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High Energy and cosmic rays physics

Preprint N 185

CONFORMAL ANOMALY IN WEYL THEORY AND ANOMALY FREE
SUPERCONFORMAL THEORIES

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CM-P00066987

Moscow 1983 June

ABSTRACT

We present the correct expression for conformal anomaly in Weyl theory implying its perturbative non-renormalisability. As a by-product it is found that non-singular cosmologies exist in quantised Weyl theory. The absence of conformal anomalies in theories incorporating local conformal invariance is proposed as necessary requirement for their consistency. Several superconformal theories, including $N = 4$ $SU_2 \times U_1$ super Yang-Mills interacting with $N = 4$ conformal supergravity, are shown to satisfy this restriction, i.e. to be completely finite (at least at one loop).

1. Introduction

It is well known that that anomalies in gauged axial current lead to inconsistency of the theory (either nonunitarity or non-renormalizability, depending on a point of view) and thus must cancel in a reasonable theory. As for conformal anomalies (CA) they are generally considered as harmless. Though classical conformal invariance can survive at the quantum level only in a UV finite theory ^{1/} this does not lead to any pathology as long as this symmetry is rigid. An example where this is not the case is string theory, i.e. 2-dim locally conformal invariant (super) gravity interacting with matter multiplets [3,4]. To maintain unitarity (i.e. general covariance) one is either to provide cancellation of CA (restricting d to be 26, 10 or 2) or to introduce new anomalous quantum degrees of freedom (like "dilaton" in 2-dim gravity) what is possible without affecting renormalizability. Here we are going to discuss the problem of CA in 4-dim locally conformal invariant power counting renormalizable theories like the Weyl theory ($\mathcal{L} = (C_{\lambda\mu\nu\rho})^2 \sqrt{g}$) and its superformal extensions [7,8] ^{2/}. In contrast with 2-dim case the account of anomalous degrees of freedom does not make these theories consistent: appearing at the first loop they spoil two-loop conformal invariant renormalizability (while their neglect leads to breakdown of unitarity) ^{3/}. Thus the only option left is

1/ namely, in a class of $N=2$ super Yang-Mills (SYM) plus $N=2$ matter theories [1], including $N=4$ SYM [2]. Note, however, that these theories are not anomaly free in external metric.

2/ thus we will not consider CA in conformal invariant extensions of Einstein gravity (see e.g. [5]) or supergravity (cf. [6]) which are naively infinite at higher loops

3/ a sort of non-unitarity due to higher derivatives will not bother us in this paper

to impose cancellation of conformal anomalies as a new restrictive principle in construction of consistent locally conformal invariant 4-dim quantum field theories. This implies complete UV finiteness and, in supersymmetric case, the automatic absence of axial anomalies. We will find below that such finite GA-free theories can be built on the basis of conformal supergravities (CSG) and are phenomenologically acceptable (e.g. contain the Einstein term R in a low energy limit without need for its dynamical inducing, cf. [9, 11]). This appears to be principally possible due to negative sign of conformal gravitino contribution in gravitational infinities [12, 13], cancellation of "topological" infinities in $N=3,4$ CSG and also due to the existence of superconformal tensor calculus [14-18]. Having provided cancellation of GA at one loop by taking particular combination of supermultiplets there are good reasons to believe that it persists through higher loops in view of applicability of superfield power counting rules to superconformal theories (e.g. $N=2,3,4$ CSG appear to be finite at $\ell \gg 2$ loops [19]).

2. GENERAL REMARKS ON CONFORMAL ANOMALY IN BACKGROUND

METRIC SECTOR. The metric dependent part of 4-dim (one-loop) divergences and GA (see e.g. [20]) has the form

$$\Gamma_{\infty} = \frac{1}{(4\pi)^2(n-4)} \int d^4x \sqrt{g} \, b_1, \quad (1)$$

$$T_{\mu}^{\mu} = \frac{2}{\sqrt{g}} g_{\mu\nu} \frac{\delta \Gamma}{\delta g_{\mu\nu}} = \frac{1}{16\pi^2} b_1, \quad (2)$$

$$b_1 = \beta_1 R R^{*R} + \beta_2 W + \beta_3 R^2 + \beta_4 \partial^2 R$$

Here $W = R_{\mu\nu}^2 - \frac{1}{2} R^2 = \frac{1}{2} C_{\mu\nu\rho}^2 - \frac{1}{2} R R^{*R}$; $\beta_1 = -\frac{13}{180} N_1 + \frac{7}{720} N_2 + \frac{N_6}{180}$
 $\beta_2 = \frac{1}{3} N_1 + \frac{1}{20} N_2 + \frac{1}{60} N_6$, $\beta_3 = -\frac{1}{10} N_1 + \frac{1}{10} N_2 + \frac{1}{180} N_6$, $\beta_4 = 0$,
 for vectors, Majorana spinors and
 scalars ($\beta = \frac{1}{6}$). At higher loops $\beta_2 \neq 0$ in ^{an} interacting theory

