PRIMORDIAL INFLATION AND SUPER-COSMOLOGY *)

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A B S T R A C T

A complete locally supersymmetric model for the early Universe is reviewed. It begins with primordial inflation just after the Planck time. The (non-trivial) breaking of SU(5) is discussed in detail, with specific emphasis on baryon generation at $T \sim O(10^7)$ GeV and monopole suppression (no longer accomplished by inflation). Gravitational effects are taken into account through $N = 1$ supergravity and play an essential rôle. What one is left with is a problem-free scenario containing all the benefits of Guth's original inflation as well as density perturbations of a desirable magnitude for the formation of galaxies, a large baryon to photon ratio, and a possibly observable flux of magnetic monopoles. By inserting only two scales, the Planck scale and the supersymmetry breaking scale, both the weak and GUT scales are produced.

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I. GUTs, COSMOLOGY AND INFLATION

The advent of Grand Unified Theories (GUTs)¹⁾ has certainly been a cornerstone for our understanding of the very early stages of the Universe. GUTs have, and continue, to resolve questions that were once thought only answerable by an initial condition. The most celebrated of these questions is the origin²⁾ of the small baryon asymmetry present in the Universe today. Pre-GUT models were only able to speculate that the net baryon to photon ratio today³⁾

$$\eta = \frac{n_b}{n_\gamma} \approx (3-6) \times 10^{-10} \quad (1.1)$$

is due to an initial condition or that initially $\eta \sim 0(1)$ and that sometime during the evolution of the Universe a large amount of entropy was produced. The GUT alternative is clearly preferred as it ascribes the origin of η to microphysical processes.

Along with its solution to the problem of baryon generation, GUTs predict the existence of magnetic monopoles (GUMs)⁴⁾. GUMs will be produced whenever a simple group such as SU(5) breaks down to a gauge group containing an explicit U(1) factor [e.g. SU(3) $\times$ SU(2) $\times$ U(1)]. The mass of these GUMs will be

$$M_{\text{GUM}} \sim M_G / \alpha_G \sim 10^{16} \text{ GeV} \quad (1.2)$$

where M_G is the GUT scale [$0(10^{15})$ GeV] and α_G is the GUT gauge coupling constant. The number density of GUMs produced during the GUT phase transition is, however, very large⁵⁾. Roughly one can estimate the density of GUMs produced by assuming that about one GUM is produced per causally connected region or horizon volume at the time of the transition

$$n_{\text{GUM}} \sim (2t_c)^{-3} \quad (1.3)$$

where t_c is related to the critical temperature by

$$t_c \sim 0.3 M_p T_c^{-2} N(T_c)^{-1/2} \quad (1.4)$$

where $M_p \approx 1.2 \times 10^{19}$ GeV is the Planck mass and $N(T_c)$ is the number of light degrees of freedom at T_c . When compared to the number of photons we have

$$\frac{n_{\text{GUM}}}{n_\gamma} \sim \left(\frac{10 T_c}{M_p} \right)^3 \quad (1.5)$$

In order to remain consistent with the cosmological bound $n_{\text{GUM}}/n_\gamma \lesssim 10^{-24}$, eq. (1.5) seems to require^{5,6)} $T_c \lesssim 10^{10}$ GeV. Naively one expects $T_c \sim M_G$, thus overproducing GUMs.

The standard big bang model, also contained a number of problems thought to be unrelated to GUTs. These include the curvature⁷⁾, isotropy⁷⁾, rotation⁸⁾ etc. problems. As we will see, GUTs offer a possible explanation to them all. These problems can be most easily understood when one considers the equations governing the evolution of Friedmann-Robertson-Walker models. The metric for this class of models is

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta d\varphi^2) \right] \quad (1.6)$$

where $a(t)$ is the scale factor and $k = 0, \pm 1$ is a measure of curvature. The Einstein equations yield

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\rho}{3M_p^2} - \frac{k}{a^2} + \frac{\Lambda}{3} + \dots \quad (1.7)$$

with possible contributions coming from anisotropies such as shear or rotation. In eq. (1.7), ρ is the total mass-energy density and Λ is the cosmological constant.

It is known that today, the universe is neither curvature-dominated nor vacuum-dominated. In particular, we know that the curvature term in (1.7) is less than the matter term so that we can write

$$\frac{k}{a^2} < \frac{8\pi\rho}{3M_p^2} \quad (1.8)$$

We also know that in an adiabatically expanding Universe $a \sim 1/T$ and we can form a dimensionless constant

$$\hat{k} \equiv \frac{k}{a^2 T^2} < \frac{8\pi\Omega\rho_c}{3M_p^2 T_0^2} \quad (1.9)$$

where $\Omega \equiv \rho/\rho_c$, $\rho_c = 1.88 \times 10^{-29} \text{ gcm}^{-3}$ is the critical energy density and $T_0 = 2.7\text{-}3 \text{ }^\circ\text{K}$ is the present temperature of the microwave background radiation. Using $\Omega < 2$ we see that

$$\hat{k} \lesssim 2 \times 10^{-58} \quad (1.10)$$

This is the curvature problem⁷⁾.

The curvature problem is typical of a problem whose solution is explained by inflation⁷⁾. Remember that this problem (1.10) arose when we assumed an adiabatically expanding Universe (i.e. $a \sim 1/T$). If however, there was some epoch in which this was not true and that we can let a grow by a factor 10^{28} with little or no change in T , the problem is solved. A temporary De Sitter phase has just this property. If the Universe at some stage where dominated by a cosmological constant eq. (1.7) indicates that $a \sim e^{Ht}$ rather than the power law type expansions associated with matter or radiation dominated Universes.

Once again we see that GUTs can save the day. Inherent in GUTs are phase transitions. In particular, these phase transitions are associated with a change in the vacuum energy density and if for some reason the Universe supercooled and remained in the false vacuum below T_c , the Universe would become vacuum-dominated. The vacuum energy density would act like a cosmological constant and the Universe would enter an approximate De Sitter phase^{7,9)}. This is the well known inflationary Universe scenario⁷⁾. The basic idea

was that when the energy density in radiation $\rho \sim T^4$, fell below the vacuum energy density V_0 of the symmetric phase [e.g. SU(5)], the Universe would begin to expand exponentially. The phase transition to the broken phase [e.g. SU(3) $\times$ SU(2) $\times$ U(1)] would then occur through the formation of bubbles whose formation rate is given by¹⁰⁾

$$P \sim D^2 e^{-B} \quad (1.11)$$

where D is a characteristic mass scale and B is the action for tunnelling. When $P \sim H^4$ the transition is over. The requirement however, that the exponential expansion last long enough (28 orders of magnitude) requires B to be very large. In fact, it has been shown¹¹⁾ that in order to have enough inflation B is so large that the transition never finishes. This results in a highly irregular Universe containing a few bubbles of the broken phase imbedded in a still exponentially expanding Universe in the symmetric phase.

The solution of this difficulty is called the new inflationary Universe^{12,13)}. In this revised scenario, the scalar field which drives the phase transition [the 24 for SU(5)] tunnels through the barrier responsible for supercooling *before* the inflation begins. Suppose the scalar potential were very flat so that the field took a long time to reach its vacuum expectation value (v.e.v.) $v = \langle 0 | \phi | 0 \rangle$ at the global minimum after tunnelling. Then inside a single bubble the vacuum energy density would still be approximately constant and the bubble would expand exponentially. If a single bubble expanded by 28 orders of magnitude, we need no longer be concerned with "finishing" the phase transition. Eventually the field will reach the global minimum and reheat the Universe by converting the vacuum energy density to radiation.

