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THE AUXILIARY FIELD TADPOLE METHOD OF EFFECTIVE POTENTIAL
EVALUATION: ITS EXTENSION TO SUSY GAUGE THEORIES

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ABSTRACT

It is shown how auxiliary field tadpoles may be used to calculate the effective potential in SUSY gauge theories. SUSY QED and SUSY QCD are given as examples.

1. INTRODUCTION

The S. Weinberg tadpole method [1] provides one of the simplest approaches in conventional (i.e. non-SUSY) field theory for the evaluation of effective potentials. Its main attributes are i) the simplicity of the underlying theory, ii) that one considers essentially the simplest loop-diagrams in the theory, and more importantly iii) it is a practical way of going beyond the one-loop level.

Recently I have considered SUSY extensions of this method, both by component fields [2] and supergraphs [3]. Although the tadpole supergraph approach is intuitively more appealing, at its present stage of development it requires a good deal more algebra than the component field approach, as was illustrated with the Wess-Zumino model for each case.

In the present paper I pursue the component field approach and show how it is implemented for SUSY gauge theories. The one-loop effective potential of SUSY QED (SQED) and SUSY QCD (SQCD) will be given as examples.

To recap, in the component field approach, referred to as AFTM (the Auxiliary Field Tadpole Method) the main trick is to work with an unconstrained SUSY. As such the S. Weinberg tadpole approach may be developed by looking at auxiliary field tadpoles rather than those of dynamical fields. This has advantages over the more obvious procedure of using constrained SUSY (eliminating auxiliary fields by their equations of motion) and looking at spin 0 tadpoles. For example, because of their dimensionality (D and F) auxiliary fields only couple to spin 0 particles in a simple way and they do not couple to spin 1 or spin $\frac{1}{2}$ fields. As a result even at a one-loop level there are fewer Feynman diagrams to consider. In the Wess-Zumino model we saw [1] how the F tadpole alone was sufficient to reconstruct the effective potential, the contribution from the spin 0 tadpole being generated automatically by use of a SUSY boundary condition. This result is found to extend to SUSY gauge theories [in the Wess-Zumino gauge^{*)}] i.e. it is sufficient to consider only D and F tadpoles, the spin 0 tadpole contributions are generated automatically by a SUSY boundary condition.

^{*)} In this paper I adopt non-SUSY gauge fixing by using the Wess-Zumino gauge supplemented with the Lorentz gauge for vector component fields.

In the case of SQED I make an explicit comparison between my result and that obtained by the Coleman-Weinberg (CW) method [4]. As a consequence (besides confirming agreement between the two results) we will see how AFTM throws new light upon the Coleman-Weinberg approach, in that the spin $1/2$ and spin 1 mass matrix contributions to the CW one-loop supertrace are generated automatically from the spin 0 mass matrix contribution; this result is a consequence of the SUSY boundary condition referred to above. Such a result is not surprising for a SUSY theory, yet on the other hand the CW supertrace expression by itself necessitates the calculation of all the mass matrices. The reason for this of course is that the CW supertrace result is valid for any gauge theory, SUSY or not.

To summarize then, this paper shows how AFTM is implemented for SUSY gauge theories (both Abelian and non-Abelian). Its principle advantages are on ample use of the underlying SUSY which simplifies the calculations and secondly that it provides a clean approach for calculating beyond the one-loop level, which is of practical interest to a class of recently proposed SUSY GUTs [5].

The paper is arranged as follows. In Section 2 a short revision of the tadpole method is presented. Section 3 begins with tree level implementation of AFTM for SQED, here the basic tricks for SUSY gauge theories are uncovered. It then proceeds to the one-loop level and concludes with a comparison to the result obtained by the CW approach. In Section 4 SQCD is dealt with in a similar manner. It is a sufficiently complex theory to allay fears that AFTM might prove difficult for such models. In Section 5 conclusions are drawn.

2. A SHORT REVIEW OF THE AUXILIARY FIELD TADPOLE METHOD (AFTM)

AFTM was described in detail in Ref. [1]. Here a short review of the main features is given.

We write the momentum space expansion for the non-fermionic part of the classical action in terms of spin 0 and auxiliary fields roughly in the fashion,

$$\Gamma(\beta_{c1}) = \sum_n \int d\tilde{p}_1 \dots d\tilde{p}_n (2\pi)^4 \delta\left(\sum_i \tilde{p}_i\right) \frac{1}{n!} \Gamma^{(n)}(p_1 \dots p_n) \beta_{c1}(p_1) \dots \beta_{c1}(p_n) \quad (2.1)$$

where $d\tilde{P}_i = d^4P_i/(2\pi)^4$. $\Gamma^{(n)}(P_i)$ are the appropriate one particle irreducible (1PI) amplitudes of the theory. In AFTM SUSY remains unconstrained in the sense that auxiliary fields are not eliminated by their equations of motion, they play a non-trivial role in the expansion of Eq. (2.1). Equation (2.1) leads [1] to the central equation of AFTM. If V denotes the effective potential then it is determined by

$$\frac{dV(b)}{db} = -\Gamma_{p_{\text{ext}}=0}^{(1)'} \quad (2.2)$$

Here b is the VEV of the field B (spin 0 or auxiliary). $\Gamma_{p_{\text{ext}}=0}^{(1)'}$ is the 1PI B tadpole evaluated at zero external momentum and belongs to the translated theory (hence the prime) defined by

$$\mathcal{L}'(B') = \mathcal{L}(B = B' + b) \quad (2.3)$$

where $\mathcal{L}(B)$ denotes the original (untranslated) Lagrangian.