(e.g. $\beta_2 \sim \lambda^5$ in ϕ^4 theory [21] and $\sim e^6$ in QED [22]). In view of flat space UV finiteness of N=4 SYM [2] it is very plausible that here the one-loop values $\beta_1 = \beta_2 = \beta_3 = 0$, $\beta_4 = \frac{1}{2}$ are in fact exact. The value of β_4 is known to be ambiguous in (2) (but not in (1)). If one defines the Weyl tensor in n-dim space then $\beta_4 = \frac{1}{2}\beta_2$ in (2) (cf. [20, 22]). Integrating (2) one can find the anomalous part of the effective action. Let $g_{\mu\nu} = e^{2\sigma} \tilde{g}_{\mu\nu}$, where $\tilde{g}$ represents a class of conformally equivalent metrics (e.g., $R(\tilde{g})=0$, so that $e^\sigma = 1 - (-\partial^2 + \frac{1}{6}R)^{-1} \frac{1}{6}R$, cf. [5]). Then

$$(4\pi)^2 \frac{\delta \Gamma}{\delta \sigma} = \left\{ b_4(\tilde{g}) + 4(\beta_2 - 2\beta_1) \tilde{\partial}_\mu [\tilde{\partial}_\nu \sigma \tilde{\partial}_\mu \tilde{\partial}_\nu \sigma - \tilde{\partial}_\mu \sigma \tilde{\partial}^2 \sigma - (\tilde{R}_{\mu\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{R}) \tilde{\partial}_\nu \sigma - \tilde{\partial}_\mu \sigma (\tilde{\partial}^2 \sigma)^2] - 6\beta_4 (\tilde{\partial}^2 \sigma - 2\tilde{\partial}^2 \sigma - 2\tilde{\partial}_\mu \sigma \tilde{\partial}_\mu \sigma) (\tilde{\partial}^2 \sigma + \tilde{\partial}_\mu \sigma \tilde{\partial}_\mu \sigma) \right\} \times \sqrt{\tilde{g}}, \quad \tilde{\partial} \equiv \partial(\tilde{g}),$$

and finally, $\Gamma = \Gamma_{anom}[\sigma, \tilde{g}] + \Gamma_{reg}[\tilde{g}]$,

$$\Gamma_{anom} = \frac{1}{16\pi^2} \int d^4x \sqrt{\tilde{g}} \left\{ (\beta_1 \tilde{R}^* \tilde{R}^* + \beta_2 \tilde{W}) \sigma + (\beta_2 - 2\beta_1) [2(\tilde{R}_{\mu\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{R}) \tilde{\partial}_\mu \sigma \tilde{\partial}_\nu \sigma + 2(\tilde{\partial} \sigma)^2 \tilde{\partial}^2 \sigma + (\tilde{\partial}_\mu \sigma \tilde{\partial}_\mu \sigma)^2] - 3\beta_4 (\tilde{\partial}^2 \sigma + \tilde{\partial}_\mu \sigma \tilde{\partial}_\mu \sigma - \frac{1}{6} \tilde{R})^2 \right\} \quad (3)$$

The first two brackets give non-local and the third is local ($\sim R^2$) parts [23] of anomaly.