Indeed, such flat potentials do exist. They are the Coleman-Weinberg¹⁴⁾ potentials which are derived by considering first order radiative corrections to the tree potential. Their properties with respect to cosmology have been discussed in detail¹⁵⁾ and here I shall only concentrate on their inflationary capabilities^{12,13)}. In general the Coleman-Weinberg potential may be written as

$$V(\phi) = A \phi^4 \left(\ln \frac{\phi^2}{v^2} - \frac{1}{2} \right) + D \phi^2 \quad (1.12)$$

where

$$A = \frac{1}{64\pi^2 v^4} \left[\sum_B g_B m_B^4 - \sum_F g_F m_F^4 \right] \quad (1.13)$$

and $g_{B(F)}$ is the number of boson (fermion) helicity states of mass $m_{B(F)}$. The expression (1.13) for A takes into account all possible first order corrections due to fermions (with a relative minus sign)

as well as bosons. The effective mass² is given by

$$D = \frac{1}{2} (m_0^2 + cT^2 + bR - 3\lambda \langle \phi^2 \rangle) \quad (1.14)$$

where m_0 is a bare mass, cT^2 is a gauge group dependent finite temperature correction, bR represents the coupling of ϕ to scalar curvature $R = R^\mu_\mu$ ($b = 1/6$ for conformal couplings), $-\lambda/4$ is the ϕ^4 self-coupling and $\langle \phi^2 \rangle$ is the quantum expectation value of ϕ^2 in curved space^{16,17}.

In order to establish whether or not sufficient inflation can occur in an SU(5) Coleman-Weinberg model, I will follow the proposal of Hawking and Moss¹⁸, that the tunnelling event occurs in a spacially homogeneous manner over a horizon volume. The tunnelling action in this case is given by¹⁸)

$$B = \frac{3M_p^4}{8} \left[\frac{1}{V_0} - \frac{1}{V_1} \right] \approx \frac{3M_p^4}{8} \left[\frac{V_1 - V_0}{V_0^2} \right] \quad (1.15)$$

for $V_1 - V_0 \ll V_0$, where V_0 is the value of the potential at the origin (or symmetric minimum)

$$V_0 = \frac{1}{2} A v^4 \quad (1.16)$$

and V_1 is the value of the potential at the local maximum (separating the two minima at $\phi = 0$ and $\phi = v$)¹⁹)

$$V_1 \approx V_0 + D^2/4A \ln \left(\frac{4v^2 A}{D} \right) \quad (1.17)$$

so that²⁰)

$$B = 3M_p^4 D^2/8A^3 v^8 \ln \left(\frac{4v^2 A}{D} \right) \quad (1.18)$$

In order for the Hawking and Moss¹⁸) picture to be correctly realized²¹), the action B must be $\gg 1$. (Otherwise the transition might take place before the De Sitter expansion begins.) Although this may not be a sufficient condition²²) (i.e. other small actions may exist) it is necessary for inflation. The second requirement for inflation is that the timescale to roll over from the local maximum to the global minimum is long enough to guarantee sufficient inflation. The roll over timescale is

$$\tau = 3H/2D \quad (1.19)$$

and to have enough inflation we must have:

$$\begin{aligned} \exp(H\tau) &> 10^{28} \\ 3H^2/2D &> 65 \end{aligned} \quad (1.20)$$

If we combine the two requirements, $B \gg 1$ and $H\tau > 65$, we see that²⁰⁾

$$\left(\frac{8A^3}{3}\right)^{1/2} \frac{v^4}{M_p^2} \ln^{1/2}\left(\frac{4v^2 A}{D}\right) < D < \frac{4\pi A v^4}{130 M_p^2} \quad (1.21)$$

where we have used

$$H^2 = \frac{8\pi V_0}{3 M_p^2} = \frac{4\pi A v^4}{3 M_p^2} \quad (1.22)$$

In standard SU(5), A is determined. The dominant contributions to A come from the gauge bosons X and Y, so that with $g_B = 36$ and $m_{X,Y} = (25/8) g^2 v^2$ we have

$$A = \frac{5625}{1024 \pi^2} g^4 \approx 5 \times 10^{-2} \quad (1.23)$$

for $g^2/4\pi = 1/41$. In the range of interest the $\ln$ term in (1.21) is about 5 and in order for the double inequality to be consistent we must have

$$A \leq \frac{1}{4} \left(\frac{\pi}{130}\right)^2 \approx 10^{-4} \quad (1.24)$$

hence the first major stumbling block for inflation in SU(5). There are, however, several other reasons why standard SU(5) will not provide an inflationary scenario. The upper limit on the effective mass² D in (1.21) implies a mass for the scalar field $O(10^8)$ GeV. This requires an enormous amount of finetuning as scalar masses will generally pick up corrections $O(10^{15})$ GeV. Linde¹⁷⁾ has also pointed out that unless λ in (1.14) is $\leq 5 \times 10^{-3}$ scalar field fluctuations will drive the transition without sufficient inflation (the ϕ^4 coupling here is an order of magnitude larger).

The most devastating blow to the inflationary scenario in standard SU(5) comes from the production of density fluctuations²³⁻²⁶⁾. GUTs do very well in producing²⁷⁾ the desired

Harrison-Zel'dovich²⁸⁾ scale independent spectrum of fluctuations. Without going into too much detail, it can be shown²³⁻²⁶⁾ that the magnitude of density fluctuations in SU(5) is:

$$\frac{\delta\rho}{\rho} \simeq \left(\frac{8A}{3\pi^3}\right)^{1/2} \ln^{3/2}(Hk^{-1}) \sim \mathcal{O}(10^{-1}) \quad (1.25)$$

for $Hk^{-1} \sim 10^{21}$ where k is the wave number of the fluctuation. Of course, what we want is a value $\delta\rho/\rho \sim \mathcal{O}(10^{-4})$ in order to remain compatible with limits on the isotropy of the background radiation and still remain strong enough to eventually produce galaxies.

In the remaining sections of this contribution, I will try to argue that although the above looks very discouraging, the inflationary scenario is by no means dead. I will begin by discussing the motivations for considering supersymmetry and supergravity in section 2. In section 3, I will discuss a specific model for inflation in the context of supergravity. Finally, I will follow through the post-inflationary period through the breaking of SU(5) in section 4. Much of what will be discussed in the following is a result of a very fruitful collaboration with J. Ellis, D.V. Nanopoulos, M. Srednicki and K. Tamvakis.

II. SUPERSYMMETRY, SUPERGRAVITY AND COSMOLOGY

Supersymmetry²⁹⁾ is a symmetry which directly relates bosons and fermions. Supersymmetric theories³⁰⁾ are classified according to the number of charges N , present in the theory. For gauge theories with maximum spin = 1 fields one is restricted to $N \leq 4$. Local supersymmetry or supergravity³¹⁾ is consistent with gauge theories for $N \leq 8$. In the simplest supersymmetric theory ($N = 1$), the particle content is divided into two types of representations. The first is a gauge supermultiplet, which associates to each gauge boson a gauge fermion. That is, in addition to the gauge bosons such as the photon, gluons, $W^\pm$ and Z , there will be photinos, gluinos, winos and zinos. The self-interactions of these particles are determined by the gauge coupling constant. The second type of representation is the chiral supermultiplet which includes the quark and lepton fields and their associated scalar partners and the Higgs fields along with their fermionic partners. The chiral superfields will have both gauge interactions as well as self-interactions which are determined by a superpotential f . For a renormalizable theory, the superpotential can be at most cubic in the fields.

