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COSMOLOGY FROM HIGHER-DIMENSIONAL GRAVITY

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A B S T R A C T

A scenario describing how the four-dimensional Friedmann cosmology may evolve from a model of gravity in $4+D$ dimensions which admits spontaneous compactification of D dimensions is presented. In particular, the possibility that the transition to the standard hot big bang model involves an inflationary phase is explored.

1. HIGHER-DIMENSIONAL COSMOLOGY

Kaluza and Klein ¹⁾ showed several decades ago how a unified description of gravity and electromagnetism becomes possible, starting from Einstein's theory of gravity in five dimensions. This idea has been generalized ²⁾⁻⁴⁾ to include also the weak and strong interactions. In these 4+D dimensional models D spacelike dimensions are spontaneously compactified. The symmetries of this "internal space" appear as gauge symmetries of the effective four-dimensional theory. Internal spaces admitting the gauge group $SU(3) \times SU(2) \times U(1)$ or grand unified groups have been proposed ^{3),4)}. Even though a fully realistic model still remains to be found, the idea is elegant enough to warrant serious investigations of some of its implications.

In this letter we discuss the cosmology of models of higher-dimensional gravity. [For an earlier discussion, see Ref. 5] For time scales sufficiently large compared with the Planck time, the cosmology is well approximated by the standard four-dimensional Friedmann universe. An attempt is made to describe part of the transition from the early D+4 dimensional universe to the standard cosmology which will be realized at later times. We will not address the problem of possible changes in the topology of space time which may happen at an even earlier time during this transition. We rather investigate the time evolution of the characteristic length scales of the internal space and three-dimensional space, assuming that at this stage of the evolution space-time has the topology of the final Friedmann universe. In particular, we explore the possibility that the universe underwent an inflationary phase ⁶⁾ during this transition. The inflation is characterized by an exponential expansion of three spacelike dimensions, while the characteristic length scale of the remaining D spacelike dimensions remains almost constant. Provided the exponential expansion lasts long enough, not only is the evolution of the asymmetry between the observed three spacelike dimensions and the internal dimensions described, but this transition also could provide a solution ^{6),7)} to several major cosmological puzzles !

A small causally connected region could have expanded sufficiently to become the whole observed universe. This would explain the observed homogeneity of our universe and the remarkable isotropy of the 3°K background radiation. The curvature of the three space dimensions decreases exponentially during the inflationary phase, thus explaining why the universe today is so flat or, equivalently, has become so old (measured in Planck time units) and why the observed mass density is so near the critical density. Our preliminary results suggest that an inflationary scenario can indeed be implemented within the framework of higher-dimensional gravity.

Higher-dimensional gravity contains, after dimensional reduction, a theory of scalars coupled to gravity. During the early evolution of the universe, the characteristic length scale of the internal space will vary with time. This corresponds to a time varying scalar field which is a singlet with respect to the four-dimensional gauge group. Only when this scalar singlet attains its final vacuum expectation value (VEV) does the effective four-dimensional cosmological constant vanish. Prior to this, however, the value of the scalar field is different from its VEV, and an effective positive cosmological constant is present for the four-dimensional gravity theory. The observed three space dimensions therefore expand exponentially in a de Sitter phase. The central idea of the inflationary universe, proposed earlier in the context of GUT phase transitions, may now be realized in the present framework.

One possible scenario depicting how "primordial inflation" of three space dimensions can arise is described as follows : at very early times the characteristic length scales of the internal space (S) and of the three-dimensional space (R_3) are of the same order of magnitude. Both may have attained, during this early evolution, a value larger than the final vacuum value for S , $\langle S \rangle$. At some time, S reaches a maximum and starts to fall again. When S is near its maximum, it changes slowly with time. During this period, the effective four-dimensional cosmological constant is positive, since S is different from $\langle S \rangle$, and R_3 expands exponentially. When S is sufficiently far away from its maximum, it may again follow a fast evolution towards $\langle S \rangle$ and oscillate around $\langle S \rangle$. These oscillations are quickly damped through the production of matter and radiation, thereby heating the universe and making a transition to the usual four-dimensional Friedmann universe. The heating produces the large entropy observed in the present universe. The latter part of the scenario is in complete analogy to a scalar field falling down from a local extremum of its "de Sitter potential" (where contributions of curvature to the scalar potential are taken into account) into the minimum of the potential at zero four-dimensional curvature. One can indeed give a description in this language.

2. THE MODEL

In order to obtain a four-dimensional Friedmann universe at later times, the $D+4$ dimensional model of gravity must fulfil several conditions ⁸⁾. For zero temperature, it must exhibit a "ground state" which is a direct product of four-dimensional Minkowski space and some internal space. The ground state solution must obey the $D+4$ dimensional field equations. Excitations of the

D+4 dimensional metric tensor above this ground state contain symmetric second rank tensors, vectors and scalars with respect to the four-dimensional general co-ordinate transformations. The four-dimensional gravitational interactions must be governed by a term linear in the four-dimensional curvature scalar with positive Planck's constant. The four-dimensional cosmological constant should vanish. The ground state must correspond at least to a local minimum of the scalar potential. Finally, all kinetic terms of the four-dimensional fields must have the right sign.