3. Conformal anomaly in Weyl theory. The one-loop divergences in this theory were established in [25]: $\beta_1 = \frac{137}{60}$, $\beta_2 = \frac{199}{15}$ (β_2 and β_1 are in general gauge dependent; in

4/ Eq. (3) gives the full effective action if the metric is conformally flat. The corresponding analog of (3) for

$\tilde{g}_{\mu\nu} = \delta_{\mu\nu}$, $\sigma = \sigma(\eta)$ was already found in [24]. Note also that (3) gives the dependence on λ of $\Gamma[\lambda^2 g_{\mu\nu}]$.

background conformal gauge $\beta_3 = 0$ [25]). The corresponding CA is thus given by (2) (where we may put $\beta_4 = \frac{1}{3}\beta_2$)⁵⁾. In obvious analogy with 2-dim gravity case [3,4] (where $\int_{\text{anom}} = \frac{26}{24\pi} \int \sigma(-D)\sigma d^2x$), here conformal degree of freedom becomes propagating ($\sigma D^2 \sigma + \dots$) at one loop (and produces non-renormalisable R^2 infinity at two loops). Let us first discuss a flat background quantization: $g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta_{\mu\nu} \varphi$, $\bar{g}_{\mu\nu} \delta^{\mu\nu} = 0$. In view of conformal invariance of the Weyl action $I[g] = I[\bar{g}]$, $\bar{g}_{\mu\nu} = \bar{g}_{\mu\nu} + \delta_{\mu\nu} \varphi$, $\delta_{\mu\nu} = \bar{g}_{\mu\nu} / (1 + \varphi)$. Taking φ as a new quantum field and choosing φ -dependent gauge for general covariance we conclude that φ drops from the theory so that conformal Ward identity is automatically satisfied ($\delta_{\mu\nu} = 0$). However, general covariance Ward identity will in general be broken (an intuitive argument is that one cannot reconcile $\delta_{\mu\nu} = 0$ with the standard covariant transformation law for the "metric" $\bar{g}_{\mu\nu}$), implying a breakdown of unitarity. The same (regularization independent) conclusion is reached if we start with $\varphi = 0$ as a quantum conformal gauge. To maintain general covariance one has to note that $\bar{g} \rightarrow g$ is not an admissible change under covariant regularisation, i.e. one must properly account for anomalous dependence on φ . This can be done systematically in the background field method with the effective action given by (see e.g. [26])

$$e^{-\Gamma[g]} = \int d\varphi d\bar{h} \exp \left\{ -I[g+h] + i \int [g, g+h] + \right. \quad (4) \\ \left. + \frac{\delta \Gamma}{\delta \bar{g}_{\mu\nu}} \bar{h}_{\mu\nu} \right\}$$

5/ this corrects the expression for CA in Weyl theory proposed in [11]. Though "topological" term can be neglected in infinities, the corresponding β_1 -term in CA is non-trivial (cf. (2), (3)). Thus the absence of CA is equivalent to the absence of all including topological, infinities.

where $\tilde{h}_{\mu\nu} = \bar{h}_{\mu\nu} + \varphi g_{\mu\nu}$, $\bar{h}_{\mu\nu} = 0$ and $\hat{I}$ includes gauge fixing and ghost terms. In general, there are two possible sources of conformal non-invariance of $\hat{I}$: the gauge term $\hat{I}$ and regularization. Being interested in the latter anomaly, we shall assume the background conformal (and coordinate) invariance of $\hat{I}$ (under $\delta g_{\mu\nu} = \epsilon g_{\mu\nu}$, $\delta \tilde{h}_{\mu\nu} = \epsilon \tilde{h}_{\mu\nu}$). The corresponding quantum coordinate gauge can be constructed as in [25] (e.g. decomposing $g_{\mu\nu} = e^{2\sigma} \tilde{g}_{\mu\nu}$, $\tilde{R} = 0$, etc), while anticipating the anomaly we must not fix the quantum conformal gauge. Then, formally, $\Gamma[\lambda^2 g_{\mu\nu}] = \Gamma[g_{\mu\nu}]$, if it was not for the anomaly, introduced by $h_{\mu\nu} \rightarrow \lambda^2 h_{\mu\nu}$. Let us first integrate over $\tilde{h}_{\mu\nu}$ (to one-loop order) to find the effective lagrangian for φ

$$e^{-\Gamma} \rightarrow \int d\varphi e^{-S[\varphi]}, \quad S = \Gamma_1[(1+\varphi)g_{\mu\nu}] - \frac{1}{2} t \Delta^{-1} t - \frac{\delta I}{\delta g_{\mu\nu}} g_{\mu\nu} \varphi, \quad t^{\mu\nu} = \frac{\varphi}{1+\varphi} \frac{\delta I}{\delta g_{\mu\nu}}, \quad \Delta^{-1} = \left\{ \frac{\delta^2 I}{\delta g_{\mu\nu} \delta g_{\rho\sigma}} + g.f. \right\} (g_{\mu\nu} \rightarrow (1+\varphi)g_{\mu\nu}) \quad (5)$$

