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A MEASUREMENT OF THE STRING TENSION NEAR THE CONTINUUM LIMIT

G. Parisi
Dipartimento di Fisica
Università di Roma II - Tor Vergata

R. Petronzio
CERN -- Geneva

and

F. Rapuano
Istituto Nazionale per la Fisica Nucleare
Sezione di Roma

ABSTRACT

The string tension has been measured at $\beta = 6$ in the quenched approximation using a new technique for minimizing the statistical errors.

The recent measurement of meson masses in the quenched approximation for lattice QCD indicates that the lattice spacing at $\beta = 6$ is about $(2 \text{ GeV})^{-1}$ ¹⁾. If one takes the conventionally assumed value for the string tension²⁾, i.e., $\sqrt{\sigma} \approx 170 \text{ MeV}$ ($a\Lambda_L$ at $\beta = 6$ is equal to 0.002346), the string tension turns out to be about 800 MeV, in contrast with the expected value of $\sim 420 \text{ MeV}$. Now, it is clear that if the lattice spacing at $\beta = 6$ is 0.1 Fm, the previous measurements of the Wilson loops extended up to 0.3 - 0.4 Fm²⁾, and they were strongly affected by systematic errors. Indeed, careful measurements at lower values of β ³⁾ suggest a substantially smaller value for the string tension. The latter measurements cannot be compared directly with the results¹⁾ at $\beta = 6$, unless one trusts the scaling law of the renormalization groups. We have therefore decided to extract the string tension at $\beta = 6$ from the same gauge field configurations of $10^3 \times 20$ points which were used for computing the mass spectrum.

Let us denote by $P(x,y,t)$ the trace of the products of the links in the z direction wrapping around the lattice (which is a torus because of periodic boundary conditions). In a lattice of size L in the z direction, it is well known that:

$$\lim_{L \rightarrow \infty} \frac{1}{L} \ln \langle P(0,0,0) P(0,0,t) \rangle_c = -V(t) \quad (1)$$

where $V(t)$ is the static potential between quarks at distance t . In our case, the lattice is sufficiently large and we suppose that:

$$\langle P(0,0,0) P(0,0,t) \rangle_c \approx \exp[-LV(t)] \quad (2)$$

Of course, systematic errors are involved in this approximation, but they should be smaller than those of previous computations.

In this note, we concentrate our attention on the string tension, i.e., the behaviour of the potential $V(t)$ at large t :

$$V(t) \sim kt \quad \text{for } t \rightarrow \infty \quad (3)$$

In the region of large t , we have

$$\langle P(0,0,0) P(0,0,t) \rangle_c \sim \exp(-L \cdot K \cdot t) \quad (4)$$

In other words, LK is the "mass" of the correlation of two P loops. It is normally useful to look to the following correlations:

$$C(t) \equiv \frac{1}{L_x L_y L_t} \sum_{x,y} \sum_{x',y',t'} \langle P(x,y,t') P(x',y',t'+t) \rangle \quad (5)$$

in order to increase the statistics. We expect that $C(t)$ should also decrease with t according to Eq. (1).

This kind of measurement, however, has been practically impossible for large lattices because of the smallness of the correlation involved. Here we use a new method to decrease the statistical errors, which is in our case fairly efficient: we gain at least a factor ten in the statistical errors, which corresponds to a factor 100 in the statistics. The idea is very simple: if A is an observable, the statistical errors are controlled by $\langle A^2 \rangle_c$. We want to find another variable B , so that

$$\langle A \rangle = \langle B \rangle \quad \text{but} \quad \langle A^2 \rangle_c \gg \langle B^2 \rangle_c \quad (6)$$

If we succeed, it is clear that it is much better to measure B instead of A . If the variable A interacts only with another set of variables (which we call C), (e.g., if A is defined inside a region and has only nearest neighbour interactions, C will be the variables on the boundary), the statistical expectation of A can be written as

$$\langle A \rangle = \langle A(C) \rangle \quad (7)$$

where $A(C)$ is the expectation value of A at fixed C . For example, in the Ising model with Hamiltonian $H = \frac{1}{2} \sum_{i,k} J_{ik} \sigma_i \sigma_k$,

$$\langle \sigma_i \rangle = \langle \text{tr} [\beta \sum_k J_{ik} \sigma_k] \rangle \quad (8)$$

