Ref.TH.3597-CERN

BROKEN SUPERSYMMETRIC QCD AND e^+e^- HADRONIC CROSS-SECTIONS

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A B S T R A C T

Effects of broken supersymmetry on some exclusive and the total inclusive e^+e^- hadronic cross-sections are studied. In particular, the first-order QCD correction to $R_{e^+e^-}$ from s-quark and gluino effects is computed and discussed with particular emphasis on the energy interval below the threshold for production of a pair of s-quarks and on the limit of infinite energies.

Ref.TH.3597-CERN

May 1983

In this note a quantitative analysis is presented of some effects induced in e^+e^- annihilation by scalar quarks (s-quarks : s_q) and gluinos ($\tilde{g}$), the conjectured supersymmetric ¹⁾ (SUSY) partners of quarks and gluons. It is well known ²⁾ that pair production of s-quarks and charged s-leptons is the most obvious signature for SUSY particles in e^+e^- annihilation. Negative results from searches for pairs of SUSY scalars at PETRA and PEP have led so far to lower limits ²⁾ on the masses given by $m_{s-\ell}, m_{s-q} \gtrsim 15+16$ GeV. In view of these results we were led to consider possible effects in the hadronic cross-sections below the threshold for s-quark pair production, i.e., at $Q \lesssim 2M$, when $Q = 2E_{\text{beam}}$ and M is the s-quark mass [for simplicity we neglect mass differences for (light) s-quarks which are expected to be small anyway ³⁾]. Assuming that gluinos are lighter than s-quarks (present limits ⁴⁾ give $m \gtrsim 2-3$ GeV, where m is the $\tilde{g}$ mass), one possibility is to look for production of the final states $q\bar{s}_q\tilde{g} + \bar{q}s_q\tilde{g}$ (Fig. 1a) or $\tilde{g}\tilde{g}, q\bar{q}\tilde{g}\tilde{g}$. The production of two $\tilde{g}$ was studied in Refs. 5) and 6), and is small being of order α_s^2 with the channel $\tilde{g}\tilde{g}$ vanishing in the limit of P and/or C conservation. A different interesting question is the effect of s-quarks and gluinos on the ratio $R_{e^+e^-}$ of the hadronic and pointlike e^+e^- cross-sections. At order α_s , in the presence of SUSY particles one has, for $Q < 2M$:

$$R_{e^+e^-} = R_0 \left[1 + \frac{\alpha_s(Q^2)}{\pi} + \delta(\eta, r) \frac{\tilde{\alpha}_s(Q^2)}{\pi} \right] \quad (1)$$

where $R_0 = 3EQ_1^2$, α_s/π is the ordinary QCD leading correction ⁷⁾, δ is the additional contribution from SUSY particles, with

$$\eta = \frac{Q^2}{M^2}, \quad r = \frac{m^2}{M^2} \quad (2)$$

$\tilde{\alpha}_s = \tilde{g}_s^2/4\pi$ where $\tilde{g}_s$ is the $qs_q\tilde{g}$ coupling constant. One expects $\alpha_s \sim \tilde{\alpha}_s$, although the two are not exactly equal in the presence of SUSY breaking.

For massless quarks, in the interval $Q \lesssim M+m$ the only contribution to δ arises from virtual s_q and $\tilde{g}$ corrections to the $q\bar{q}$ channel (Fig. 1b). For $M+m \lesssim Q \lesssim 2M$ the contribution to δ from the real production diagrams of Fig. 1a is also present. Note that for Q of order M , δ is expected to be of order 1, so that the QCD leading correction could be quite significantly altered producing a detectable effect somewhat below the threshold for open SUSY scalar pair production. On the other hand, the accuracy of the data on the total hadronic e^+e^- cross-section is by now sufficient to extract a significant, although not very precise, determination of the first order QCD correction, which at present

leads to ⁸⁾ $\alpha_s(Q \approx 30 \text{ GeV}) \approx (0.20 \pm 0.06)$ and is about to be further improved. It is thus interesting to evaluate δ and study its behaviour as a function of Q^2 , M^2 and m^2 . For completeness, we also analyze δ above threshold for pair production, where the additional contributions from the diagrams shown in Fig. 1c also become relevant. In particular, we derive the asymptotic value of δ at values of $Q \gg M, m$ (i.e., the massless limit). Finally, the cross-section and distributions for the real production processes in Fig. 1a are derived and discussed.