where $\Gamma_1[g_{\mu\nu}]$ coincides with one-loop effective action computed in background coordinate and $\varphi=0$ conformal gauges [25]. The second non-local term in (5) demonstrates that there is no quantum conformal degeneracy off shell ($\frac{\delta I}{\delta g} \neq 0$). The last term provides cancellation of linear in φ term in S as needed for consistency. To establish the explicit dependence on φ we are

to use eq. (3) with $1+\varphi = e^{2\sigma}$, $\tilde{g}_{\mu\nu} = g_{\mu\nu}$ and $\beta_1 = \frac{1}{2}\beta_2$

$$S = \frac{1}{(4\pi)^2} \int d^4x \sqrt{g} \left\{ -\beta_2 (\partial^2 \sigma)^2 + (\beta_2 - 2\beta_1) (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) \cdot \right. \quad (6)$$

$$\left. \partial_\mu \sigma \partial_\nu \sigma + \frac{1}{2} \beta_2 R (\partial \sigma)^2 + [\beta_1 R R^* + \beta_2 (W + \frac{1}{3} \partial^2 R)] \sigma^2 \right\} + \text{non-local and } O(\sigma^3) \text{ terms.}$$

The subsequent (one-loop) integration over σ produces additional divergences easily found employing the algorithm for 4-th order operator [25] (see (1)): $\Delta\beta_1 = \frac{1}{90} + a$, $\Delta\beta_2 = \frac{1}{24}(a-1)(2a+3)$, $\Delta\beta_3 = \frac{1}{288}(866a^2 + a - 27)$, $a = \beta_1/\beta_2 = 137/796$.

They represent the infinities of the subset of two-loop diagrams, containing one $\bar{h}$ and one $\mathcal{P}$ loop. The appearance of R^2 -infinity implies two-loop non-renormalizability of Weyl theory. This non-conformal-invariant infinity is hardly surprising in an interacting theory (cf. [21, 22]). The peculiarity of the present theory is that $\beta_3 \neq 0$ already at two loops and is independent of coupling constant.

Let us now stress perturbative non-renormalizability of the theory, $\mathcal{L} = C^2 + F_{\mu\nu}^2 + \bar{\psi}\hat{\partial}\psi$, recently proposed [11] as a basis for induced gravity approach [9] (see also [27]). Quantizing first only matter fields we find R^2 -infinity at 4-th loop. As was noted in [10], the coefficient $\mathcal{P}$, of the R^2 term in effective action may turn to be finite non-perturbatively if gauge theory is asymptotically free. This does not, however, save renormalizability, because R^2 -infinities, known to be present in $C^2 + R^2$ -theory [25], here cannot be absorbed in a finite (non-bare) $\mathcal{P}$ ^{6/}. Finally, it was made a suggestion [11] that non-perturbatively $\mathcal{P}$ may be finite in the Weyl theory itself (in view of its asymptotic freedom [25]). Though this may help to make sense of the Weyl theory, it should be understood that this proposal goes beyond the standard criterion of perturbative renormalizability (even Einstein theory may be non-perturbatively finite). At present it seems more constructive to demand cancellation of CA in a

^{6/} actually, the finite $-\frac{1}{12}\beta_4 R^2$ local anomalous term (cf. (3)) is present already in one-loop matter effective action, or in the leading $1/N$ approximation. Note also that $\mathcal{P}$ may turn to be strictly zero (e.g. in N=4 SYM). In this case perturbative non-renormalizability is completely due to that of pure Weyl theory.

hope to find a theory which is finite and conformal invariant in each loop order.

4. Quantum cosmology in Weyl theory. Here we are going to present a simple application of one-loop expression for CA in Weyl theory formally neglecting its higher-loop inconsistency (with a hope that some features of the discussion may survive in a more realistic context). Namely, let us suppose that quantum Weyl theory (with matter) correctly describes evolution of the Universe at times when the Einstein term is not yet induced. In contrast with the Einstein theory the problem of quantum cosmology here is well-posed (at one loop) due to asymptotic freedom. Following refs. [24, 28] (where was studied the possibility to avoid initial singularity taking into account quantized matter corrections) we shall assume that quantum cosmology is described by a homogeneous isotropic spatially flat metric $g_{\mu\nu} = a^2(\eta) \delta_{\mu\nu}$ where a is to be determined from the effective equation. Then one-loop effective action is completely given by its anomalous part (3).

