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QUANTUM CHROMODYNAMICS ON A SIMPLICIAL LATTICE

R.W.B. Ardill and J.-P. Clarke

Department of Mathematics,
Royal Holloway College, Englefield Green,
Surrey, TW20 OEX, U.K.

J.-M. Drouffe *)

CERN -- Geneva

and

K.J.M. Moriarty

Department of Mathematics,
Royal Holloway College, Englefield Green,
Surrey, TW20 OEX, U.K.

ABSTRACT

Monte Carlo simulations on a 6^4 simplicial lattice are used to calculate Wilson loops, and hence the string tension, for pure SU(3) gauge theory. The asymptotic freedom scale parameter Λ is measured for both triangles and squares and universality is established.

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Permanent address: S. Ph. T., CEN Saclay,
91190 Gif-sur-Yvette, Cedex, France.

Lattice gauge theories are usually formulated on a hypercubical lattice.⁽¹⁾ Recently, a number of alternative lattice formulations have been put forward. These include the random lattice⁽²⁾, the body-centered tesseract lattice⁽³⁾ and the simplicial lattice^(4,5). One of the motivations for studying gauge theory on at least one of these alternative lattices is to test the principle of universality which asserts that the continuum physics is independent of the lattice on which it is formulated. Previously, we studied $SU(2)$ ⁽⁴⁾ and $U(2)$ ⁽⁵⁾ gauge theory on a simplicial lattice and found no surprises over the results found on a hypercubical lattice. However, the gauge group for Quantum Chromodynamics (QCD), the theory of strong interactions, is $SU(3)$. It thus seems reasonable to extend our previous analysis to $SU(3)$. In the present paper, we study $SU(3)$ on a simplicial lattice, the lattice having six sites in each space-time direction. The Wilson loops for both triangles and squares are calculated, the associated string tensions are extracted and the asymptotic-freedom scale parameter Λ is evaluated.

We define our system on a four-dimensional Euclidean space-time simplicial lattice⁽⁴⁾. Nearest-neighbour lattice sites are denoted by i and j and are joined to form a link denoted by $\{i,j\}$, on which sits an $N \times N$ unitary-unimodular matrix $U_{ij} \in SU(N)$ such that

$$U_{ji} = (U_{ij})^{-1}.$$

Our partition function is defined by

$$Z(\beta) = \int \left(\prod_{\{i,j\}} dU_{ij} \right) \exp(-\beta S[U]),$$

where dU_{ij} is the normalized invariant Haar measure for $SU(N)$ and β is the inverse coupling constant squared defined by

$$\beta = \frac{12}{\sqrt{5} g_0^2},$$

where g_0 is the bare coupling constant and the action S is the sum over all unoriented triangles such that

$$S[U] = \beta \sum_{\Delta} E_{\Delta} = \beta \sum_{\Delta} \left(1 - \frac{1}{N} \text{Re Tr } U_{\Delta} \right),$$

where U_{Δ} is the parallel transporter around a triangle. Periodic boundary conditions are used throughout our calculations. The method of Metropolis et al.⁽⁶⁾ is used to equilibrate our lattice.

Following the calculational techniques outlined in Refs. 4 and 5 and using standard techniques⁽⁷⁻⁹⁾, we find the average action for

$$\begin{aligned} \langle E_{\Delta} \rangle = & 1 - \frac{\beta}{18} - \frac{\beta^2}{216} - \frac{\beta^3}{2916} - \frac{25}{279936} \beta^4 - \frac{259}{15116544} \beta^5 - \\ & - \frac{469}{906992640} \beta^6 + \frac{24187}{97955205120} \beta^7 + \frac{455}{26121388032} \beta^8 \\ & + \frac{58453}{63474972917760} \beta^9 + O(\beta^{10}). \end{aligned} \quad (1)$$

and the average action for squares to be

$$\langle E \rangle = 1 - 2\left(\frac{\beta}{18}\right)^4 - 16\left(\frac{\beta}{18}\right)^6 + O(\beta^7). \quad (2)$$

We also obtain the strong-coupling expansions for the string tension for triangles and squares to be

$$K_{\Delta} = -\frac{4}{\sqrt{3}} \left(\ln\left[\frac{\beta}{18}\right] + \frac{\beta}{12} + \frac{7}{2592} \beta^2 + \frac{1}{1296} \beta^3 - \frac{61}{6718464} \beta^4 + O(\beta^5) \right), \quad (3)$$

and

$$K_{\square} = -4 \left(\ln \left[\frac{\Gamma(\frac{3}{4})\beta}{36\Gamma(\frac{3}{4})} \right] + \frac{\beta}{12} + \frac{5}{5776} \beta^2 + O(\beta^3) \right), \quad (4)$$

respectively.