The virtual contribution from the diagrams in Fig. 1b below threshold for pair production ($\eta < 4$) is found to be

$$\delta_{\text{VIRTUAL}} = C_F \left\{ \frac{3}{4} + \frac{1}{2(1-r)} + \frac{r^2}{2(1-r)^2} \ln r - \sqrt{\frac{4-\eta}{\eta}} \cdot \text{arctg} \sqrt{\frac{\eta}{4-\eta}} + \int_0^1 dy \frac{r+(1-r)y}{\eta y + r - 1} \ln \left[\frac{1-\eta y(1-y)}{r+(1-r)y} \right] \right\} \quad (3)$$

where $C_F = (N_C^2 - 1)/2N_C = \frac{4}{3}$ and η, r are defined by Eq. (2). Note that δ_{VIRTUAL} vanishes at $Q = 0$ as a consequence of the Ward identities of QED. For $m = 0$ and small values of Q^2 , an expansion of Eq. (3) in η leads to ^{*})

$$\delta_{\text{VIRTUAL}} \sim \frac{C_F}{18} \eta + O(\eta^2) \quad (4)$$

This small value for the positive slope in Q^2 is responsible for the very slow increase of δ_{VIRTUAL} with Q^2 , as seen from Fig. 2, so that, even at $Q \sim 2M$, $\delta_{\text{VIRTUAL}} \lesssim 0.5$. Thus in the interval $Q \lesssim M+m$, where $\delta \equiv \delta_{\text{VIRTUAL}}$, the effects of SUSY particles in $R_{e^+e^-}$ turn out to be particularly small and practically invisible.

At $Q > M+m$ the real contribution from the final states $q\bar{s}_q \tilde{g} + \bar{q}s_q \tilde{g}$ also comes into play (Fig. 1a). The double differential cross-section for $q\bar{s}_q \tilde{g}$ ^{**)} can be written down in the form :

$$\frac{1}{\sigma_0} \frac{d\sigma}{dx_q dx_s} = \Theta[Q-M-m] \frac{\tilde{\alpha}_s}{8\pi} C_F \left\{ \frac{(1-x_s)(1-4\tilde{M}^2+4\tilde{m}^2) + (\tilde{M}^2-\tilde{m}^2)(1-4\tilde{M}^2)}{(1-x_s)^2 + \tilde{M}^2 \frac{\Gamma^2}{Q^2}} + \frac{1}{1-x_q} \left[2x_s - \frac{4(1-x_s)[\tilde{M}^2 + (\tilde{M}^2-\tilde{m}^2)(\tilde{M}^2-\tilde{m}^2-x_s)]}{(1-x_s)^2 + \tilde{M}^2 \frac{\Gamma^2}{Q^2}} \right] - 2 - \frac{2(\tilde{M}^2-\tilde{m}^2)}{(1-x_q)^2} \right\} \quad (5)$$

^{*}) For $m = M$ one has : $\delta_{\text{VIRTUAL}} \sim (C_F/24) \cdot \eta + O(\eta^2)$.

^{**)} For only one scalar out of the two associated with each fermion.

where $\sigma_0 = R_0(4\pi\alpha^2/3Q^2)$ is the lowest-order cross-section, $x_{q,s} = (2E_{q,s}/Q)$, $\tilde{M} = M/Q$, $\tilde{m} = m/Q$. Γ is the width for $s_q \rightarrow q\tilde{g}$ given by

$$\Gamma = C_F \frac{\tilde{\alpha}_s}{2} \frac{(M^2 - m^2)^2}{M^3} \quad (6)$$

which is introduced in order to shift the pole corresponding to an on-shell s-quark, which, in the subsequent evaluation of δ_{real} , approaches the integration limit when $Q \rightarrow 2M$. Note that in the limit $M = m = 0$, Eq. (5) can be cast in the form :