Taking $\sigma = \ln a$ we find from (3)

$$\Gamma = \frac{V_3}{(4\pi)^2} \int d\eta (-3\beta_1 \sigma'^2 + p \sigma'^4) + \text{const} \quad (7)$$

where $\sigma' = d\sigma/d\eta$, $p = \beta_2 - 2\beta_1 - 3\beta_1$, and β_i are total (Weyl + matter) coefficients. Integrating $\delta\Gamma/\delta\sigma = 0$ twice we get two simple equations: either (A) $y' = \text{const}$ or (B) $y'^2 + \lambda y^4 + Fy = E$, $y \equiv \sigma'$, $E, F = \text{const}$, $\lambda = p/3\beta_1$.

Eq.(A), i.e. $\sigma = C_1 \eta^2 + C_2 \eta + C_3$, $C_1 > 0$, describes non-singular bouncing Universe (cf. [24]). The solutions of eq.(B) (having obvious mechanical interpretation) with $\lambda < 0$ in general correspond to singular metrics ("infinite motion")^{7/}.

for instance, if $F = E = 0$ we find $y = -|\lambda|^{-1/2} \eta^{-1}$, which is non-singular (de Sitter) solution only if $|\lambda| = 1$. Note also that in contrast to refs. [24, 28] we have no classical (Einstein) term in

(7), which essentially complicates analogous problem in [24, 28].

If $\lambda > 0$ ("finite motion") γ and ϕ are periodic functions of τ , describing non-singular oscillating quantum Universe. It remains to be seen whether the inclusion of the Einstein term, particle production, etc., can drive the Universe from this "breathing" quantum state (with the Planck scale of oscillations) to some expanding state. As for the actual value of λ , it is likely to be positive (because $\beta_4 < \frac{1}{3}\beta_2$ in a system including vectors, cf. Sect.2).

5. Superconformal theories. Let us start with recalling contributions in one-loop infinities of various fields of conformal supergravities [12, 13]. In Table 1 ν is the number of dynamical degrees of freedom, β_1 and β_2 are coefficients in (1), (2), $h_{\mu\nu}$ is conformal graviton, ψ_μ is conformal gravitino, V_μ , χ and E are ordinary vector, Majorana spinor and complex scalar, $T_{\mu\nu}$ is (non-gauge) antisymmetric tensor, A and C are higher derivative $(\bar{A} \hat{\partial}^3 A, C^* \square^2 C)$ Majorana spinor and complex scalar. The only correction to refs. [12, 13] is $\beta_1^{\text{new}}(T) = \beta_1^{\text{old}}(T) + \frac{1}{2}$ which is just the value for the laplacian on 2-forms (extra 1/2 is due to zero modes introduced by decomposition, used in [13], cf. [29]). The results of Table 1 were independently checked [30], using the de Sitter background and ζ -function techniques of ref. [31]. The field content [7,8] and total infinities in extended CSG [12, 13] are collected in Table 2. In both Tables we assumed that the (presently unknown) value of parameter γ of non-minimal coupling of G to the square of Weyl tensor [12, 13] is zero (in general, $\beta_1(N=4) = -\gamma$, $\beta_2(N=4) = -2 - 2\gamma$). Though $\gamma = -1$ implies finiteness of $N=4$ CSG (up to topological infinity) [12, 13], $\beta_1 \neq 0$ leads to non-zero CA (even assuming that β_4 , which cannot depend on γ , cancels in $N=4$ CSG). An argument in favour of the simplest $\gamma=0$ choice is that it provides vanishing of β_1 ("topological") part

of CA in N=4 theory just as in N=3 CSG. ^{8/} But then $\beta_2 \neq 0$ and thus to obtain a CA-free theory we need some "improvement" of N=4CSG.