We recall that the 'true' effective potential is obtained by the replacement $b \rightarrow B_{c1}$ once $V(b)$ is calculated via Eq. (2.2). This is often left understood. For gauge theories in general a *caution* must be given. The validity of Eq. (2.2) assumes that $\mathcal{L}(B)$ itself has no b dependence, i.e. all b dependence arises through the translation in Eq. (2.3). If one employs an R_ξ gauge then this is not always true since here VEV's are introduced prior to the field translations (strictly speaking the gauge fixing term is added to the translated theory to avoid spin 0 vector mixing. However one can work backwards and view gauge fixing at the untranslated level which corresponds to a gauge fixing term containing VEVs prior to translation). In this paper we do not employ an R_ξ gauge. The problems of vector spin 0 mixing are avoided by imposing the Lorentz condition supplementing the Wess-Zumino gauge.

3. SQED AND AFTM

3.1 Preliminaries

Supersymmetric QED (SQED) provides a simple but non-trivial model for which we can display the necessary procedures in evaluating V for a SUSY gauge theory by AFTM.

SQED may be defined [6] as a SUSY $U(1)$ gauge theory with one vector (V) and two chiral (S & R) superfields. The action may be written as

$$S = \int dz \bar{S} e^{gV} S + \bar{R} e^{-gV} R + \left\{ \int d\sigma \frac{1}{4} WW + mSR \right\} + H.C. \quad (3.1)$$

where $dz = dx d\theta^2 d\bar{\theta}^2$, $d\sigma = dx d\theta^2$ and the conventions of Wess and Bagger [6] are followed (in particular $W^\alpha = -\frac{1}{4} \bar{D}^2 D^\alpha V$).

A component field expansion for Eq. (3.1) may be found in Ref. [6]. However AFTM (up to the one-loop level) only requires a knowledge of the kinetic terms for spin 0 fields (here A_S and A_R) and also all D and F (here D , F_S , and F_R) auxiliary field terms in $\mathcal{L}$ *). Spin 0 kinetic terms are standard while the D and F terms which appear in $\mathcal{L}$ can be obtained from the D and F equations of motion formulae. For example the superpotential $P(S,R)$ is just

$$P(S,R) = mSR \quad (3.2)$$

hence

$$\bar{F}_{S(R)} = - \frac{\partial P(A)}{\partial A_{S(R)}} = -m \frac{\partial (A_S A_R)}{\partial A_{S(R)}} = -m A_{R(S)} \quad (3.3)$$

*) This is a consequence of employing the Wess-Zumino gauge supplemented with a Lorentz gauge for the spin 1 components.

which actually originates in

$$\frac{\partial \mathcal{L}}{\partial F_{S(R)}} = 0 \quad (3.4)$$

hence performing the integration and symmetrizing in $F \leftrightarrow \bar{F}$ we see $\mathcal{L}$ must contain

$$\{ \bar{F}_S F_S + m(A_R F_S + \bar{A}_R \bar{F}_S) \} + \{ S \leftrightarrow R \} \quad (3.5)$$

and no other F dependence. In writing down the 'well-known' [7] equations of motion for D auxiliary fields it is well to keep in mind how we have already defined the gauge coupling [by Eq. (3.1)] hence in this case,

$$D = -\frac{g}{2} (\bar{A}_S A_S - \bar{A}_R A_R) \quad (3.6)$$

the origin of which is in $\partial \mathcal{L} / \partial D = 0$, thus all D auxiliary terms in $\mathcal{L}$ are given by,

$$\frac{1}{2} D^2 + \frac{g}{2} D (\bar{A}_S A_S - \bar{A}_R A_R) \quad (3.7)$$

The terms of importance are thus summarized by

$$\begin{aligned} \tilde{\mathcal{L}} = & \bar{A}_S \square A_S + \bar{A}_R \square A_R \\ & + \{ [\bar{F}_S F_S + m(A_R F_S + \bar{A}_R \bar{F}_S)] + [R \leftrightarrow S] \} \\ & + \{ \frac{1}{2} D^2 + \frac{g}{2} D (\bar{A}_S A_S - \bar{A}_R A_R) \} \end{aligned} \quad (3.8)$$

(an η metric signature $(-1, 1, 1, 1)$ is used throughout this paper).