This identity was derived by Callan⁴⁾ a long time ago, and is a particular case of the DLR⁵⁾ equation. It is evident that the fluctuation of the $[\beta \sum_k J_{ik} \sigma_k]$ are smaller than those of σ_i . An extreme case is at $\beta = 0$: σ_i fluctuates while the other is identically zero, which is the correct answer.

The same technique can be used for measuring the correlation function of two mutually independent variables A and A' (i.e., if we define C and C' as before: $C \cap A = C \cap A' = \emptyset$). If the interaction is nearest neighbour, this requires that A and A' are at least at distance two. This trick is particularly useful in our case, because each of the variables U which belong to the P loops are mutually independent of each other if the distance of the P loops is at least two. We can finally write:

$$\langle \prod_{i \in P} U(i) \prod_{i' \in P'} U(i') \rangle = \langle \prod_{i \in P} \bar{U}(i) \prod_{i' \in P'} \bar{U}(i') \rangle \quad (9)$$

where the $\bar{U}(i)$ are defined by:

$$\bar{U} = \frac{\int dU U \exp(-\beta H)}{\int dU \exp(-\beta H)} \quad (10)$$

where the integration is done over only one $SU(3)$ matrix and depends on the neighbouring links. In the case of the $U(3)$ group or $O(3)$ ⁶⁾, the integral can be expressed in terms of Bessel functions. The corresponding expression for $SU(3)$ is not very handy⁶⁾. We have then decided to evaluate $\bar{U}$ numerically by using a heat bath method⁷⁾. We have done N heated hits with $N = 14$; although the correct results are obtained for any value of N , the statistical fluctuations decrease with N down to a plateau value which is reached for $N \sim O(10)$.

We have analyzed 35 configurations which have been produced in the study of the meson spectrum; they are separated by 100 Monte Carlo sweeps. After having omitted those initial configurations for which $\langle P \rangle$ is different from zero above the statistical fluctuations, (this may be a remnant of the initial cold start), we have obtained the result in Fig. 1. The correlation shows a nice exponential (hyperbolic cosine) behaviour which can be fitted with $K = 0.039 \pm 0.004$. This value of K corresponds, with a lattice spacing of 0.1 Fermi, to $\sqrt{K} = 400 \pm 20$ MeV, which is now in agreement with the supposed value of 420 MeV.

This result makes us confident about the estimate of the lattice spacing at $\beta = 6$.

Let us comment on the consequences of this evaluation. If we use the value of the 0^+ glueball estimated by Berg and Billoire⁸⁾, we get, by extrapolating their data at $\beta = 6$, $m_{0^+} \sim 1300$ MeV, which is a more reasonable value than the one obtained with the old string tension. Furthermore, the confinement-deconfinement transition is now at $T \approx 370$ MeV⁹⁾, which is a value higher than the normally estimated one. The most striking result is on the latent heat of the deconfinement transition which, for pure gauge fields, turns out to be of ~ 10 GeV/Fm³¹⁰⁾. The corresponding constant B of the vacuum pressure of the bag model is then ~ 0.08 GeV⁴, which is quite different from the one estimated by the people who studied the bag model: 2×10^{-4} GeV⁴¹¹⁾. The bag must have a cigar-like or a flux-tube-like shape. In a very simplified model, in which $K = (\pi/4)D^2B$ where D is the diameter of the flux-tube, D turns out to be ~ 0.3 Fm. Notice that the diameter of the string turns out to be very near to the inverse of the effective gluon mass estimated in Ref. 12).

The CPU time used for the analysis of the configurations produced on the CRAY has been of about 6hs of CDC 7600.

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FIGURE CAPTION

Figure : The quantity defined in Eq. (5) as a function of t .