A crucial observation is that one can consistently couple N=4 CSG to N=4 SYM by gauging the global superconformal group of the latter. ^(and completely - in d=10) On linearized level this was done in [8]. The lagrangian of N=4 SYM in external field of N=4 CSG has the following structure

$$\begin{aligned} \mathcal{L} = & \frac{1}{8} e^{-\frac{1}{2}\varphi} \text{Re } X + \frac{1}{8} B \text{Im } X + \frac{1}{2} |\mathcal{D}\phi_i|^2 + \frac{1}{2} \bar{\psi} \not{D} \psi + \\ & \times \left(\delta_i^k \delta_j^l R/6 - D^{kl}{}_{ij} + \bar{\psi} \not{D} \psi + \varphi \not{D} \varphi + B \not{D} B + \bar{\Lambda} \not{\partial} \Lambda + \dots \right) + \\ & + i \bar{\lambda}^i \not{D} \lambda_i + \bar{\Lambda}^i (\not{G} \cdot F^-) \lambda_i + \bar{\psi}_\mu^i (\not{G} \cdot F^-) \gamma_\mu \lambda_i + \quad (8) \\ & + \bar{\psi}_\mu^i F_{\mu\nu} \gamma_\nu \not{D} \phi_i + \bar{\lambda} \lambda E + \bar{\lambda} (\not{G} \cdot T^-) \lambda + \bar{\psi} \not{D} \psi + \\ & + \text{higher-order terms including standard } \bar{\lambda} \lambda \phi \text{ and } \phi^4, \text{ where} \\ X = & (F_{\mu\nu})^2 - \frac{1}{2} F_{\mu\nu}^+ T_{\mu\nu}{}^{ij} \phi_i \phi_j - \frac{1}{16} (T_{\mu\nu}{}^{ij} \phi_i \phi_j)^2, \quad F_{\mu\nu}^\pm = F_{\mu\nu} \pm i F_{\mu\nu}^*, \\ \not{D}_\mu = & \partial_\mu + A_\mu + V_\mu; \quad A_\mu^a, \lambda_i^a, \phi_i^a \quad \text{are fields of N=4 SYM (in} \end{aligned}$$

(8) we suppressed summation over index of adjoint representation of gauge group G), $D^{kl}{}_{ij}$ is auxiliary field of N=4 CSG and (in analogy with SU_4 formulation of PSG [33]) we substituted C by a pair real scalars ($C^* \not{D}^2 C \rightarrow \varphi \not{D}^2 \varphi + e^\varphi B \not{D}^2 B$). Complete form of (8) has much in common with the lagrangian for (N=2 SYM) (N=2 CSG) case [16]. Direct analysis of (8) shows that: (i) coupling of N=4 SYM to N=4 CSG does not modify (one-loop) β -function of the former, known to be zero (this non-trivial fact is due to special "chiral-dual" structure of the coupling, cf. [12]); (ii) one-loop infinities of combined system are given by (1) with β_i being sums of partial CSG and SYM contributions, $\beta_1 = 0 + 0$.

^{8/} note the analogy with (ungauged) Poincare supergravities (PSG) where one can make $\beta_1 = 0$ for $N \geq 3$ [29, 32]. In N=4 CSG we have the "right" number of antisymmetric tensors by construction and need no "duality rotations".

$\beta_1 = 0$, $\beta_2 = -2 + \frac{1}{2} \epsilon$, $\epsilon = \dim G$. Thus we conclude that the system of N=4 (SU_4) conformal supergravity and $SU_4 \times U_1$ or U_1^4 (N=4) N=4 SYM is completely free from (one-loop) infinities (in a proper background gauge) and hence is anomaly free.

This conclusion is likely to be valid also at higher loops (cf. [19]). This theory formally contains enough vectors ($SU_4 \times SU_2 \times U_1$) and spinors to have chances to be phenomenologically acceptable. Its status depends on resolution of the problem of ghosts and of correspondence with Einstein theory. Here we want to stress that there is a natural solution to the second problem, which avoids speculation on dynamical origin of the Einstein term.

To demonstrate this, let us take the U_1 N=4 vector multiplet to be ^aghost one in (8), $\mathcal{L} = -\frac{1}{2} \partial_\mu \phi_{ij} / \partial_\mu \phi_{ij} - \frac{1}{12} \partial_\mu \phi_{ij} / \partial_\mu \phi_{ij} (R + \bar{\psi} \not{\partial} \psi + \dots)$.

Making the (non-anomalous!) Weyl transformation of all the fields,

$|\phi_{ij}|' = \sqrt{12/K}$, $g'_{\mu\nu} = \frac{K}{\sqrt{12}} |\phi_{ij}| g_{\mu\nu}, \dots$ we are left with the gauge-equivalent theory, $\mathcal{L} = -\frac{1}{K^2} R +$ ("G-model" term for ϕ_{ij}) + ... , possessing the desired low-energy limit ^{9/}.