In pure D+4 dimensional gravity theories, the above conditions can only be met ⁸⁾ if terms containing more than two derivatives are included in the action ^{*}). As an example ⁸⁾, we shall discuss the following action :

$$S = - \frac{1}{V_0} \int d^4x \hat{e} (\alpha \hat{R}^2 + \beta \hat{R}_{\mu\nu} \hat{R}^{\mu\nu} + \gamma \hat{R}_{\mu\nu\lambda\sigma} \hat{R}^{\mu\nu\lambda\sigma} + \delta \hat{R} + \epsilon) \quad (1)$$

$\hat{R}_{\mu\nu\lambda\sigma}$ is the d dimensional curvature tensor and $\hat{e} = \sqrt{|g|}$ is the determinant of the vielbein. Co-ordinates are denoted by $x^{\hat{\mu}} = (x^{\mu}, y^{\alpha}) = (t, x^i, y^{\alpha})$ where x^i and y^{α} are spacelike co-ordinates of the three-dimensional space and the D dimensional internal space. The normalization constant V_0 is chosen to be the volume of the internal space for $S = \langle S \rangle$. Then $\delta(\epsilon)$ has dimension mass^2 (mass^4) and α, β, γ are dimensionless.

The action (1) should be considered as an effective action at some distance scale close to the Planck scale. All contributions from quantum fluctuations which can be cast in the form (1) are supposed to be included (they may even be mainly responsible for the appearance of higher derivative terms). Of course, the full effective theory would also contain even higher derivative terms like R^4 , etc. We should consider Eq. (1) as an approximation which is valid so long as R is sufficiently smaller than the characteristic length scale $\delta^{-\frac{1}{2}}$. (It is difficult, at this stage, to give a precise estimate of the limits of validity, since this would require some knowledge about the coefficients of R^4 terms, etc., in the effective action.) Assuming that we can neglect these invariants, the remaining effects of the inclusion of quantum fluctuations will result in a scale dependence of the parameters $\alpha, \beta, \gamma, \delta$ and ϵ and in contributions violating general co-ordinate invariance in d dimensions due to spontaneous compactification. Both effects may be neglected here. The cosmology of this model is governed by the classical field equations derived from this action ⁹⁾ :

^{*}) An alternative possibility is the introduction of supplementary matter fields in the D+4 dimensional action. We expect that some aspects of the discussion will also apply to this kind of model.

$$\begin{aligned}
 & 2\alpha \hat{R} \hat{R}_{,\mu\nu} + 2\beta \hat{R}_{\mu\sigma} \hat{R}_{,\nu}^{\hat{\sigma}} + 2\gamma \hat{R}_{\mu\sigma\tau\varphi} \hat{R}_{,\nu}^{\hat{\sigma}\hat{\tau}\hat{\varphi}} + \delta \hat{R}_{,\mu\nu} \\
 & - \frac{1}{2} \hat{g}_{\mu\nu} (\alpha \hat{R}^2 + \beta \hat{R}_{\hat{\sigma}\hat{\tau}} \hat{R}^{\hat{\sigma}\hat{\tau}} + \gamma \hat{R}_{\hat{\sigma}\hat{\tau}\hat{\varphi}\hat{\lambda}} \hat{R}^{\hat{\sigma}\hat{\tau}\hat{\varphi}\hat{\lambda}} + \delta \hat{R} + \epsilon) \\
 & + 2\alpha (\hat{g}_{\mu\nu} \hat{R}_{,\hat{\sigma}}^{\hat{\sigma}} - \hat{R}_{,\mu\nu}) + \beta (\hat{g}_{\mu\nu} \hat{R}_{,\hat{\sigma}\hat{\tau}}^{\hat{\sigma}\hat{\tau}} + \hat{R}_{\mu\nu,\hat{\sigma}}^{\hat{\sigma}} \\
 & - \hat{R}_{\mu,\hat{\sigma}}^{\hat{\sigma}} - \hat{R}_{\nu,\hat{\sigma}}^{\hat{\sigma}}) + 2\gamma (\hat{R}_{\mu\nu,\hat{\sigma}\hat{\tau}}^{\hat{\sigma}\hat{\tau}} + \hat{R}_{\mu,\hat{\sigma}\hat{\tau}}^{\hat{\sigma}\hat{\tau}} - \hat{R}_{\nu,\hat{\sigma}\hat{\tau}}^{\hat{\sigma}\hat{\tau}}) = \frac{1}{2} \hat{T}_{\mu\nu}
 \end{aligned} \tag{2}$$

The quantity $\hat{T}_{\mu\nu}$ on the right-hand side of Eq. (2) is the D+4 dimensional generalization of the energy momentum tensor. It includes all incoherent excitations and vanishes for zero entropy.

