, this result rests crucially on the assumption of "minimal" Lagrangian, which in turn implies $c_\alpha = 0$ in Eq. (3.6). On the other hand, the continuous R-symmetry of $\mathcal{L}(\text{supersymmetric}; f)$ in Eq. (3.6), with f as in Eq. (4.4), which would keep λ_g and λ_γ massless to all orders of perturbation theory, is explicitly broken by the analytic soft breaking terms in Eq. (3.6). In concrete models the breaking of the R-invariance is transmitted to the y -fields by the supergravity couplings to the x -fields where R-symmetry is explicitly broken. In particular, at the one-loop order we get contributions to the photino and gluino masses^{38, 39, 44, 45}:

$$\left(m_\gamma, m_g \right) = 0 \left(\frac{\alpha}{2\pi}, \frac{\alpha_s}{2\pi} \right) m_{3/2} \times \min \left[1, \left(\frac{m}{m_{3/2}} \right)^2 \right], \quad (4.18)$$

where m is the mass of the virtual fermion in the loop and the running scale in α_s is set by $\max(m_{3/2}, m)$.

Let us first consider the gluino mass. The lower limit on a light charged w -ino sets, through Eq. (4.13), an upper limit on the gravitino mass, $m_{3/2} \lesssim 350$ GeV. In the framework of the minimal $SU(3) \times SU(2) \times U(1)$ model with three families, only a heavy top, $m_t \approx 150$ -200 GeV, with a comparably heavy gravitino can give a sizeable mass to the gluino, $m_g \approx 1.5$ GeV, which would also have a relatively long lifetime, $\tau \approx 1$ cm. A gluino with these properties might have escaped detection⁴⁷. On the other hand, each hypothetical heavy-coloured fermion of some new sequential family and/or of grand unified extensions of the minimal model would give an additional contribution

proportional to $m_{3/2}$, making the gluino substantially heavier^{42,48}.

In the case of the photino mass, already in the minimal model, there is a charged w-ino which is heavier than the gravitino, giving a contribution³⁸

$$m_Y \approx \frac{\alpha}{3\pi} m_{3/2} \log 4 \quad (4.19)$$

in the 10-100 MeV region^{*)} depending on the value of $m_{3/2}$. In any case, the photino is going to be the lightest of the new particles, and as such, essentially stable.

This concludes the description of the new particle spectrum. Its main features are: i) a light charged w-ino or light quark and lepton scalar partners; ii) the approximate $SU(2)_{L+R}$ classification symmetry. This same symmetry, which acts on $\vec{W}_\mu$ as a vector, keeps small, $O(10^{-3})$, the one-loop radiative corrections to the electroweak ρ parameter from gaugino-Higgsino and Higgs scalar exchanges⁴³. The only important contribution might come from scalar top and bottom exchanges, with a heavy top, since this essentially duplicates the conventional top-bottom contribution to ρ and sets an upper limit on the top mass if $m_t \lesssim 200$ GeV.

Our discussion has been confined to models based on the minimal low-energy group $SU(3) \times SU(2) \times U(1)$ and tree-level breaking of $SU(2) \times U(1) \rightarrow U_{em}(1)$. In the context of GUTs, one may consider the breaking as being induced by one-loop radiative corrections triggered, for example by

*) Such a low value for m_{ph} may give cosmological problems. See L. Alvarez-Gaumé et al., Ref. 39. Equation (4.19) is an asymptotic formula for $m_{3/2} \gg M_W$ which, however, gives a good interpolation of the exact one-loop formula for lower values of $m_{3/2}$.

a heavy top mass $m_t = O(100 \text{ GeV})$. For this point of view we refer to the original literature³⁹.

From a general phenomenological point of view, the "minimal" Lagrangian that we have been discussing in this section may be favoured against the most general one, Eq. (3.6), especially (but not only) because of the universal supersymmetry-breaking scalar mass term. An arbitrary mass matrix, as in Eq. (3.6), would in fact at least lead to unacceptably large flavour-changing neutral-current effects⁴⁶. On the other hand, as already discussed in the previous section, the minimal Lagrangian is not strictly renormalizable. This calls for the introduction of counter terms corresponding to the terms of the most general effective Lagrangian but with a size estimated from the radiative corrections themselves cut off at $\Lambda \lesssim M_P$. In most instances these terms will be negligibly small, as happens, for example, in the gaugino case for the minimal model³⁸.

In conclusion, the models based on the low-energy effective Lagrangians obtained after supersymmetry breaking from the $N = 1$ supergravity Lagrangians look promising. In Section 1 we have expressed the view that in a *weak* coupling theory with *fundamental* scalar Higgses, supersymmetry and its local extension, supergravity, is almost unavoidable. The arguments for the relevance, at some stage, of $N = 1$ supergravity are in our opinion admittedly less stringent. Recently, investigations of the possible bridge between $N = 1$ and higher- N supergravity theories were started⁴⁹. Leaving aside aesthetical considerations, at least the better ultraviolet convergence properties of higher- N theories make them extremely interesting⁵⁰. From a general

point of view there is certainly one reason for not being satisfied with the theories that we have considered, namely the very accurate fine tuning which remains to be done to make the cosmological constant very small and which is not stable against radiative corrections. We are clearly assuming that a possible dynamical origin for this effect can be incorporated in a suitable extension of the models under consideration, without modifying at least their "low-energy" consequences.

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