3.2 Implementation of AFTM at the tree level

This is a trivial calculation for which we already know the answer (Wess-Zumino gauge)

$$V_0 = \frac{1}{2} D_{cl}^2 + \bar{F}_{S,cl} F_{S,cl} + \bar{F}_{R,cl} F_{R,cl} \quad (3.9)$$

For this very reason it is useful for illustrating the methods involved. First we construct the translated theory. From Eq. (3.8) we obtain*)

*) The only terms of interest are $\tilde{\mathcal{L}}$ itself, all newly generated contributions to D and F tadpoles and all newly generated D , F , and A quadratic terms. Because we impose the Lorentz gauge AV quadratic terms ($\sim A \partial_\mu V^\mu$) do not appear, hence we avoid both AV mixing and the necessity to include the V terms of $\mathcal{L}$.

$$\begin{aligned}
\tilde{\mathcal{L}} \rightarrow \tilde{\mathcal{L}} + \{ [F_S(\bar{f}_S + m a_R) + \bar{F}_S(f_S + m \bar{a}_R)] + [S \leftrightarrow R] \} \\
+ D \{ d + \frac{g}{2} (\bar{a}_S a_S - \bar{a}_R a_R) \} \\
+ \frac{g}{2} (a_S D \bar{A}_S + \bar{a}_S D A_S + d \bar{A}_S A_S) \\
- \frac{g}{2} (a_R D \bar{A}_R + \bar{a}_R D A_R + d \bar{A}_R A_R) \\
+ \text{terms of no interest.}
\end{aligned} \tag{3.10}$$

Since the tree level effective action is just $\Gamma_0 = S(\phi_{cl})$ (as illustrated in Fig. 1) then using Eqs. (2.1), (2.2) and (3.10) we conclude

$$\frac{\partial V_0}{\partial f_S} = -(\bar{f}_S + m a_R) \tag{3.11}$$

Recalling that we have *not* constrained the theory so that f , $\bar{f}$, d , a and $\bar{a}$ are all independent parameters, Eq. (3.11) integrates to give,

$$\begin{aligned}
V_0(f_{S(R)}, \bar{f}_{S(R)}, d, a_{S(R)}, \bar{a}_{S(R)}) = -(\bar{f}_S f_S + m a_R f_S) \\
+ K(f_R, \bar{f}_{S(R)}, d, a_{S(R)}, \bar{a}_{S(R)})
\end{aligned} \tag{3.12}$$

The $f_R, \bar{f}_{S(R)}$ dependence of the integration constant is determined by considering $\partial V_0 / \partial f_R$ and $\partial V_0 / \partial \bar{f}_{S(R)}$ integrations and using the standard trick associated with partial differential equations. However this simply reduces to the rule that we must have $S \leftrightarrow R$ and $f \leftrightarrow \bar{f}$ symmetry, hence Eq. (3.12) implies

$$\begin{aligned}
V_0 = - \{ (\bar{f}_S f_S + m a_R f_S + m \bar{a}_R \bar{f}_S) + (S \leftrightarrow R) \} \\
+ K'(d, a_{S(R)}, \bar{a}_{S(R)})
\end{aligned} \tag{3.13}$$

Clearly the d dependence of V_0 will be obtained by considering $\partial V_0 / \partial d$.

Equations (2.1), (2.2) and (3.10) imply

$$\frac{\partial V_0}{\partial d} = - \left(d + \frac{g}{2} (\bar{a}_S a_S - \bar{a}_R a_R) \right) \tag{3.14}$$

integration followed by the standard trick mentioned above for determining the integration constant gives

$$\begin{aligned}
 V_0 = & -\{ (\bar{f}_s f_s + m a_R f_s + m \bar{a}_R \bar{f}_s) + (s \leftrightarrow R) \} \\
 & - \{ \frac{1}{2} d^2 + \frac{g}{2} d (\bar{a}_s a_s - \bar{a}_R a_R) \} \\
 & + K'' (a_{s(R)}, \bar{a}_{s(R)})
 \end{aligned} \tag{3.15}$$

The important observation which really makes AFTM so useful is that we *know* the effective potential satisfies (at any order in perturbation theory utilising the Wess-Zumino gauge)

$$V(f = \bar{f} = d = 0) = 0 \tag{3.16}$$

equally valid for each order in perturbation theory (which is just another way of stating the fact that if unbroken at the tree level SUSY will not be broken at any higher order in perturbation theory). As a result $K'' = 0$ (so we see that consideration of f and d tadpoles *will always be sufficient*). With this Eq. (3.15) is the correct form for the unconstrained tree level effective potential of SQED, i.e. applying the equations of motion in Eqs. (3.5) and (3.6), Eq. (3.15) reduces to,

$$\begin{aligned}
 V_0 = & - \{ (\bar{f}_s f_s - \bar{f}_s f_s - f_s \bar{f}_s) + (s \leftrightarrow R) \} \\
 & - \{ \frac{1}{2} d^2 - d^2 \} \\
 = & \bar{f}_s f_s + \bar{f}_R f_R + \frac{1}{2} d^2
 \end{aligned} \tag{3.17}$$

which is the familiar form given in Eq. (3.9). I reiterate that the intention here was simply to illustrate the method and various tricks which may be employed (particularly in connection with the determination of integration constants). It is not until we go to the one-loop level that we obtain non-trivial results.