In a globally supersymmetric theory²⁹⁾, the effective potential describing the self-interactions of the chiral superfields is derived from the superpotential f as follows

$$V(\phi_i) = \sum_i |f_{\phi_i}|^2 \quad (2.1)$$

where

$$f_{\phi_i} = \partial f / \partial \phi_i \quad (2.2)$$

for all fields ϕ_i . Global supersymmetry is unbroken if

$$f_{\phi_i} = 0 \quad \forall \phi_i \quad (2.3)$$

(in the present discussion, I will neglect all gauge interactions; similar conditions for these exist as well).

In a locally supersymmetric theory, the effective potential³²⁾ is somewhat more complicated

$$V(\phi_i) = \left[\sum_i |f_{\phi_i}|^2 - \frac{3}{M^2} |f|^2 \right] \exp \left[\xi |\phi_i|^2 / M^2 \right] \quad (2.4)$$

where the generalized derivative f_{ϕ_i} is now

$$f_{\phi_i} = \partial f / \partial \phi_i + \phi^* f / M^2 \quad (2.5)$$

where $8\pi M^2 = M_P^2 = G_N^{-1}$ is the inverse of the gravitational constant. Note that regardless of the superpotential f , the exponential in (2.4) renders $V(\phi_i)$ non-renormalizable. Thus, because of the non-renormalizability of supergravity, there is no longer any use to the restriction to, at most, cubic terms in f . Indeed, we shall make much use of this extra freedom. Local supersymmetry is also unbroken for $f_{\phi_i} = 0$.

Aside from its aesthetic value, supersymmetry is of course very useful for curing maladies in particle physics and as we shall see, in cosmology. Non supersymmetric gauge theories seem to require scalar fields for spontaneous symmetry breaking. Apart from this, there is no other reason (theoretical or experimental) for their existence. Supersymmetry has them already built in. But the immense popularity of supersymmetry is because of the possibility it offers of solving the gauge hierarchy problem³³⁾. Stated simply, the gauge hierarchy problem is twofold. On the one hand, one would like to understand why there is the apparent discrepancy in the mass scales

$$M_W \ll M_G \ll M_P \quad (2.6)$$

Secondly, in ordinary GUTs, when one calculates radiative corrections to scalar masses (which are responsible for generating the scales M_W and M_G) they tend to always push the light scalars (with

masses M_W) up to the heavy ones (with masses M_G). In order to preserve the hierarchy, an enormous amount of finetuning must take place.

Supersymmetry has the possibility of solving³³⁾ the second aspect of the hierarchy problem. Because of the existence of so-called non-renormalization theorems³⁴⁾, which guarantee cancellations (to all orders in perturbation theory) when calculating radiative corrections in an exactly supersymmetric theory. Thus, once a particular value of a parameter is set at the tree level, it stays there. In a broken supersymmetric theory there will be corrections proportional to the mass splitting between bosons and fermions

$$m_B^2 - m_F^2 = M_S^2 \epsilon \quad (2.7)$$

where M_S is the scale of supersymmetry breaking and ϵ is some coupling constant. These cancellations have also been employed in attempts to cure the strong CP problem³⁵⁾.

In analogy with spontaneous symmetry breaking of gauge theories, global supersymmetry is broken with the appearance of a goldstino. In local supersymmetry, the goldstino is eaten by the gravitino, the superpartner of the spin 2 graviton, through the superhiggs effect^{36,37)}. The mass of the gravitino is determined in much the same way as that of gauge boson in ordinary gauge theories

$$m_B = g \langle \phi \rangle \quad (2.8)$$

where g is a gauge coupling constant and $\langle \phi \rangle$ is the v.e.v. of the scalar field breaking the gauge symmetry. The corresponding order parameter for supersymmetry is M_S^2 and the gravitino mass is given by³⁷⁾

$$m_{3/2} \sim G_N^{1/2} M_S^2 \sim M_S^2 / M_P \quad (2.9)$$

In particular ϵ in (2.7) should be $O(m_{3/2}/M_P)$ so that mass splittings and mass corrections should be³⁸⁾

$$\delta m \sim M_S^2 / M_P \lesssim M_W \quad (2.10)$$

in order to preserve the hierarchy. In fact in this way, broken supersymmetry generates^{39,40)} the weak interaction scale. In section 4, we will see how the GUT scale is also generated by M_S .

Let us now see how supersymmetry affects the cosmological inflationary scenario^{20,41)}. If one recalls, one of the major difficulties with the new inflationary model stemmed from the discrepancy between (1.23) and (1.24). Radiative corrections in a supersymmetric theory, however, are greatly altered. In exact supersymmetry, the no-renormalization theorems³⁴⁾ tell us that first order radiative corrections of the Coleman-Weinberg type are exactly zero. Indeed by glancing at the expression (1.13) for the ϕ^4

self-coupling A , we realize that for each boson of mass m_B and total helicity g_B , there is an associated fermion of mass $m_F = m_B$ and total helicity $g_F = g_B$ (for each $N = 1$ chiral superfield there are two scalars per fermion and for gauge superfields, one gauge boson per gauge fermion). Hence with exact supersymmetry

$$A \equiv 0 \quad (2.11)$$

We know, however, that supersymmetry must be broken in nature and the mass degeneracy between bosons and fermions be lifted. The splitting between masses is just (2.7) which we will leave arbitrary for the moment. [It may even be different for different supermultiplets^{39,40}].] The superheavy gauge boson contribution to (1.13) then becomes²⁰⁾

$$\begin{aligned} A &= \frac{g_{B(F)}}{64\pi^2 v^4} [m_B^4 - m_F^4] \approx \frac{g_{B(F)}}{32\pi^2 v^4} [m_B^2 - m_F^2] m_B^2 \\ &\approx \frac{g_{B(F)}}{32\pi^2 v^4} m_B^2 M_s^2 \epsilon \end{aligned} \quad (2.12)$$

For the SU(5) gauge bosons X and Y, we have²⁰⁾

$$A = \frac{75}{32\pi^2} g^2 \frac{M_s^2}{v^2} \epsilon \quad (2.13)$$

The twin constraint (1.21) which insures sufficient inflation translates to

$$\frac{1}{2} g^3 \frac{M_s^3 \epsilon^{3/2} v}{M_p^2} \lesssim D \lesssim 2 \times 10^{-2} g^2 \frac{M_s^2 \epsilon v^2}{M_p^2} \quad (2.14)$$

or to remain consistent

$$g M_s \epsilon^{1/2} \lesssim 5 \times 10^{-2} v \sim 10^{14} \text{ GeV} \quad (2.15)$$

which is trivially satisfied in almost any supersymmetric model.

The remaining problems discussed in the previous section are also easily seen to disappear. The technical problem of finetuning the mass terms in (1.14) to low values is cured by realizing that the deadly corrections can now be kept small $O(M_s^2 \epsilon)$. Even the constraint coming from the overproduction of density perturbations disappears. Equation (1.25) becomes

$$\frac{\delta\rho}{\rho} \sim \frac{5}{2\pi^{3/2}} \cdot \frac{g M_S \varepsilon^{1/2}}{\nu} \cdot \ln^{3/2}(H k^{-1}) \quad (2.16)$$

which for $\nu \sim 10^{15}$ GeV and $g \sim 0.7$ will yield $\delta\rho/\rho \sim 10^{-4}$ for $M_S \varepsilon^{1/2} \sim 2 \times 10^{10}$ GeV. Hence a glimmer appears that inflation and supersymmetry will go well together.