From the renormalization group, we obtain a relationship between the asymptotic string tension σ , the lattice string tension K and the asymptotic-freedom scale parameter Λ of

$$K \sim \frac{\sigma}{\Lambda^2} \left| \frac{4\pi^2\sqrt{5}\beta}{33} \right|^{\frac{102}{121}} \exp \left| -\frac{4\pi^2\sqrt{5}\beta}{33} \right|. \quad (5)$$

To perform our calculations we carried out a total of 136 iterations through the simplicial lattice with 20 Monte Carlo updates per link. The first 16 iterations were used for equilibrating the lattice. We then averaged over the next 120 iterations through our lattice. However, in order to reduce correlations between events every second lattice configuration was eliminated. Thus we used 60 lattice configurations in our averages. The crossover point between strong and weak-coupling is $\beta \simeq 4.8$. As a result we used disordered starting lattices for $\beta \leq 4.8$ and ordered starting lattices for $\beta \geq 5.0$. Equilibration was enhanced by storing the lattice configurations from the run with the previous β value and starts were initiated at $\beta = 9.0$ (ordered) and $\beta = 1.0$ (disordered).

Fig. 1. shows the average action for triangles $\langle E_{\Delta} \rangle$ as a function of the inverse coupling constant squared. We also show the 8th and 10th order strong-coupling expansions of Eqn. 1 in Fig. 1.

The Monte Carlo results for the triangular Wilson loops are shown in Fig. 2. The numbering scheme is taken from Fig. 3 of Ref. 1. In Fig. 2, we also show the leading-order strong-coupling expansion of Eqn. 1

We present the results for the logarithmic ratios

$$\begin{aligned} \chi_1^\Delta &= \frac{2}{\sqrt{3}} \ln \left| \frac{W_4^\Delta}{W_6^\Delta} \right|, \\ \chi_2^\Delta &= \frac{1}{\sqrt{3}} \ln \left| \frac{W_9^\Delta}{W_{13}^\Delta} \right|, \end{aligned} \tag{6}$$

and

$$\chi_3^\Delta = \frac{2}{\sqrt{3}} \ln \left| \frac{W_4^{\Delta^2}}{W_1^\Delta W_9^\Delta} \right|,$$

in Fig. 3. We show the truncated strong-coupling expansion of Eqn. 3 and its Padé approximant. We also display a curve which corresponds to the behaviour of Eqn. 5 with

$$\Lambda = (3.0 \pm 0.5) \times 10^{-3} \sqrt{\sigma},$$

which is about one half of the hypercubical lattice result⁽¹⁰⁾.

In Fig. 4 we show the average action for squares $\langle E_\square \rangle$ for pure SU(3) gauge theory as a function of the inverse coupling constant squared β . Also shown is the strong-coupling expansion of Eqn. 2. All Wilson loops of size up to 3×3 are shown in

Fig. 5 where we have followed the numbering scheme of Fig.3 of Ref. 4. We also show the leading-order strong-coupling behaviour of Eqn. 2.

Fig. 6 shows the quantities $\chi_1^\square$ and $\chi_2^\square$ defined by

$$\chi_1^\square = \ln \left| \frac{w_3^\square}{w_4^\square} \right|, \quad (7)$$

$$\chi_2^\square = \frac{1}{2} \ln \left| \frac{w_4^{\square 2}}{w_1^\square w_9^\square} \right|,$$

as a function of the inverse coupling constant squared β . We also show in Fig. 6 the strong-coupling expansion of Eqn. 4.

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FIGURE CAPTIONS

Fig. 1. The average action per plaquette $\langle E_{\Delta} \rangle$ for pure SU(3) gauge theory on a simplicial lattice as a function of the inverse coupling constant squared β . The curve (A) represents the strong-coupling expansion of Eqn. 1 and curve (B) represents the [4/4] Padé approximation to the expansion.

Fig. 2. The Wilson loops for pure SU(3) gauge theory on a simplicial lattice as a function of the inverse coupling constant squared β . The full upward triangles represent W_1^{Δ} , the open circles represent W_2^{Δ} , the full circles represent W_3^{Δ} , the crosses represent W_4^{Δ} , the open upward triangles represent W_6^{Δ} , the full downward triangles represent W_9^{Δ} and the open downward triangles represent W_{13}^{Δ} .

Fig. 3. The string tension in triangular planes for pure SU(3) gauge theory on a simplicial lattice as a function of the inverse coupling constant squared β . The full upward triangles represent χ_1^{Δ} , the full circles represent χ_2^{Δ} and the open circles represent χ_3^{Δ} . Also shown in the diagram are the behaviours of Eqn. 3 and 5.

Fig. 4. The average action per plaquette $\langle E_{\square} \rangle$ for pure SU(2) gauge theory on a simplicial lattice as a function of the inverse coupling constant squared β . The curve (A) represents the strong-coupling expansion of Eqn. 2.

Fig. 5. The Wilson loops for pure SU(3) gauge theory on a simplicial lattice as a function of the inverse coupling constant squared β . The full upward triangles represent $W_1^{\square}$, the open circles represent $W_2^{\square}$, the crosses represent $W_3^{\square}$, the open upward triangles represent $W_4^{\square}$, the full downward triangles represent $W_6^{\square}$ and the full circles represent $W_9^{\square}$.

Fig. 6. The string tension in quadratic planes for pure SU(3) gauge theory on a simplicial lattice as a function of the inverse coupling constant squared β . The full upward triangles represent $\chi_1^{\square}$ and the full circles represent $\chi_2^{\square}$. Also shown in the diagram are the behaviours of Eqn. 4 and 5.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6