Ref.TH.3591-CERN

STRING TENSION IN SU(3) GAUGE THEORY
ON A 16^4 LATTICE

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ABSTRACT

Wilson loop expectation values are measured with improved statistics at $\beta = 5.40$ and 5.70 by Monte Carlo method. In this coupling constant region the (dimensionless) string tension drops more rapidly than required by the one-loop renormalization formula.

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In the glue sector of non-perturbative QCD the string tension is the only quantity for which definite experimental information is available. Therefore - apart from its role as an order parameter for confinement - the tension determined on the lattice gives the lattice distance in physical units setting the scale for other quantities in the gluon sector. In the quenched approximation this is the scale of hadron spectrum calculations also.

The string tension in SU(3) was first measured by Creutz and Pietarinen, followed by further, somewhat improved Monte Carlo calculations¹⁾. Nevertheless, the situation is less satisfactory here than in the case of SU(2)²⁾. Additionally, recent results on the meson spectrum on large lattices³⁾ seem to prefer lattice distance values which are different from that predicted by the existing tension calculations.

By measuring the string tension from the ratio of Wilson loops of increasing size, the final result is probably an upper bound for the string tension (apart from statistical errors, of course). The ratio

$$\chi(i) = -\ln \left[\frac{W(i,i) W(i-1,i-1)}{W(i-1,i) W(i,i-1)} \right] = \left[-\ln \frac{W(i,i)}{W(i-1,i)} \right] - \left[-\ln \frac{W(i,i-1)}{W(i-1,i-1)} \right] \quad (1)$$

can be interpreted approximately as the derivative of the static potential at a distance i : $V(i) - V(i-1)$. For all the phenomenological potentials ($\sim$ Coulomb like at short distances, $\sim$ linear for large separations) this is a decreasing function of i , converging towards the string tension.

The existing MC data¹⁾ show clearly this behaviour: at a fixed coupling $\chi(i)$ is decreasing for increasing i . The problem is that there is not much sign of convergence and it is questionable how well the tension is represented by $\chi(i_{\max})$ in this case.

In this paper, we report on the results obtained on a large, 16^4 lattice at $\beta = 5.40$ and 5.70 . The largest loop we measured is 3×3 and 4×4 at $\beta = 5.40$ and 5.70 , respectively. These Wilson loop expectation values are given in Table 1. The corresponding $\chi(i)$, $x(i) \equiv 10^3 \Lambda^{\text{latt}} / (\text{tension})^{1/2}$ and the lattice unit $a(i)$ in $1/\text{GeV}$ are summarized in Table 2.

The numbers contained in Table 1 were obtained from 55 and 45, 16^4 gauge field configurations at $\beta = 5.40$ and 5.70 , respectively. These configurations were separated by 10 full sweeps each using the Metropolis method with 10 hits per link.

The Wilson loop values in Table 1 are expected to be largely free of finite size effects. The 1×1 , 1×2 and 2×2 expectation values can be compared with the numbers obtained in a recent, high statistics measurement on a $5^3 \times 8$ lattice⁴⁾. At $\beta = 5.70$ even these small loops show a definite, systematic decrease as the volume of the lattice is increased.

Being unable to measure loops larger than 4×4 , our result on the tension is not very stringent. On the basis of Table 2 one obtains

$$\begin{aligned} \Lambda^{\text{latt}} &\geq (6.1 \pm 0.1) \cdot 10^{-3} (\text{tension})^{1/2}, \quad \beta = 5.40, \\ \Lambda^{\text{latt}} &\geq (6.9 \pm 0.5) \cdot 10^{-3} (\text{tension})^{1/2}, \quad \beta = 5.70, \end{aligned} \quad (2)$$

where Λ^{latt} is defined via the usual one-loop formula. In Eq. (2) the error refers to the statistical error, while the inequality sign indicates that by looking at the sequence of $X(1), X(2), \dots$, further decrease of the dimensionless tension is expected as the distance is increased.

Given the sequence $X(1), X(2), \dots$, an extrapolation can be attempted by assuming something about the form of the potential $V(i)$. Taking an (ad hoc) linear + Coulomb form the extrapolated value of $x(\infty)$ would be (6.25 ± 0.1) and (7.5 ± 0.2) at $\beta = 5.40$ and 5.70 , respectively.

Equation (2) suggests that $(a^2 \cdot \text{tension})$ drops more rapidly between $\beta = 5.40$ and 5.70 than it is required by the one-loop renormalization group formula. A similar behaviour was found by Celik et al.⁵⁾ in measuring the critical temperature of the deconfining phase transition in this coupling constant region. It is interesting to note that the ratio $T_c/(\text{tension})^{1/2}$ is consistent of being β independent.

The 16^4 gauge field configurations used in this analysis were created at CERN and at DESY for the purpose of hadron spectrum calculations, and were taken from tapes. Calculating the Wilson loop expectation values took ~ 45 hours CPU time on the CERN IBM.

We have learned of a recent analysis⁶⁾ on a $10^3 \times 20$ lattice, predicting an even larger value for x at $\beta = 6.0$, indicating that additional careful work is needed to settle the important question of the scaling behaviour and numerical value of the tension.

ACKNOWLEDGEMENTS

We thank R. Petronzio for discussions. Z.K. thanks the Theoretical Division of CERN for its kind hospitality.

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Table 1

Measured values of Wilson loops at $\beta = 5.4$ and $\beta = 5.7$.

Type	$\beta = 5.40$	$\beta = 5.70$
W(1,1)	1.4153 ± 0.0003	1.6471 ± 0.0008
W(1,2)	0.6947 ± 0.0003	0.9734 ± 0.0007
W(2,2)	0.1813 ± 0.0003	0.3958 ± 0.0007
W(2,3)	0.0490 ± 0.0002	0.1720 ± 0.0004
W(3,3)	0.0075 ± 0.0002	0.0580 ± 0.0003
W(3,4)	-	0.0206 ± 0.0002
W(4,4)	-	0.0058 ± 0.0002

Table 2

Derived values of $\chi(I)$, [see Eq. (1)], $x(i) = 10^3 \Lambda^{\text{latt}} / (\text{tension})^{1/2}$ and the lattice distance $a(i)$. The numbers $a(i)$ were calculated with $(\text{tension})^{1/2} = 0.45 \text{ GeV}$.

I \	1	2	3	4	β
$\chi(I)$	0.7512 ± 0.0002	0.632 ± 0.002	0.57 ± 0.02	-	5.4
$x(I)$	5.3072 ± 0.0008	5.788 ± 0.008	6.1 ± 0.1	-	5.4
$a(I)$	1.9261 ± 0.0003	1.766 ± 0.002	1.68 ± 0.03	-	5.4
$\chi(I)$	0.5996 ± 0.0005	0.374 ± 0.002	0.253 ± 0.006	0.22 ± 0.04	5.7
$x(I)$	4.245 ± 0.002	5.38 ± 0.02	6.53 ± 0.08	6.9 ± 0.5	5.7
$a(I)$	1.7208 ± 0.0007	1.359 ± 0.004	1.12 ± 0.01	1.05 ± 0.08	5.7