3.3 Implementation at the one-loop level

The first question to answer is, what diagrams contributing to the one-loop auxiliary field tadpoles of the translated theory? Before we can answer this we

must know how D and F auxiliary fields couple to the other component fields. This information is contained in Eq. (3.8) [Eq. (3.10) contains no new D or F interaction terms]. At a glance we see that F auxiliary fields have no coupling. Immediately we can conclude that there are *no* one-loop (or higher!) contributions to the F tadpoles hence [using Eq. (2.2)]

$$\frac{\partial V_1}{\partial \bar{f}_{S(R)}} = \frac{\partial V_1}{\partial f_{S(R)}} = 0 \quad (3.18)$$

(V_1 is the one-loop contribution to V) concluding that

$$V_1 \equiv V_1(a_{S(R)}, \bar{a}_{S(R)}, d) \quad (3.19)$$

Equation (3.8) does reveal $D\bar{A}\bar{A}$ vertices in the theory. Consequently we immediately deduce the one-loop contributions to the D tadpole, as illustrated in Fig. 2 along with the necessary vertex factors. (Note that I adopt the procedure of Grisaru et al. [8] where one formulates Feynman rules for the euclidean effective action. One will then proceed to Eq. (2.2) via Eq. (2.1). The vertex factors in Fig. 2 are in accordance with this procedure.)

Before one can write down the corresponding mathematical expression for Γ_1 (see Fig. 2) we must know at least the $\bar{A}_S A_S$ and $\bar{A}_R A_R$ propagators of the theory.

With the immediate aim of calculating these we extract from Eq. (3.10) the quadratic part of the spin 0 auxiliary field action^{*)},

$$\begin{aligned} S_0 = \int dx \left(\frac{1}{2} D^2 + \bar{A}_S \square A_S + \bar{A}_R \square A_R \right. \\ \left. + \frac{g}{2} [a_S D \bar{A}_S + \bar{a}_S D A_S + d \bar{A}_S A_S] \right. \\ \left. - \frac{g}{2} [S \leftrightarrow R] \right) + \text{Source terms} \end{aligned} \quad (3.20)$$

*) At this stage I restrict attention to massless SQED as it is simpler to handle. As a result the F 'propagators' decouple from the spin 0 propagators. Because of the Lorentz gauge the same happens for the vector fields V^m , this explains why F and V^m terms are not needed in Eq. (3.20).

which may be written as

$$S_0 = \int dx \frac{1}{2} \bar{\Psi} A \Psi + \bar{\Psi} B \quad (3.21)$$

with

$$\bar{\Psi} = (\bar{A}_S \ A_S \ \bar{A}_R \ A_R \ 0) \quad (3.22)$$

and B a vector of sources

$$\bar{B} = (\bar{J}_S \ J_S \ \bar{J}_R \ J_R \ 1) \quad (3.23)$$

A is hermitian and given by

$$A = \begin{pmatrix} \square + \frac{d}{2} g & 0 & 0 & 0 & \frac{g a_S}{2} \\ 0 & \square + \frac{d}{2} g & 0 & 0 & \frac{g \bar{a}_S}{2} \\ 0 & 0 & \square - \frac{d}{2} g & 0 & -\frac{g a_R}{2} \\ 0 & 0 & 0 & \square - \frac{d}{2} g & -\frac{g \bar{a}_R}{2} \\ \frac{g \bar{a}_S}{2} & \frac{g a_S}{2} & -\frac{g \bar{a}_R}{2} & -\frac{g a_R}{2} & 1 \end{pmatrix} \quad (3.24)$$

It follows that the generating functional Z_0 is given by

$$\ln Z_0 = -\frac{1}{2} \int \bar{B} A^{-1} B \, dx \quad (3.25)$$

and by looking at $\delta^2 \ln Z_0 / \delta J_{S(R)} \delta \bar{J}_{S(R)}$ one can readily extract the appropriate propagators. It is useful to illustrate this step to allay fears that calculating A^{-1} restricts the usefulness of AFTM to relatively simple models. A^{-1} can be constructed using the *method of minors*. It follows

$$A^{-1} = \frac{1}{\Delta} \begin{pmatrix} m_{11} & \cdots & m_{15} \\ \vdots & & \vdots \\ m_{51} & \cdots & m_{55} \end{pmatrix} \quad (3.26)$$

where $m_{ij} = (-)^{i+j} n_{ji}$ *) is defined as the i - j th minor of A . Δ is just $\text{Det } A$. With Eq. (3.26) in Eq. (3.25) we obtain the formal $\bar{A}_S A_S$ and $\bar{A}_R A_R$ propagators illustrated in Fig. 3 (note $m_{11} = m_{22}$ and $m_{33} = m_{44}$). Figure 2 may now be evaluated.

$$\Gamma_i^{(0)} = \int d\tilde{k} (2\pi)^4 \delta(k) \int d\tilde{p} \left\{ \left(\frac{g}{2}\right) \cdot 1 \cdot \left(-\frac{m_{11}}{\Delta}\right) + \left(-\frac{g}{2}\right) \cdot 1 \cdot \left(-\frac{m_{33}}{\Delta}\right) \right\} D(k) \quad (3.27)$$

(the 'ones' denote loop factors). Using Eqs. (2.1) and (2.2) we conclude

$$\frac{\partial V_i}{\partial d} = \frac{g}{2} \int d\tilde{p} \left(\frac{m_{11} - m_{33}}{\Delta} \right) \quad (3.28)$$