Finally, it is interesting to note that integration over the auxiliary field D^{ij} $\in \mathfrak{e}$ modifies N=4 SYM scalar potential $|f^{abc} \phi_{ij}^b \phi_{ic}^c|^2$ by extra $(1/\phi_{ij}^a)^2$ term, which may be important for spontaneous supersymmetry breaking.

The discussed theory is unique finite one among $N \leq 4$ superconformal ones. To have a finite theory with more matter $\rightarrow$ fields we are to increase the number of gravitinos. Though it is generally believed that $N > 4$ CSG do not exist (at least, without higher spins), the standard arguments [34, 8] can hardly be considered as convincing (e.g. they assume the existence of N=5 Poincare tensor calculus). That is why, in what follows shall

^{9/} in other words, we are only to assume that scale symmetry is spontaneously broken what in any case is necessary to give phy-
(see next page)

proceed heuristically, making a hypothesis that higher N CSG exist and contain only fields of the Table 1^{for N=8}. For example, for $N=5$ it may seem natural to choose the following (U_5) spectrum: $h, 5\psi, 25V, 10T, \dots$ (cf. Table 2)... From the conditions of zero number of degrees of freedom and $\beta_2 = 0$ we find: $N_\psi + 3N_h = 52$, $N_V + 2N_C = 14$. Adding the restriction (valid for $N \leq 4$ [18, 8]) that N_h and N_C are equal to the numbers of spinors and scalars in corresponding ($N=5$) PSG we get: $N_h = 11$, $N_C = 5$, $N_\psi = 19$, $N_V = 4$ and thus $\beta_2 = -\frac{5}{2} - \frac{1}{3}N_C - \frac{1}{6}N_h = -6$. If it turned to be possible to construct this theory and to couple it to $N=4$ SYM with $z = 12$ (e.g. $G = SU_3 \times SU_2 \times U_1$) then we would have one more example of a finite theory.

A more systematic way to study higher N theories is to employ lower N multiplets as "building blocks". An attractive choice is to use $N=3$ superconformal multiplets: $[2]_3 = (N=3 \text{ CSG}) [\frac{3}{2}]_3 = (N=4 \text{ CSG}) - (N=3 \text{ CSG}) = (\psi, 6V, 3T, 11\chi, 3A, 7E, C)$; $[1]_3 = (V, 4\chi, 3E) = (N=4 \text{ vector multiplet})$. All of them have $\beta_2 = 0$ (i.e. zero "topological" term in CA, cf. the construction of CA-free PSG [32]) and $\beta_2 = 1; -3; 1/2$ (see Tables 1,2). Equation $\beta_2^{tot} = 0$ gives $N_1 = 2(3N-10)$, $N_{12} = N-3$; e.g. $N=5$, $N_1 = 10$ or

usual interpretation to conformal invariant theory. It is useful to recall here that coupling of (gauged) SO_N PSG to U_1 CSG can be described (in "Poincare gauge") as coupling of: $N=1$ chiral multiplet to $N=1$ CSG [11]; SO_2 $N=2$ SYM plus $N=2$ tensor (or scalar) multiplet to $N=2$ CSG [16-18]; SO_N $N=4$ SYM to $N=3,4$ CSG in $N=3,4$ cases. These particular members of general class of superconformal theories are not anomaly free (see Table 3).

$N=8, N_1 = 28$. If we additionally require that it should be $\frac{1}{2}N(N-1)$ vector multiplets (e.g. that they form SO_N $N=4$ SYM, as in PSG + CSG system, see footnote 9) then the only solutions are $N=5$ and $N=8$. These finite theories with rich spectra may be con-

sidered as superconformal extensions of $N=5$ and $N=8$ PSG. The same program can be realized starting with $N=2$ multiplets (with an advantage of having $N=2$ superfield formulation and of assigning $N=2$ matter to fundamental representations, cf. [2]): $[2]_2 = (N=2 \text{ CSG})$;

$$[\frac{3}{2}]_2 = (N=3 \text{ CSG}) - (N=2 \text{ CSG}); [1^*]_2 = (N=4 \text{ CSG})$$

$$-[2]_2 - 2[\frac{3}{2}]_2 = (V, T, 4X, 2A, 4E, C); [1]_2 = (V, 2X, E); [\frac{1}{2}]_2 = (2X, 2E); [\frac{1}{2}^*]_2 = (2X, 3\phi, t_{\mu\nu})$$

(as tensor multiplet).