The major effects, until now, of supersymmetry were that it i) produced a larger barrier to keep the action B large and insure sufficient supercooling and ii) by lowering the vacuum energy density of the symmetric state, it produced a flatter potential allowing for a longer roll-over time scale. The above, however, was not a supersymmetric model for inflation, but rather an indication of which way things will go. In the following sections, I shall look closely at real supersymmetric models and their potential for supplying a complete and problem-free cosmological scenario.

III. PRIMORDIAL INFLATION

As we have seen, in order to preserve³⁸⁾ the gauge hierarchy, any realistic supersymmetric model will need relatively small mass splittings $O(M_W)$. This would imply a supersymmetry breaking scale

$$M_S^2 \sim M_W M_P \sim (10^{10} \text{ GeV})^2 \quad (3.1)$$

At energy scales at or above the GUT scale, the effects of supersymmetry breaking will be negligible. It seems reasonable, therefore to consider inflation in the context of (nearly) exact supersymmetry. First order corrections to the tree potential may then be done away with entirely. Let us furthermore, for reasons which will become clear shortly, consider the field which drives inflation to be a chiral superfield gauge singlet ϕ (the inflaton) which receives a v.e.v. $\langle 0|\phi|0 \rangle = \mu$ with

$$M_G < \mu < M_P \quad (3.2)$$

Before specifying a particular superpotential for the inflaton, suppose that the derived effective zero temperature potential can be put in the polynomial form⁴¹⁾

$$V(\phi) = \alpha \phi^4 - \beta \phi^3 + \gamma \phi^2 + \delta \quad (3.3)$$

Starting with a general potential of this form (which can hopefully be derived in a supersymmetric theory), we can derive what conditions must be imposed on the parameters α , β , γ , and δ in order to achieve inflation. To begin with, we assume that all subsequent

phase transitions will produce a change in the vacuum energy density which is negligible when compared with the one discussed here. This will be true, of course, if the inflation scale is much larger than the GUT scale for example. Thus at the global minimum of the potential (at $\langle 0|\phi|0\rangle = \mu$) we will require that the cosmological constant vanish. The effective cosmological constant (after the transition) is just the value of the potential at the minimum

$$\Lambda = \frac{8\pi}{3} V(\mu) / M_p^2 \quad (3.4)$$

thus we require

$$V(\mu) = 0 \quad (3.5)$$

We must also insure that at $\phi = \mu$ the potential has a minimum i.e.,

$$\frac{\partial V}{\partial \phi}(\mu) = 0 \quad \text{and} \quad \frac{\partial^2 V}{\partial \phi^2}(\mu) > 0 \quad (3.6)$$

The vacuum energy density at the origin or "symmetric" minimum is just $V(0) = \delta$. Finally, the potential must possess a local maximum near the origin. Its position is solved to be at

$$\phi_1 \approx 2\gamma/3\beta \quad (3.7)$$

The value of the potential at the local maximum is just

$$V_1(\phi_1) \approx \frac{4}{27} \frac{\gamma^3}{\beta^2} + \delta \quad (3.8)$$

Up to this point, I have not discussed anything but the zero temperature potential. In general, however, one must consider finite temperature corrections to the potential of the form

$$\delta V_T = \frac{T^2}{8} \partial^2 V / \partial \phi \partial \phi^* \quad (3.9)$$

(In general the field ϕ will be complex; it is considered real here only for simplicity). At high temperatures prior to the onset of inflation, these are of course important and must be considered. For all temperatures such that $T^2 \ll \gamma/\alpha$, the corrections take on a simple form,

$$\delta V_T = c T^2 \phi^2 \quad (3.10)$$

$$c = 3\alpha - \frac{9}{4}\beta^2/\gamma \quad (3.11)$$

Furthermore, during inflation, these may be neglected if at the Hawking temperature

$$T_H^2 = \frac{H^2}{4\pi^2} = \frac{2}{3\pi} \frac{\delta}{M_P^2} \quad (3.12)$$

which is the lowest temperature attainable during the De Sitter phase^{18,42,43}), we have $cT_H^2 \ll \gamma$.

Let us now determine under what conditions we have inflation. The first requirement²¹⁾ was that the action be large. In this case, we have

$$B = \frac{M_P^4 \gamma^3}{18 \beta^2 \delta^2} \gg 1 \quad (3.13)$$

while the condition for a long roll-over time scale is

$$\frac{\gamma}{\delta} < \frac{4\pi}{65 M_P^2} \quad (3.14)$$

To get an idea of what these conditions (3.13) and (3.14) mean, it is useful to scale the parameters β , γ , δ by dimensionless quantities

$$\hat{\beta} = \beta/\mu ; \quad \hat{\gamma} = \gamma/\mu^2 ; \quad \hat{\delta} = \delta/\mu^4 \quad (3.15)$$

Equation (3.13) then becomes

$$B = \frac{M_P^4 \hat{\gamma}^3}{18 \mu^4 \hat{\beta}^2 \hat{\delta}^2} \gg 1 \quad (3.16)$$

and eq. (3.14) becomes

$$\frac{\hat{\gamma}}{\hat{\delta}} < \frac{4\pi}{65} \left(\frac{\mu^2}{M_P^2} \right) \quad (3.17)$$

where μ remains the v.e.v. of the field ϕ at the global minimum.

We can clearly see now the effect of the inflationary scale. If we choose $\mu \sim 10^{15}$ GeV, we must then fine tune $\hat{\gamma}/\hat{\delta} \lesssim 10^{-9}$ and furthermore $\hat{\beta}^2/\hat{\delta} \lesssim 10^{-14}$! Thus if we suppose at best that $\alpha \sim \hat{\beta} \sim \hat{\delta}$, [remember eq. (3.6)], all of these must be $O(10^{-14})$ and $\hat{\gamma} \sim 10^{-25}$. Clearly, rather absurd values for coupling constants. Notice, however, what happens when $\mu = M_P$. Equation (3.17) tells us that $\hat{\gamma}/\hat{\delta} \lesssim 10^{-1}$ and $\hat{\beta}^2/\hat{\delta} \lesssim 10^{-6}$ which is more typical of known Yukawa couplings for example. These observations are what we feel to be the motivations for primordial inflation⁴¹⁾. As the v.e.v. of the field which drives inflation becomes larger than the GUT scale and moves towards the Planck scale fine tunings disappear while choices for couplings become more natural.

Before moving on to a specific model, it is important to note what happens to the magnitude of density fluctuations⁴⁴⁾. The basic property that these perturbations are essentially scale-independent remains intact. What we find, however, is that it is not longer the ϕ^4 coupling which governs the magnitude, but rather the cubic coupling β . We find that for the potential of the form (3.3)

$$\frac{\delta\rho}{\rho} \approx \frac{1}{(2\pi^3)^{1/2}} \frac{\beta}{H} \ln^2(Hk^{-1}) \quad (3.18)$$

In terms of $\hat{\beta}$, we must require $\hat{\beta}^2 \sim 10^{-12} (\mu/M_p)^2 \hat{\delta}$ to have $\delta\rho/\rho \sim 10^{-4}$. This again is a very stiff requirement especially for $\mu \ll M_p$. As we will see shortly, supergravity will ease this constraint as well as previous ones.

As primordial inflation implies $\mu \sim M_p$, we must ask ourselves if first order gravitational (FOG) effects will start to become important. Indeed, Planck time physics remains very uncertain. One possibility, however, is considering the effects of local supersymmetry or supergravity. The hope, of course, is that eventually all quantum gravitational effects will be understood in the context of, for example, an $N = 8$ supergravity theory. In some, as yet unknown, way at scales at or below the Planck scale, one may be left, however, with a residual $N = 1$ supergravity theory. We will make, therefore, a large assumption; all FOG (and higher) effects are taken into account when one works in the context of supergravity. In particular, gravitational interactions will be included by considering non-renormalizable terms in the superpotential⁴⁵⁾ and at scales below M_p we are left with a workable effective theory.