A surprising simplification now results. Using the product rule for determinant differentiation applied to $\det A$ [see Eq. (3.24)] it is clear that

$$\begin{aligned} \frac{\partial \Delta}{\partial d} &= \frac{\partial \text{Det } A}{\partial d} = \frac{g}{2} (m_{11} + m_{22} - m_{33} - m_{44} + 0) \\ &= g (m_{11} - m_{33}) \end{aligned} \quad (3.29)$$

In Eq. (3.28) we obtain

$$\frac{\partial V_i}{\partial d} = \frac{1}{2} \int d\tilde{p} \frac{\partial \text{Ln } \Delta}{\partial d} \quad (3.30)$$

This may be integrated to give

$$V_i = \frac{1}{2} \int d\tilde{p} \text{Ln } \Delta(d) + K(a_{s(R)}, \bar{a}_{s(R)}) \quad (3.31)$$

where the constant of integration has no f dependence [see Eq. (3.19)]. Finally we apply our boundary condition $V_i(d=f=0) = 0$; hence we obtain

$$V_i = \frac{1}{2} \int d\tilde{p} [\text{Ln } \Delta(d) - \text{Ln } \Delta(0)] \quad (3.32)$$

To see that this is correct and to observe certain advantages of AFTM it is useful to compare with the result obtained via the Coleman-Weinberg (CW) method [4].

*) n_{ij} is the det of the matrix resulting from the deletion of row i and column j in matrix A .

3.4 Comparison with the CW result

The usually quoted CW result [7] for the finite part of the effective potential

$$V_1 = \frac{1}{64\pi^2} \text{Str } M^4 \text{Ln } M^2 \quad (3.33)$$

is of course specific to a special renormalization scheme, however one can trace its unique origin to the unregularized result^{*)},

$$V_1 = \frac{1}{2} \int d\tilde{p} \text{Str } \text{Ln} \left(1 + \frac{M^2}{p^2} \right) \quad (3.34)$$

The mass matrices of the general SUSY gauge theory have been calculated by Barbieri et al. [4] (see also Ferrara et al. [9]). For massless SQED they reduce to the following results

$$M_o^2 = \frac{g^2}{4} \begin{pmatrix} \bar{a}_s a_s - \frac{2d}{g} & a_s a_s & -a_s \bar{a}_R & -a_s a_R \\ \bar{a}_s \bar{a}_s & \bar{a}_s a_s - \frac{2d}{g} & -\bar{a}_s \bar{a}_R & -\bar{a}_s a_R \\ -a_R \bar{a}_s & -a_s a_R & \bar{a}_R a_R + \frac{2d}{g} & a_R a_R \\ -\bar{a}_s \bar{a}_R & -a_s \bar{a}_R & \bar{a}_R \bar{a}_R & \bar{a}_R a_R + \frac{2d}{g} \end{pmatrix} \quad (3.35)$$

while both $M_{1/2}^2$ and M_1^2 are d independent.

We thus see that Eq. (3.34) may be written as

$$\frac{\partial V_1}{\partial d} = \frac{1}{2} \int d\tilde{p} \frac{\partial}{\partial d} \text{Tr } \text{Ln} (-p^2 - M_o^2) \quad (3.36)$$

Let us define

$$\tilde{\Delta} = \text{Det} (-p^2 - M_o^2) \quad (3.37)$$

*) $\text{Str } x \equiv \sum_J (-)^{2J} \text{Tr } x_J$, m_J thus denotes the appropriate spin J mass matrix.

Then Eq. (3.36) becomes

$$\frac{\partial V_1}{\partial d} = \frac{1}{2} \int d\tilde{p} \frac{\partial}{\partial d} L_n \tilde{\Delta} \quad (3.38)$$

This equation has a striking similarity to Eq. (3.30) despite the fact that $\tilde{\Delta}$ is the determinant of a 4×4 matrix compared to Δ , the determinant of a 5×5 matrix. In fact we may observe that

$$\frac{\partial L_n \Delta}{\partial d} = \frac{\partial L_n \text{Det } A}{\partial d} = \frac{\partial L_n \text{Det}(-p^2 - M_{0A}^2)}{\partial d} \quad (3.39)$$

where m_{0A}^2 is the spin 0 auxiliary field mass matrix. So we see that AFTM and CW give almost identical results in appearance, the difference being that AFTM uses unconstrained SUSY, hence has a larger (*but structurally simpler*) mass matrix than the pure spin 0 (constrained) mass matrix of CW.

Finally to show that the answers are identically the same it remains to evaluate Δ and $\tilde{\Delta}$ (Δ is a lot simpler to evaluate than $\tilde{\Delta}$ despite its extra dimension), the result is

$$\Delta = \tilde{\Delta} = (p^4 - \rho^2)((p^4 - \rho^2) + 2(p^2 - \rho)\frac{\rho^2}{4}\bar{a}_R a_R + 2(p^2 + \rho)\frac{\rho^2}{4}\bar{a}_S a_S) \quad (3.40)$$

where $\rho = gd/2$.