$$\frac{1}{\sigma_0} \frac{d\sigma}{dx_s dx_g} = \frac{\tilde{\alpha}_s}{8\pi} C_F \frac{(1-x_s)(1-x_g) + x_s x_g}{(1-x_s)(x_s+x_g-1)} \quad (7)$$

with $x_g = 2E_g/Q = 2-x_s-x_q$, which coincides with the result of Ref. 9) where only the massless limit was studied. The massless distribution is obviously a drastic over-estimate of the realistic effect expected from massive s-quarks and gluinos at available energies. But it is useful for a simple comparison with the analogous three-jet distribution ⁷⁾ in ordinary QCD. One immediately notices that the shape is different and the normalization $\tilde{\alpha}_s/8\pi$ is smaller by a factor of 4, which indicates that this effect is also very small. The decays $s_q \rightarrow q\tilde{g}$ and $\tilde{g} \rightarrow q\bar{q}\tilde{\gamma}$ ¹⁰⁾ eventually produce a complicated topology for this kind of event.

By performing one integration of Eq. (5) and correctly normalizing, one finds, for δ_{real} , the following result :

$$\begin{aligned} \delta_{\text{REAL}} = & \Theta[Q-M-m] C_F \int_{2\tilde{M}}^{1+\tilde{M}^2-\tilde{m}^2} dx_s \left\{ \left[-1 + \frac{1+\tilde{M}^2-\tilde{m}^2-x_s-4(1+\tilde{M}^2-x_s)(\tilde{M}^2-\tilde{m}^2)}{2[(1-x_s)^2+\tilde{M}^2\tilde{m}^2/Q^2]} \right] \right. \\ & \cdot \left(1 - \frac{\tilde{m}^2}{1+\tilde{M}^2-x_s} \right) \sqrt{x_s^2-4\tilde{M}^2} + \frac{\tilde{m}^2\sqrt{x_s^2-4\tilde{M}^2}}{\tilde{m}^2+(1+\tilde{M}^2-\tilde{m}^2-x_s)(\tilde{M}^2-\tilde{m}^2)} - \sqrt{x_s^2-4\tilde{M}^2} + \\ & \left. + \left[x_s - \frac{2(1-x_s)[\tilde{m}^2+(1+\tilde{M}^2-\tilde{m}^2-x_s)(\tilde{M}^2-\tilde{m}^2)]}{(1-x_s)^2+\tilde{M}^2\tilde{m}^2/Q^2} \right] \ln \frac{2\tilde{m}^2+(x_s+\sqrt{x_s^2-4\tilde{M}^2})(1+\tilde{M}^2-\tilde{m}^2-x_s)}{2\tilde{m}^2+(x_s-\sqrt{x_s^2-4\tilde{M}^2})(1+\tilde{M}^2-\tilde{m}^2-x_s)} \right\} \end{aligned} \quad (8)$$

The behaviour of δ_{real} and of $\delta = \delta_{\text{virtual}} + \delta_{\text{real}}$ is shown in Fig. 2. The dominant contribution is from δ_{virtual} and δ reaches a sizeable magnitude only in the vicinity of the threshold for pair production. Thus the chances of obtaining useful bounds on the masses of SUSY particles from this effect are slim.

Above threshold for pair production of s-quarks, the new contributions in Fig. 1c become relevant. We first discuss the limit $Q \rightarrow \infty$ where the massless limit is approached. In this limit, to first order in α_s , $R_{e^+e^-}$ becomes :

$$R_{e^+e^-} = R_0 \left[\frac{3}{2} + \frac{\alpha_s(Q^2)}{\pi} (1 + \delta') + \frac{\tilde{\alpha}_s(Q^2)}{\pi} \delta \right] \quad (9)$$

where the zeroth order value is modified with respect to Eq. (1) by the production of two pairs of scalars for each flavour, δ' arises from the contributions of gluons coupled to s-quarks and δ contains, in addition to the terms from Figs. 1a and 1b already considered, also the new contribution δ_s from the virtual correction to $s_q \bar{s}_q$ production :

$$\begin{aligned} \delta' &= \delta_{\text{VIRTUAL}}^{\text{S-QCD}} + \delta_{\text{REAL}}^{\text{S-QCD}} \\ \delta &= \delta_{\text{VIRTUAL}} + \delta_{\text{REAL}} + \delta_s \end{aligned} \quad (10)$$