To treat inflation in simple supergravity⁴⁶⁾, let us start with the most general superpotential for a single scalar field, the inflaton ϕ ,

$$f_I = m^3 \left(\sum_n \frac{\lambda_n}{n+1} \left(\frac{\phi}{M} \right)^{n+1} + \lambda' \right) \quad (3.19)$$

where m is an (as yet) unspecified mass parameter, $M = M_p/\sqrt{8\pi}$ and the λ_i are all dimensionless coupling constants. From eqs. (2.4) and (2.5), we can derive the effective potential and put it in the form (3.3). There will, of course, be terms higher than fourth order, but at scales $\phi \ll M$ we can neglect (?) these. In order for the superpotential (3.19) to provide a satisfactory inflationary picture, we must insure that at the origin we have a true minimum and that there is a second minimum at $\langle 0|\phi|0 \rangle = \mu$ which we will take to be equal to M . A supersymmetric minimum at M requires⁴⁶⁾

$$\sum_n \frac{\lambda_n}{n+1} = \sum_n \lambda_n = 0; \quad \sum_n n \lambda_n \neq 0 \quad (3.20)$$

A minimum at the origin is found if $\lambda' = 2\lambda_1$.

Without passing through all the details, a similar analysis to the general one given above yields the necessary constraints on the parameters

$$\lambda_0 > 260 \lambda_2 \quad (3.21a)$$

$$-\lambda_3 m^3 \ll 4 \times 10^{-3} \quad (\lambda_3 < 0) \quad (3.21b)$$

$$\lambda_0 m^3 \ll 10^{-3} \quad (3.21c)$$

for the choice $\lambda_1 = -\lambda_3$; and this easily achieved for $m = 10^{-2} M$, $\lambda_0 \approx 1$, $\lambda_1 = -\lambda_3 \approx 10^{-1}$ and $\lambda_2 \approx 10^{-3}$. This is of course only a single choice for parameters of which many are possible and still give density perturbations $\delta\rho/\rho \sim 10^{-4}$. Hence, we see that in the context of supergravity one can have an acceptable inflationary scenario without any severe adjustments to parameters.

At this point, we must remember however, that although primordial inflation can solve the standard cosmological problems such as curvature, isotropy etc., it says nothing about the monopole problem. Previously inflation was occurring during the SU(5) phase transition. In this case, because the number of GUMs per bubble is O(1) and we live in a single bubble, there is only about one GUM in our visible Universe. Primordial inflation is not associated with SU(5) and care must be taken with respect to the production of GUMs. In fact, the whole problem of breaking SU(5) and baryon generation is not trivial in the context of supersymmetry. In the following section, I will look at this problem as well as the evolution of the Universe subsequent to primordial inflation.

IV. SUPER-COSMOLOGY

Globally supersymmetric theories, all have one thing in common; the lowest vacuum state is the supersymmetric one with zero vacuum energy⁴⁷⁾. At zero temperature, supersymmetric GUTs may have several degenerate minima⁴⁸⁾, e.g. SU(5), SU(3) $\times$ SU(2) $\times$ U(1), SU(4) $\times$ U(1) etc. all with zero vacuum energy density. Unlike gauge symmetries, supersymmetry is broken at finite temperature⁴⁹⁾ due to the different spin statistics of bosons and fermions. As the Universe cools, it may eventually choose a supersymmetric minima⁵⁰⁾, but which one?

In a non-supersymmetric GUT, at very high temperatures, generally only a symmetric minimum [e.g. SU(5)] exists. As the temperature

drops, broken minima [e.g. $SU(3) \times SU(2) \times U(1)$ or $SU(4) \times U(1)$] begin to appear. At a critical temperature, T_c , the value of the potential at both the symmetric minimum and broken minimum are equal. (There may be more than one critical temperature.) At lower temperatures, the broken minima drop far below the symmetric state and a phase transition occurs. What one tends to find⁴⁸⁾ in supersymmetric GUTs is that the symmetric state remains the global minimum as the temperature drops. In a zero temperature, exactly supersymmetric GUT, this state of affairs would (naïvely) remain until $T = 0$, hence the Universe would never be able to get out of the $SU(5)$ phase. One might hope that because supersymmetry must (somehow) get broken, things might change. But in order to preserve the gauge hierarchy, these effects should only be felt below M_W , thus making baryon generation practically impossible.

Something else does occur, however. As the Universe cools, at some point $SU(5)$ interactions become strong. Indeed, when one calculates the renormalization group equations for the $SU(5)$ fine structure constant, $\alpha = g^2/4\pi$, one can find⁴⁸⁾ that $\alpha \sim 0(1)$ at a temperature $T \sim \Lambda_{SU(5)} \sim 10^9 - 10^{10}$ GeV. To see what happens as T approaches $\Lambda_{SU(5)}$, one has to look at the form of the finite temperature effective potential

$$V_T(\Sigma) = V(\Sigma) + c_1 \Sigma^2 T^2 - c_2 T^4 \quad (4.1)$$

where Σ is the 24 breaking $SU(5)$, c_1 can be derived from the superpotential yielding $V(\Sigma)$ and $c_2 > 0$ is a constant depending on the particle content⁴⁹⁾. At high temperatures, $T > \Lambda_{SU(5)}$, $SU(5)$ has many more light particle degrees of freedom than either $SU(3) \times SU(2) \times U(1)$ or $SU(4) \times U(1)$ and $c_2(5) > c_2(3,2,1)$, $c_2(4,1)$. When T approaches $\Lambda_{SU(5)}$, the theory becomes "confining" and particle degrees of freedom begin to disappear, thus lowering $c_2(5)$ relative to the other states. At this point the phase transition becomes possible. One may also arrange the Higgs sector in such a way as to make $c_2(3,2,1) > c_2(4,1)$ so that $SU(3) \times SU(2) \times U(1)$ is actually preferred over $SU(4) \times U(1)$ ⁵¹⁾. [This choice in fact is also preferred from the point of view of generating a baryon asymmetry⁵²⁾.]

Although a transition is now in principle possible because the vacuum energy density of the broken state is lower than the symmetric one, a large barrier separating the two phases may prohibit the transition^{50, 53)}. For example, if one starts with the most general (renormalizable) superpotential for the adjoint Σ alone⁴⁸⁾

$$f(\Sigma) = \frac{1}{2} M_\Sigma \text{Tr}(\Sigma^2) + \frac{1}{3} \lambda \text{Tr}(\Sigma^3) \quad (4.2)$$

where $M_\Sigma \sim M_G \sim 10^{16}$ GeV, one finds that the bubble formation rate of the broken phase is very small:

$$P \approx M^4 e^{-B} \quad (4.3)$$

where the action B in this case is⁵³⁾

$$B \approx .04 (M_\Sigma / T)^9 \quad (4.4)$$

P is largest when the transition first becomes possible, i.e. at $T \sim \Lambda_{\text{SU}(5)}$ so that

$$P \lesssim 10^{-10^{52}} \quad (4.5)$$

and hence no transition occurs.

In the context of global supersymmetry, this situation can be remedied by taking a very small value for both λ and $M^{51)}$. For small enough values, the barrier separating the broken and symmetric phases can be made arbitrarily small. In particular, we would like the barrier height to be $\lesssim 0(10^{10})$ GeV so that strong coupling phenomena at a temperature of the same order could drive the transition. To do this, however, one needs⁴¹⁾ $M_\Sigma \sim M_W$ and $\lambda \sim M_W/M_G$. Although such an assignment is technically possible, it seems very unnatural and contrary to the desire to keep all couplings $0(10^{-3}-1)$.