Before closing this section a number of observations are worthwhile. The most significant of these is that AFTM shows how the underlying SUSY can be exploited so that V_1 is determined by the spin 0 mass matrix alone. By this I mean that working from the CW result in Eq. (3.34) we can construct $\partial V_1 / \partial d$ as in Eq. (3.36) where the contributions from the spin $1/2$ and spin 1 mass matrices disappear. When integrated Eq. (3.36) uniquely determines V_1 upon applying our boundary conditions (learnt from AFTM), namely Eqs. (3.19) and (3.16). In other words the boundary conditions automatically generate the spin $1/2$ and spin 1 mass matrix contributions from the spin 0 contribution. This feature is typical of SUSY and is lost if one simply uses the CW results in Eqs. (3.33) and (3.34) which are valid for a general gauge theory, SUSY or not.

The next observation is that it is not surprising to find exact agreement with CW for V_1 of SQED. The CW result is specific to an assumed Wess-Zumino gauge supplemented with covariant Landau gauge, while the result here is specific to a Wess-Zumino gauge supplemented with the Lorentz condition; they are identical gauge choices. (It is well known the effective potential is a gauge dependent object [10] which can become apparent even at the tree level [10-12]).

Finally a minor observation is that although AFTM leads to larger dimensional mass matrices (due to the inclusion of auxiliary fields) compared to CW spin 0 mass matrices, it happens that the larger dimensional matrices have a much simpler structure than the smaller dimensional ones. This leads to significant simplification in the evaluation of determinants even in a relatively simple model like SQED.

How does AFTM perform for more complex SUSY gauge theories where mass matrices can have arbitrary dimensions? We study SQCD to answer this question.

4. AFTM AND SQCD

In this section AFTM is applied to SQCD, essentially the non-Abelian analog of SQED considered above.

The superfield formulation of this theory is given by

$$\begin{aligned} S = \int d^4z [& \bar{S} e^{gV} S + \bar{R} e^{-g\tilde{V}} R] \\ & + \{ (\int d^4x \frac{1}{4k} \text{Tr} (W^\alpha W_\alpha) + m R^T S) + \text{H.c.} \} \end{aligned} \quad (4.1)$$

where Wess-Bagger [6] conventions are adopted. Superfix, T, denotes a vector transposition and, $\sim$, a matrix transposition (the distinction is needed later).

Further

$$\begin{aligned} V &= V^a T^a \\ W_\alpha &= W_\alpha^a T^a \end{aligned} \quad (4.2)$$

T^a defines the fundamental representation of the gauge group and they are normalized so that $\text{Tr}(T^a T^b) = k\delta^{ab}$.

Once more gauge fixing will proceed via the (non-SUSY) choice of Wess-Zumino gauge supplemented with the Lorentz condition on the vector component of the vector superfield ($\partial^m v_m^a = 0$). The associated ghost sector does not affect our results at the one-loop level and need not be specified.

The initial requirement is to extract the relevant component field terms. This is done as described in subsection 3.1. The superpotential is just

$$P(R, S) = m R_i S_i \quad (4.3)$$

This gives equations of motion for the F auxiliary fields, equally derivable from $\partial \mathcal{L} / \partial F = 0$, hence we learn that $\mathcal{L}$ contains

$$[\bar{F}_R^i F_R^i + m (\bar{A}_S^i \bar{F}_R^i + A_S^i F_R^i)] + [S \leftrightarrow R] \quad (4.4)$$

where i is the colour index [as in Eq. (4.3)]. The D^a equations of motion [in Wess-Zumino gauge and consistent with my definition of the gauge coupling in Eq. (4.1)] are given by

$$D^a + \frac{g}{2} (\bar{A}_S T^a A_S - \bar{A}_R \tilde{T}^a A_R) = 0 \quad (4.5)$$

which gives upon integration the contribution to $\mathcal{L}$

$$\frac{1}{2} D^a D^a + \frac{g}{2} D^a (\bar{A}_S T^a A_S - \bar{A}_R \tilde{T}^a A_R) \quad (4.6)$$

Finally one has standard spin 0 kinetic terms,

$$\bar{A}_S^i \square A_S^i + \bar{A}_R^i \square A_R^i \quad (4.7)$$

Thus we define

$$\begin{aligned} \tilde{\mathcal{L}} = & [\bar{F}_R^i F_R^i + m (A_S^i F_R^i + \bar{A}_S^i \bar{F}_R^i)] + [S \leftrightarrow R] \\ & + [\frac{1}{2} D^a D^a + \frac{g}{2} D^a (\bar{A}_S T^a A_S - \bar{A}_R \tilde{T}^a A_R)] \\ & + \bar{A}_S^i \square A_S^i + \bar{A}_R^i \square A_R^i \end{aligned} \quad (4.8)$$