The corresponding (β_1, β_2) -values are: $(\frac{11}{24}, \frac{12}{3})$; $(-\frac{11}{24}, -\frac{12}{3})$, $(\frac{11}{24}, \frac{1}{3})$; $(-\frac{1}{24}, \frac{1}{3})$; $(\frac{1}{24}, \frac{1}{6})$; $(\frac{12}{24}, \frac{1}{6})$ so that $\beta_1^{tot} = \beta_2^{tot} = 0$ are satisfied if, e.g., $N_1^* = 0$ and $N_2 + 9N_2^* = 14N_{\frac{1}{2}} - 16$, $N_2 = 11 - 11N_{\frac{1}{2}} + 13N_{\frac{1}{2}}^* + N_{\frac{1}{2}}^*$, $N_{\frac{1}{2}} = N - 2$. For instance, $N=5, N_1 = 12, N_{\frac{1}{2}} = 8, N_{\frac{1}{2}}^* = 2$ is a candidate for $N=5$ CSG (again it admits

$SU_3 \times SU_2 \times U_1$ as $N=2$ SYM gauge group). Another solution is $N=8, N_1 = 25, N_{\frac{1}{2}} = 41, N_{\frac{1}{2}}^* = 3$. Interestingly, the number of hypermultiplets (44) is nearly equal to that (45) needed to describe three (SU_5) generations (as well as their "mirror" partners). 10/

In conclusion, we have given arguments for existence of finite anomaly free theories, which incorporate gravity and possess maximal local superconformal symmetry at the quantum level.

10/ If we were not assuming overall N -supersymmetry (what seems necessary in view of probably higher spin inconsistencies for separate $\frac{3}{2}$ -multiplets) we were also to provide vanishing of $N=2$ SYM β -function cf. [2]. It is interesting to note that 10 hypermultiplets in fundamental representation of $SU_3 \times SU_2 \times U_1$ just do the job in $N=5$ solution, while $40 = 4 \times 5 + 2 \times 10$ are enough to compensate SU_5 gauge contribution in $N=8$ case.

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Table 1: Degrees of freedom and contributions in infinities of fields of conformal supergravities

	h	ψ	V	T	χ	Λ	E	C
$\mathcal{D}$	6	-8	2	6	-2	-6	2	4
β_1	$\frac{137}{60}$	$-\frac{173}{180}$	$-\frac{13}{180}$	$\frac{11}{30}$	$\frac{7}{720}$	$\frac{21}{720}$	$\frac{1}{90}$	$\frac{1}{45}$
β_2	$\frac{199}{18}$	$-\frac{149}{30}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{20}$	$-\frac{1}{60}$	$\frac{1}{30}$	$-\frac{4}{15}$

Table 2: Field content and infinities of extended conformal supergravities

	$h_{\mu\nu}$	ψ_μ^i	$V_\mu^{\dot{A}}$	$T_{\mu\nu}^{\dot{A}\dot{B}}$	$\chi_{\mu\dot{A}}$	$\Lambda_{\dot{A}\dot{B}}$	$E_{\dot{A}\dot{B}\dot{C}}$	$C_{\dot{A}\dot{B}\dot{C}\dot{D}}$	β_1	β_2
$N=1$	1	1	1	0	0	0	0	0	$\frac{5}{4}$	$\frac{17}{2}$
$N=2$	1	2	4	1	2	0	0	0	$\frac{11}{24}$	$\frac{13}{3}$
$N=3$	1	3	9	3	9	1	3	0	0	1
$N=4$	1	4	15	6	20	4	10	1	0	-2

Table 3: Infinities of combined system of Poincare and conformal supergravity

	($N=1$ chiral multiplet)+ ($N=1$ CSG)	($SO_2, N=2$ SYM)+ ($N=2$ scalar or tensor multiplet)+ ($N=2$ CSG)	($SO_2, N=4$ SYM)+ ($N=3$ CSG)	($SO_2, N=4$ SYM)+ ($N=4$ CSG)
β_1	$71/48$	$\frac{11}{24}$ or $\frac{11}{24} + \frac{1}{2}$	0	0
β_2	$103/12$	$29/3$	$5/2$	1

Т - 14170

Подписано в печать 4 июля 1983 года

Заказ № 357. Тираж 100 экз. л/л 0,9

Отпечатано в Редакционно-издательском

отделе ЦИАН СССР

Москва, В-312, Ленинский проспект, 53