It is at this point that supergravity and the use of non-renormalizable terms can save us. Consider the superpotential⁵⁴⁾

$$f_S = \frac{a_1 X^4}{M} + \frac{a_2 X^2 \text{Tr}(\Sigma^3)}{M^2} + \mu^2(z + \Delta) \quad (4.6)$$

where $M = M_P/\sqrt{8\pi}$ and X and z are SU(5) singlets. The couplings a_1 and a_2 will be taken $0(1)$. The third term in (4.6) is needed to break local supersymmetry and will not be discussed here. The mass scale $\mu \sim M_S \sim 10^{10}$ GeV. Once again from eqs. (2.4) and (2.5), one can derive the effective potential for the fields X and Σ . Indeed, (4.6) was chosen so as to mimic the behaviour of (4.2). When one solves for the various minima of the effective potential one finds that in fact the $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$ minimum is the lowest. Furthermore, it occurs when Σ takes the v.e.v.

$$\langle \Sigma \rangle \approx (\mu M)^{1/2} \quad (4.7)$$

This is in reality a prediction of the GUT scale

$$M_G^2 \approx (\mu M) \sim M_S M_P \sim (10^{15} \text{ GeV})^2 \quad (4.8)$$

At this minimum, the field X has an expectation value $\langle X \rangle = (\mu/M)^{3/4} M$ and gives a barrier height of $(\mu/M)^{5/4} M \sim 10^8$ GeV.

Thus we have been able to reproduce the scenario in global supersymmetry without small parameters by considering supergravity.

To complete the model of the evolution of the early Universe in $N = 1$ supergravity, there must still be a mechanism for generating a baryon asymmetry²⁾ and a solution to the monopole problem⁵⁾. The solution to the monopole problem is rather simple in this case. Because the transition has been delayed down to $T \sim \Lambda_{SU(5)} \sim 10^9 - 10^{10}$ GeV, this is in effect the critical temperature for the $SU(5)$ transition and the temperature at which monopoles are produced. Equation (1.5) would then say that

$$\frac{n_{GUM}}{n_\gamma} \sim \left(\frac{10 \Lambda_{SU(5)}}{M_P} \right)^3 \sim 10^{-24} - 10^{-27} \quad (4.9)$$

which is consistent with the cosmological limit. In fact, coupled together with the new inflationary scenario, primordial inflation and supercosmology might be the only way to get an observable number of monopoles. As monopoles will in general not be produced thermally, the standard new inflationary scenario^{12,13)} would predict only one monopole per visible region.

To generate a baryon asymmetry is in general not quite so easy. Detailed calculations^{55,56)} of the baryon asymmetry produced in standard models, tend to show that a large asymmetry becomes more and more difficult as the scale at which the asymmetry is produced is lowered. In standard $SU(5)$ one has⁵⁶⁾

$$\frac{k n_B}{s} \approx 2 \times 10^{-3} (\Delta B) [(3K)^{1/2} + 1]^{-1} \quad (4.10)$$

where s/k is the specific entropy, ΔB is the net baryon number produced by a decay of a pair of Higgses $H, \bar{H}$ and $K = 2.9 \times 10^{17} \alpha_H / M_H$ is the ratio of the decay rate to the expansion rate at $T = M_H$. The observed baryon to photon ratio today is related to $k n_B / s$ by

$$\frac{n_B}{n_\gamma} \approx 7 (k n_B / s) \quad (4.11)$$

In the standard picture, if the lightest Higgs boson were to have a mass $O(10^{15})$ GeV with a Yukawa coupling $\alpha_H \sim 10^{-4}$ we would have a suitable asymmetry $k n_B / s \sim 10^{-3} \Delta B$.

In supercosmology, however, to have any chance of generating an asymmetry at 10^{10} GeV one needs Higgs bosons with masses of that order that have baryon number violating interactions. In a non-supersymmetric theory, this would result in a (too) low asymmetry $k n_B / s \sim 4 \times 10^{-8} \Delta B$. In supersymmetric models, the situation at first sight seems even worse⁵⁷⁾. The increased number of particle degrees of freedom and larger couplings along with dimension 5 operators³⁰⁾, drive things towards equilibrium and not away from it. Naïvely, one might then expect⁵⁷⁾ the baryon asymmetry to become negligible in supersymmetric models with 10^{10} GeV Higgses.

This approach does not take into account a possibility which is available, i.e. to couple the H and $\bar{H}$ to an $SU(5)$ singlet⁵¹⁾. In this case, one can show that it is relatively simple to get a large asymmetry with low mass Higgses in supersymmetry.

To complete the scenario, consider the superpotential^{54,58)}

$$f_B = a_3 \frac{X Y \bar{H} H}{M} + a_4 M \bar{\theta} \theta + a_5 \frac{\bar{\theta} \Sigma^2 H}{M} + a_6 \frac{H \Sigma^2 \theta}{M} + a_7 \mu Y^2 + a_8 Y^3 + \text{Yukawa couplings} \quad (4.12)$$

where H and $\bar{H}$ are a 5 and $\bar{5}$ and Y is the additional $SU(5)$ singlet which will be used to generate the baryon asymmetry. θ and $\bar{\theta}$ are a 50 and $\bar{50}$ used to separate the mass scales of the doublet and triplet in H and $\bar{H}$ ⁵⁹⁾. All couplings a_3 - a_8 will be $O(1)$. The complete superpotential for the "world" will then be

$$f = f_I + f_S + f_B \quad (4.13)$$

Subsequent to inflation, as ϕ reaches its v.e.v., the Universe begins to fill up with the inflatons ϕ at an initial temperature

$$T_\phi \approx 10^{15} \text{ GeV} \quad (4.14)$$

Through a gravitational coupling which is generated when one considers the full superpotential (4.13), one can see that the inflatons can decay predominantly into the singlets Y through

$$\left(\frac{m_\phi}{M}\right) a_8 \phi Y^3 \quad (4.15)$$

with a rate given by

$$\Gamma_\phi = m_\phi^3 / M^2 \quad (4.16)$$

for

$$m_\phi = \frac{\partial^2 f}{\partial \phi^2} = \frac{m^3}{M^2} \sum_n n \lambda_n \approx 10^{13} \text{ GeV} \quad (4.17)$$

The decay $\phi \rightarrow 3Y$ takes place at a temperature around 10^{10} GeV.

Shortly afterwards, the $SU(5)$ transition takes place at $T = \Lambda_{SU(5)} \sim 10^9 - 10^{10}$ GeV and the net baryon number of the Universe is $n_B/n_\gamma \approx 0$. Through the " a_3 " coupling in (4.12), the singlets Y (which now dominate the Universe) decay only into an $H, \bar{H}$ pair. The decay rate for this process is⁵⁸⁾:

$$\begin{aligned}\Gamma_Y &\sim a_3^2 \langle X \rangle^2 m_Y / M^2 \\ &\sim a_3^2 (\mu/M)^{5/2} M\end{aligned}\tag{4.18}$$

for $m_Y \sim \mu$ [from the " a_7 " term in (4.12)]. This decay ($Y \rightarrow H, \bar{H}$) occurs at $T \approx 10^7$ GeV. Recall however, that prior to the decay $n_Y/n_\gamma \approx 1$ and so afterwards we must have $n_H/n_\gamma \approx 1$ as well. For $m_H \sim \mu \sim 10^{10}$ GeV at $T = 10^7$ GeV, this is very far from equilibrium. (The equilibrium value would go as $e^{-m_H/T} \ll 1$.) The resultant baryon asymmetry will be its maximum value $n_B/n_\gamma \approx 10^{-2} \Delta B$. It should be stressed that this value is not the result of fine tunings or small coupling constants, but rather a judicious coupling of a singlet⁵¹⁾ and the use of non-renormalizable interactions^{54,58)}.