Under translation ($F \rightarrow F + f$, $D \rightarrow D + d$, $A \rightarrow A + a$) we find

$$\begin{aligned}
 \tilde{\mathcal{L}} \rightarrow \tilde{\mathcal{L}} &+ \{ [\bar{F}_R^i (f_R^i + m \bar{a}_S^i) + \text{H.c.}] + [S \leftrightarrow R] \} \\
 &+ \{ D^a [d^a + \frac{g}{2} (\bar{a}_S T^a a_S - \bar{a}_R \tilde{T}^a a_R)] \} \\
 &+ \frac{g}{2} \{ D^a (\bar{a}_S T^a A_S + \bar{A}_S T^a a_S) + d^a \bar{A}_S T^a A_S \\
 &\quad - D^a (\bar{a}_R \tilde{T}^a A_R + \bar{A}_R \tilde{T}^a a_R) - d^a \bar{A}_R \tilde{T}^a A_R \} \\
 &+ \text{Terms of no interest.}
 \end{aligned} \tag{4.9}$$

From the central Eq. (2.2) we find

$$\frac{\partial V_0}{\partial \bar{f}_R^i} = - (f_R^i + m \bar{a}_S^i) \tag{4.10}$$

which may be integrated with $f \leftrightarrow \bar{f}$ and $S \leftrightarrow R$ symmetrization performed. Also we have

$$\frac{\partial V_0}{\partial d^a} = - (d^a + \frac{g}{2} [\bar{a}_S T^a a_S - \bar{a}_R \tilde{T}^a a_R]) \tag{4.11}$$

which may be integrated to extract the d^a -dependence of the integration constant of Eq. (4.10).

Finally one applies $V_0(f = d = 0) = 0$ to obtain

$$\begin{aligned}
 V_0 = & - \{ [\bar{f}_R^i f_R^i + m (\bar{f}_R^i \bar{a}_S^i + f_R^i a_S^i)] + [S \leftrightarrow R] \} \\
 & - \{ \frac{1}{2} d^a d^a + \frac{g}{2} d^a (\bar{a}_S T^a a_S - \bar{a}_R \tilde{T}^a a_R) \}
 \end{aligned} \tag{4.12}$$

which is the correct unconstrained tree level effective potential as is seen by application of f and d equations of motion,

$$V_0 = \bar{f}_R^i f_R^i + \bar{f}_S^i f_S^i + \frac{1}{2} d^a d^a \tag{4.13}$$

We now proceed to the one-loop level. Equations (4.8) and (4.9) show us that F 's do not couple to other component fields (just as in SQED. The origin of this feature is of course the absence of chiral self-interactions in these two models.)

We conclude therefore that there are no one-loop contributions to f tadpoles (see Fig. 4), hence by the central Eq. (2.2)

$$\frac{\partial \bar{V}_i}{\partial \bar{f}_{S(R)}^i} = \frac{\partial V_i}{\partial f_{S(R)}^i} = 0 \quad (4.14)$$

or

$$V_i \equiv V_i (d^a, \bar{a}_{S(R)}^i, a_{S(R)}^i) \quad (4.15)$$

Equations (4.8) and (4.9) do however reveal $\bar{D}\bar{A}A$ couplings in the theory. Hence we do obtain contributions to the one-loop D -tadpoles, these are shown in Fig. 4 along with the $\bar{D}\bar{A}A$ vertex factors.

The calculation proceeds in a very similar way to that of SQED discussed in Section 3.3. We define vectors $\underline{A}_S$ simply denoted by A_S itself, so $(A_S)^i = A_S^i$, $(D)^a = D^a$, etc. We define vectors ψ and $\mathbb{B}$ by

$$\psi = \begin{pmatrix} A_S \\ \bar{A}_S^T \\ A_R \\ \bar{A}_R^T \\ D \end{pmatrix} \quad \mathbb{B} = \begin{pmatrix} \bar{J}_S^T \\ J_S \\ \bar{J}_R^T \\ J_R \\ K \end{pmatrix} \quad (4.16)$$

Restricting attention to massless SQCD for simplicity (where F auxiliary field propagators do not mix with those of spin 0 or D component fields) we may write the quadratic part of the action in the form of Eq. (3.21), where this time ψ and $\mathbb{B}$ are defined in Eq. (4.16) and matrix $\mathbb{A}$ is given in 'block' form by

$$\mathbb{A} = \begin{pmatrix} \square + \frac{g}{2}(d^T T) & 0 & 0 & 0 & \frac{g}{2}(T^T a_S) \\ 0 & \square + \frac{g}{2}(d^T \tilde{T}) & 0 & 0 & \frac{g}{2}(\tilde{T}^T \bar{a}_S^T) \\ 0 & 0 & \square - \frac{g}{2}(d^T \tilde{T}) & 0 & -\frac{g}{2}(\tilde{T}^T a_R) \\ 0 & 0 & 0 & \square - \frac{g}{2}(d^T T) & -\frac{g}{2}(T^T \bar{a}_R^T) \\ \frac{g}{2}(\bar{a}_S^T T) & \frac{g}{2}(a_S^T \tilde{T}) & -\frac{g}{2}(\bar{a}_R^T \tilde{T}) & -\frac{g}{2}(a_R^T T) & 1 \end{pmatrix} \quad (4.17)$$

Just as D^a defines the components of D then T^a defines the components of T . For T we have two distinct transpositions, T^T is the vector

$$T^T = (T^1 \ T^2 \ \dots \dots \dots) \quad (4.18)$$

while $\tilde{T}$ will denote T with each matrix entry transposed, hence

$$\tilde{T}^T = (\tilde{T}^1 \ \tilde{T}^2 \ \dots \dots \dots) \quad (4.19)$$

For example $D^a T^a \equiv D^T T \equiv T^T D$.