We computed δ and δ' in two different ways.: first directly in the massless theory by using dimensional regularization, and then in the massive theory by taking the limit $\tilde{M}, \tilde{m} \rightarrow 0$ at the end, after cancellation of mass singularities between real and virtual diagrams. The second method is considerably more complicated but it allows the numerical computation of δ and δ' in the intermediate region where the approach to the asymptotic limit can be studied.

In the massless theory, one finds :

$$\left. \begin{aligned} \delta_{\text{VIRTUAL}} &= C_F \left(\frac{1}{2\epsilon} + \frac{3}{2} \right) \\ \delta_{\text{REAL}} &= C_F \left(-\frac{1}{\epsilon} - \frac{13}{4} \right) \\ \delta_s &= C_F \left(\frac{1}{2\epsilon} + 1 \right) \end{aligned} \right\} \Rightarrow \delta = -1 \quad (11)$$

$$\left. \begin{aligned} \delta_{\text{VIRTUAL}}^{\text{S-QCD}} &= \frac{1}{2} C_F \left(-\frac{1}{\epsilon^2} - \frac{2}{\epsilon} - 4 + \frac{\pi^2}{2} \right) \\ \delta_{\text{REAL}}^{\text{S-QCD}} &= \frac{1}{2} C_F \left(\frac{1}{\epsilon^2} + \frac{2}{\epsilon} + 7 - \frac{\pi^2}{2} \right) \end{aligned} \right\} \Rightarrow \delta' = 2 \quad (12)$$

where $2\epsilon = 4-n$ and a common factor $(\mu^2/Q^2)^{2\epsilon}(1/\Gamma(2-2\epsilon))$ is omitted (which is one for $\epsilon \rightarrow 0$). δ and δ' are separately regular in the limit $\epsilon \rightarrow 0$, as must be the case because in general $\alpha_s \neq \tilde{\alpha}_s$. One thus finds :

$$\begin{aligned} R_{e^+e^-} &= R_0 \left[\frac{3}{2} + \frac{3 \alpha_s(Q^2)}{\pi} - \frac{\tilde{\alpha}_s(Q^2)}{\pi} \right] \\ &\Rightarrow R_0 \left[\frac{3}{2} + 2 \frac{\alpha_s(Q^2)}{\pi} \right] \end{aligned} \tag{13}$$

where the last relation follows for $\alpha_s \approx \tilde{\alpha}_s$. The approach to the asymptotic limit for the contribution of SUSY particles to the correction to $R_{e^+e^-}$ is illustrated in Fig. 3. We notice that for massless gluinos the deviations from the asymptotic values of the various terms almost exactly compensate so that the results of the massless theory are remarkably accurate in this case for all values of Q^2 .

In conclusion, we have once more found that the effects of SUSY particles at low energies are remarkably suppressed and that supersymmetry is very effective in concealing its possible influence on measurable quantities.

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FIGURE CAPTIONS

Figure 1 Diagrams contributing to the hadronic cross-section at first order in α_s .

Figure 2 The correction δ to $R_{e^+e^-}$, defined by Eq. (1), below threshold for production of pairs of s-quarks, for various indicative values of the ratio of the gluino mass m to the s-quark mass M :
 $m/M = 0$ (dashed), $m/M = \frac{1}{2}$ (dot-dashed), $m/M = 1$ (solid).

Figure 3 The total correction $\delta + \delta'$ to $R_{e^+e^-}$, where δ and δ' are defined by Eq. (9), above threshold for production of pairs of s-quarks, for various indicative values of the ratio of the gluino mass m to the s-quark mass M : $m/M = 0$ (dashed), $m/M = \frac{1}{2}$ (dot-dashed), $m/M = 1$ (solid).

(a)

(b)

(c)

fig.1

fig. 2

fig.3