Finally, it is interesting to note, that without any extra effort, the gravitino problem⁶⁰⁾ is also solved^{58,61)}. This problem arose⁶⁰⁾ when one realizes that gravitinos which decay only gravitationally, do so very late, and if sufficiently abundant, produce too much entropy, possibly ruining the successes of big bang baryosynthesis and/or big bang nucleosynthesis. If, however, the initial abundance of gravitinos were reduced by inflation for example, the problem could be avoided⁶¹⁾. In the above scenario, there is no gravitino problem for an inflaton mass $m_\phi < 10^{15}$ GeV⁵⁸⁾. Baryon generation required an inflaton mass $m_\phi \gtrsim 10^{13}$ GeV (in order to be able to produce Y 's in abundance). We hope that it is not a coincidence that the parameters derived from demanding density perturbations $\delta\rho/\rho \sim 10^{-4}$ gave an inflaton mass $m_\phi \approx 10^{13}$ GeV!

V. SUMMARY AND DISCUSSION

What I have tried to show in the previous sections, is that supersymmetry, supergravity and cosmology all fit very nicely together. The big bang model itself has been able to explain the observed expansion of the Universe, the existence of the microwave background radiation and the origin of the light elements through nucleosynthesis. Before the inception of GUTs, it was not able to give any clue as to why the present density is so close to the critical density while at the same time being so old. It was also not able to explain the high degree of isotropy nor the small observed baryon asymmetry. It is by now no secret that GUTs have radically changed our understanding of the very early stages of the Universe. Baryon number violating interactions led to the solution of the baryon asymmetry, while postulating the existence of a past inflationary period can solve many of the remaining problems. Notwithstanding, the monopole problem and gravitino problem created by GUTs and supersymmetry can also be solved in the same framework.

What I have not discussed up until now is the value of the cosmological constant. Herein lies a remaining cosmological problem, perhaps even more severe than previous ones. Why is the cosmological constant so small? Put in dimensionless form, we know that in order that the expansion today is not dominated by Λ we must have

$$\frac{\Lambda}{M_P^2} \lesssim 10^{-121} ! \quad (5.1)$$

Surely an astounding piece of fine tuning if there ever was one. The trouble is we cannot even say that this was an initial condition which stayed there. Each vacuum phase transition leaves a change in the value of Λ . $N = 8$ supergravity⁶²⁾ generally begins with $\Lambda \sim -M_P^2$ (an anti-De Sitter space). This very large negative somehow must have become very positive just around the Planck time (if we want to keep the inflationary scenario). Whether or not this change was due to the breakdown of the higher supersymmetries by the formation of condensates is as yet unknown. In the present model, the inflationary epoch begins with $\Lambda/M_P^2 \sim 10^{-12}$ (a long way away from its present value). At the GUT phase transition Λ would have been down already to $\Lambda/M_P^2 \sim 10^{-40}$. The next change might have been at the $SU(2) \times U(1)$ transition which saw $\Lambda/M_P^2 \sim 10^{-68}$ followed perhaps by the quark hadron transition which left an effect, $\Lambda/M_P^2 \sim 10^{-80}$ until somehow it arrived at its present value (5.1). This can not be an accident. Is there a symmetry which guarantees a low value today? This question remains a still unanswered goal for cosmology.

Apart from the cosmological constant, (which is tuned here to be zero today), the model presented appears to be devoid of problems involving small parameters or fine tunings. There have, of course, been other attempts at supersymmetric inflationary⁶³⁾ and GUT scenarios⁶⁴⁾, most notably, those which are based on the reverse hierarchy scheme⁶⁵⁾. These scenarios have many features in common with this one (e.g. flat potentials) but seem to fail to produce an acceptable picture after inflation. General difficulties with this type of model will be discussed elsewhere⁶⁶⁾. The main features introduced in the present model are i) at high enough energies ($\gg M_S$), supersymmetry appears to be exact and we may do away with radiative corrections; and ii) the extensive use of non-renormalizable terms leading to effective interactions at scales lower than M_P . If supergravity is on the right track for unifying gravity with standard gauge interactions, then it should not seem surprising that a variety of small parameters previously not understood, stem from these non-renormalizable terms which scale with inverse powers of M_P . One can only be optimistic, that the so-far rapid success of applying supergravity to GUTs and cosmology is just beginning.

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REFERENCES

- 1) For reviews, see: J. Ellis, CERN preprint TH.3174 (1981), to be published in Proc. 1981 Les Houches Summer School;
P. Langacker, Phys. Rep. 72C (1981) 185;
D.V. Nanopoulos, in *Theory of Fundamental Interactions*, ed. G. Costa and R. Gatto, Proc. of the Int. School of Physics Enrico Fermi LXXXI (Soc. Italiana di Fisica, Bologna) 1982, p. 156.
- 2) A.D. Sakharov, Zh. Eksp. Teor. Fiz. Pisma Red. 5, (1967) 32.
- 3) K.A. Olive, D.N. Schramm, G. Steigman, M.S. Turner and J. Yang, Ap. J. 246 (1981) 557;
J. Yang, M.S. Turner, G. Steigman, D.N. Schramm and K.A. Olive, Chicago EFI preprint (1983), *Primordial Nucleosynthesis: A critical comparison between theory and experiment*.
- 4) G. 't Hooft, Nucl. Phys. B79 (1974) 276;
A.M. Polyakov, Zh. Eksp. Teor. Fiz. Pisma Red. 20 (1974) 194.
- 5) Ya.B. Zel'dovich and M.Y. Khlopov, Phys. Lett. 79B (1979) 239;
J.P. Preskill, Phys. Rev. Lett. 43 (1979) 1365.
- 6) A.H. Guth and S.H. Tye, Phys. Rev. Lett. 44 (1980) 631.
- 7) A.H. Guth, Phys. Rev. D23 (1981) 347.
- 8) J. Ellis and K.A. Olive, CERN preprint TH.3399, to be published in Nature (1983).
- 9) D. Kazanas, Ap. J. 241 (1980) L59.
- 10) S. Coleman, Phys. Rev. D15 (1977) 2929;
C. Callan and S. Coleman, Phys. Rev. D16 (1977) 1762.
- 11) A.H. Guth and E. Weinberg, Phys. Rev. D23 (1981) 826, Nucl. Phys. B212 (1983) 321.
- 12) A.D. Linde, Phys. Lett. 108B (1982) 389.
- 13) A. Albrecht and P.J. Steinhardt, Phys. Rev. Lett. 48 (1982) 1220.