A can formally be inverted by the method of minors ($\Delta = \text{Det } A$)

$$A^{-1} = \frac{1}{\Delta} \begin{pmatrix} \tilde{m}^{s\bar{s}} & \dots & \dots & \dots \\ \vdots & m^{s\bar{s}} & & \\ & & \tilde{m}^{r\bar{r}} & \\ & & & m^{r\bar{r}} \\ & & & & \ddots \end{pmatrix} \quad (4.20)$$

which is also written in block form and only the relevant minors on the diagonals are displayed. With Eq. (4.20) in Eq. (3.25) the propagators (needed for the evaluation of Fig. 4), are displayed in Fig. 5. Figure 4 is then straightforward to evaluate

$$\Gamma_1^{(0)} = \int d\tilde{k} (2\pi)^4 \delta(k) \left[-\frac{g}{2} \int d\tilde{p} \frac{1}{\Delta} (T_{ij}^a m_{ij}^{s\bar{s}} - \tilde{T}_{ij}^a m_{ij}^{r\bar{r}}) \right] D^a(k) \quad (4.21)$$

hence from the central equation of Eq. (2.2) we deduce

$$\frac{\partial V_1}{\partial d^a} = \frac{g}{2} \int d\tilde{p} \frac{1}{\Delta} \text{Tr} (\tilde{T}^a m^{s\bar{s}} - T^a m^{r\bar{r}}) \quad (4.22)$$

Once more the structure of this may be simplified by the following theorem. If A is a matrix with elements linear in a variable d , so we can write

$$A = C + Ld \quad (4.23)$$

(C and L matrices) then

$$\frac{d \text{Det } A}{d d} = \text{Tr} (L M) \quad (4.24)$$

where M is the matrix of minors belonging to A . As a result it is easy to show

$$\frac{\partial \Delta}{\partial d^a} = g \text{Tr} (\tilde{T}^a m^{s\bar{s}} - T^a m^{a\bar{a}}) \quad (4.25)$$

In Eq. (4.22) we obtain

$$\frac{\partial V_1}{\partial d^a} = \frac{1}{2} \int d\tilde{p} \frac{1}{\Delta} \frac{\partial \Delta}{\partial d^a} \quad (4.26)$$

which upon integration, using the boundary conditions $V_1(f = d = 0) = 0$ and

Eq. (4.15) gives

$$V_1 = \frac{1}{2} \int d\tilde{p} [L_n \Delta(d) - L_n \Delta(0)] \quad (4.27)$$

exactly the analog of the SQED result Eq. (3.32) where, just as in Eq. (3.39)

$$\Delta = \text{Det } A = \text{Det} (-p^2 - M_{0A}^2) \quad (4.28)$$

the difference being that here the spin 0 auxiliary field mass matrix is defined in Eq. (4.17).

5. CONCLUSIONS

In this paper AFTM has been extended to SUSY gauge theories. It is sufficiently developed that one can now realistically consider its application to complex SUSY GUTs. Furthermore the SUSY effective potential beyond the one-loop level is now open to investigation with this approach which could provide interesting results.

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Figure captions

- Fig. 1 : The tree level effective action of translated SQED determining the tree level effective potential. F_i , $\bar{F}_i$ and D are external (classical) auxiliary fields and k is the external momentum.
- Fig. 2 : The one-loop contribution to the effective action of SQED. No contributions come from F tadpoles.
- Fig. 3 : Propagator Feynman rules for $\bar{A}_S A_S$ and $\bar{A}_R A_R$ propagators in SQED.
- Fig. 4 : The one-loop contribution to the effective action of SQCD. The $D\bar{A}A$ vertex factors are indicated. F tadpoles are absent as F 's do not couple to A 's in models without chiral self-interactions.
- Fig. 5 : Propagator Feynman rules for $\bar{A}_S^i A_S^j$ and $\bar{A}_R^i A_R^j$ propagators in SQCD.

$$\Gamma_0 = \sum_{i=s,r} \left(\frac{F_i}{k} \times + \frac{\bar{F}_i}{k} \times \right) + \frac{D}{k} \times$$

+ (other terms of no interest)

Fig. 1

$$\Gamma_1 = \frac{D}{k} \frac{g}{2} \text{ (circle with } p \text{)}^{\bar{A}_s A_s} + \frac{D}{k} \frac{-g}{2} \text{ (circle with } p \text{)}^{\bar{A}_r A_r}$$

+ (other terms of no interest)

Fig. 2

$$\frac{\bar{A}_s A_s}{p} = \frac{-M_{11}}{\Delta}$$

$$\frac{\bar{A}_r A_r}{p} = \frac{-M_{33}}{\Delta}$$

Fig. 3

$$\Gamma_i = \frac{D^a}{k} \frac{g}{2} T_{ij}^a + \frac{D^a}{k} \frac{-g}{2} T_{ji}^a + (\text{other terms of no interest})$$

Fig. 4

Fig. 5