- 14) S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 1888.
- 15) L. Abbott, Nucl. Phys. B185 (1981) 233;
M. Sher, Phys. Rev. D24 (1981) 1699;
P. Hut and F. Klinkhamer, Phys. Lett. 104B (1981) 439;
A. Billoire and K. Tamvakis, Nucl. Phys. B200 [FS4] (1982) 329;
K. Tamvakis and C.E. Vayonakis, Phys. Lett. 109B (1982) 283;
G.P. Cook and K.T. Mahanthappa, Phys. Rev. D23 (1981) 1321 and
D25 (1982) 1154;
M.D. Pollock and M. Calvani, Phys. Lett. 117B (1982) 392.
- 16) A. Vilenkin, Phys. Lett. 115B (1982) 91;
A. Vilenkin and L.H. Ford, Phys. Rev. D26 (1982) 1231.
- 17) A.D. Linde, Phys. Lett. 116B (1982) 335.
- 18) S.W. Hawking and I.G. Moss, Phys. Lett. 110B (1982) 35;
See also S.W. Hawking, Cambridge-DAMTP preprint *Euclidean
approach to the inflationary universe* (1983).
- 19) C.-G. Källman, private communication (1982).
- 20) J. Ellis, D.V. Nanopoulos, K.A. Olive and K. Tamvakis, Phys.
Lett. 118B (1982) 335.
- 21) P.J. Steinhardt, Univ. of Pennsylvania preprint UPR-0192T
(1982).
- 22) S. Parke, Phys. Lett. 121B (1983) 313.
- 23) S.W. Hawking, Phys. Lett. 115B (1982) 295;
See also S.W. Hawking and I.G. Moss, Cambridge-DAMTP preprint
Fluctuations in the inflationary universe (1982).
- 24) A.H. Guth and S.-Y. Pi, Phys. Rev. Lett. 49 (1982) 1110.
- 25) A.A. Starobinski, Phys. Lett. 117B (1982) 175.
- 26) J.M. Bardeen, P.J. Steinhardt and M.S. Turner, Chicago EFI
preprint 83-13 (1983).
- 27) W.H. Press, Phys. Scr. 21 (1980) 702.
- 28) E.R. Harrison, Phys. Rev. D1 (1970) 2726;
Ya.B. Zel'dovich, Mon. Not. R. Astron. Soc. 160 (1972) 1P.
- 29) Ya.A. Gol'fand and E.P. Likhtman, Pisma Zh. Eksp. Theor. Fiz.
13 (1971) 323;
D. Volkov and V.P. Akulov, Phys. Lett. 46B (1973) 109;
J. Wess and B. Zumino, Nucl. Phys. B70 (1974) 39;

- For a review see P. Fayet and S. Ferrara, Phys. Rep. 32C (1977) 249.
- 30) For a review, see D.V. Nanopoulos, *Grand Unification and physical supersymmetry*, Lectures given at the XIIIth GIFT International Seminar on Theoretical Physics, Girona, Spain 1983, ed. J.A. Grifols, A. Mendez and A. Ferrando (World Scientific Pub. Co., Singapore 1983).
 - 31) For a review, see P. van Nieuwenhuizen, Phys. Rep. 68C (1981) 189.
 - 32) E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and P. van Nieuwenhuizen, Nucl. Phys. B147 (1979) 105;
E. Cremmer, S. Ferrara, L. Girardello, and A. Van Proeyen, Phys. Lett. 116B (1982) 231 and Nucl. Phys. B212 (1983) 413.
 - 33) L. Maiani, in Proc. of the Summer School of Gif-sur-Yvette (1979) p. 3.
E. Witten, Nucl. Phys. B188 (1981) 513;
S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981) 150;
N. Sakai, Zeit. für Phys. C11 (1982) 153.
 - 34) J. Wess and B. Zumino, Phys. Lett. 49B (1974) 52;
J. Iliopoulos and B. Zumino, Nucl. Phys. B76 (1974) 310;
S. Ferrara, J. Iliopoulos and B. Zumino, Nucl. Phys. B77 (1974) 413;
M.T. Grisaru, W. Siegel and M. Rocek, Nucl. Phys. B159 (1979) 420.
 - 35) J. Ellis, S. Ferrara and D.V. Nanopoulos, Phys. Lett. 114B (1982) 231.
 - 36) D.V. Volkov and V.A. Soroka, JETP Lett. 18 (1973) 312.
 - 37) S. Deser and B. Zumino, Phys. Rev. Lett. 38 (1977) 1433.
 - 38) J. Ellis and D.V. Nanopoulos, Phys. Lett. 116B (1982) 133.
 - 39) R. Barbieri, S. Ferrara and D.V. Nanopoulos, Z. Phys. C13 (1982) 267; Phys. Lett. 116B (1982) 16.
 - 40) J. Ellis, L. Ibanez and G.G. Ross, Phys. Lett. 113B (1982) 983, CERN preprint TH.3382 (1982).
M. Dine and W. Fischler, Nucl. Phys. B204 (1982) 346.
 - 41) J. Ellis, D.V. Nanopoulos, K.A. Olive and K. Tamvakis, CERN preprint TH.3404, Nucl. Phys. B in press (1983).

- 42) G. Gibbons and S.W. Hawking, Phys. Rev. D15 (1977) 2738.
- 43) R. Brandenberger and R. Kahn, Phys. Lett. 119B (1982) 75.
- 44) J. Ellis, D.V. Nanopoulos, K.A. Olive and K. Tamvakis, Phys. Lett. 120B (1983) 331.
- 45) J. Ellis, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. 121B (1983) 123;
J. Ellis, J. Hagelin, D.V. Nanopoulos and K. Tamvakis, SLAC preprint PUB-3042 (1983).
- 46) D.V. Nanopoulos, K.A. Olive, M. Srednicki and K. Tamvakis, Phys. Lett. 123B (1983) 41.
- 47) B. Zumino, Nucl. Phys. B89 (1975) 535.
- 48) M. Srednicki, Nucl. Phys. B202 (1982) 327;
D.V. Nanopoulos and K. Tamvakis, Phys. Lett. 110B (1982) 449.
- 49) L. Girardello, M. Grisaru and P. Salomonson, Nucl. Phys. B178 (1981) 331.
- 50) J. Ellis, C.H. Llewellyn Smith and G.G. Ross, Phys. Lett. 114B (1982) 227.
- 51) D.V. Nanopoulos, K.A. Olive and K. Tamvakis, Phys. Lett. 115B (1982) 15.
- 52) D.V. Nanopoulos and K. Tamvakis, Phys. Lett. 114B (1982) 235.
- 53) M. Srednicki, Nucl. Phys. B206 (1982) 132.
- 54) D.V. Nanopoulos, K.A. Olive, M. Srednicki and K. Tamvakis, Phys. Lett. 124B (1983) 171.
- 55) E.W. Kolb and S. Wolfram, Nucl. Phys. B172 (1980) 224; Phys. Lett. 91B (1980) 217.
- 56) J.N. Fry, K.A. Olive and M.S. Turner, Phys. Rev. D22 (1980) 2953 and 2977; Phys. Rev. Lett. 45 (1980) 2074.
- 57) J.N. Fry and M.S. Turner, Chicago EFI preprint 83-10 (1983).
- 58) D.V. Nanopoulos, K.A. Olive and M. Srednicki, CERN preprint TH.3555 (1983).
- 59) C. Kounnas, D.V. Nanopoulos, M. Quiros and M. Srednicki, CERN preprint TH.3559 (1983).

- 60) S. Weinberg, Phys. Rev. Lett. 48 (1982) 1303.
- 61) J. Ellis, A.D. Linde and D.V. Nanopoulos, Phys. Lett. 118B (1982) 59.
- 62) B. De Wit and H. Nicolai, Nucl. Phys. B188 (1981) 98; Nucl. Phys. B208 (1982) 323.
- 63) A. Albrecht, S. Dimopoulos, W. Fischler, E. Kolb, S. Raby and P. Steinhardt, in preparation (1983);
C.E. Vayonakis, Phys. Lett. 123B (1983) 396.
- 64) S. Dimopoulos and S. Raby, Los Alamos preprint LA-UR-82 (1982).
- 65) E. Witten, Phys. Lett. 105B (1981) 267.
- 66) J. Ellis, D.V. Nanopoulos, K.A. Olive, M. Srednicki and K. Tamvakis, in preparation (1983).