The conditions stated previously for obtaining a four-dimensional Friedmann universe are met for the parameter values ⁸⁾

$$\begin{aligned}
 \mathcal{G} &= D(D-1)\alpha + (D-1)\beta + 2\gamma > 0 \\
 \beta + \frac{4}{D-2} \gamma &> 0 \\
 \gamma > 0, \quad \delta > 0 \\
 \epsilon &= \frac{1}{4} \delta^2 \frac{D(D-1)}{\mathcal{G}} \\
 (D-1)\beta + 2\gamma &> 0
 \end{aligned} \tag{3}$$

The field equations (2) admit the manifold $S^D \times \mathcal{M}^4$ as a zero temperature solution, where the radius $\langle S \rangle$ of the internal D dimensional hypersphere S^D is given by

$$\langle S \rangle^2 = \frac{2\mathcal{G}}{\delta} \tag{4}$$

A four-dimensional effective theory is obtained by integrating out the coordinates of S^D . This effective theory exhibits $SO(D+1)$ gauge invariance. The ground state corresponds to an absolute minimum of the scalar potential. The Planck mass M_P is given by

$$M_P^2 = 16\pi [(D-1)\beta + 2\gamma] \delta / \mathcal{G} \tag{5}$$

and the four-dimensional cosmological constant vanishes.

3. DE SITTER SOLUTIONS

For early times, R_3 and S are both of the order of the Planck length and the cosmology is best described by using directly the D+4 dimensional field equations (2). In general, this system of equations is quite complicated and admits a large variety of solutions. For simplicity, we will restrict ourselves to solutions where space is homogeneous and isotropic separately in the usual three dimensions and in the internal D dimensions. At a given time, space is a direct product of S^3 and S^D , but the radii $R_3(t)$ of S^3 and $S(t)$ of S^D are time dependent. [The discussion can easily be adapted to an open four-dimensional universe. $R_3(t)$ corresponds to the scale factor in the Robertson-Walker metric.] With this ansatz, one finds

$$\begin{aligned}\hat{R}_{\alpha\beta\gamma\delta} &= -(\dot{S}^{-2} + \ddot{S}^2 S^{-2})(\hat{g}_{\alpha\gamma} \hat{g}_{\beta\delta} - \hat{g}_{\alpha\delta} \hat{g}_{\beta\gamma}) \\ \hat{R}_{ijkl} &= -(R_3^{-2} + \dot{R}_3^2 R_3^{-2})(\hat{g}_{ik} \hat{g}_{jl} - \hat{g}_{il} \hat{g}_{jk}) \\ \hat{R}_{\alpha i \beta j} &= -\dot{S} S^{-1} \dot{R}_3 R_3^{-1} \hat{g}_{\alpha\beta} \hat{g}_{ij} \\ \hat{R}_{\alpha\beta\gamma 0} &= -\ddot{S} S^{-1} \hat{g}_{\alpha\beta} \\ \hat{R}_{i0j0} &= -\ddot{R}_3 R_3^{-1} \hat{g}_{ij}\end{aligned}\tag{6}$$

($\hat{R}_{\mu\nu\alpha\lambda} = 0$ for all other index combinations except those obtained by index permutations from the above expressions.) Inserting these expressions in the field equation (2), we obtain three independent equations ^{*)} for $S(t)$ and $R_3(t)$.

Since we are especially interested in an exponential expansion of the usual three space dimensions, we first look for de Sitter solutions in four-dimensional space time :

$$\begin{aligned}S(t) &= S_H = \text{const.} \\ \dot{R}_3^2 / R_3^2 &= H^2 - R_3^{-2}\end{aligned}\tag{7}$$

*) In the following we will concentrate on solutions where temperature effects can be neglected ($\hat{T}_{\mu\nu} = 0$). For a more general discussion, the time dependent temperature is a third independent function and the three equations will have a still larger variety of solutions, depending on the initial conditions.

Indeed, such solutions do exist ^{*}) (for $\hat{T}_{\mu\nu} = 0$) and the field equations (2) are fulfilled for

$$H^2 = \frac{1}{24} \delta \frac{D(D-1)}{\rho} (1-z)^2 (1+\sigma z)^{-1} \quad (8)$$

where

$$z = \frac{\langle S \rangle^2}{S_H^2} \quad (9)$$

is determined by

$$a_1 z^3 + a_2 z^2 + a_3 z + a_4 = 0 \quad (10)$$

with

$$\begin{aligned} a_1 &= D(D-1) + D(D-1)\rho + [D(D-1)-12]\sigma - 12\sigma^2 \\ a_2 &= -3D(D-1) - 3D(D-1)\rho - \frac{3}{D}[D^2(D-1)-4D+16]\sigma + 12\frac{D-4}{D}\sigma^2 \\ a_3 &= \frac{3}{D}[D^2(D-1)-4D-16] + 3D(D-1)\rho + \frac{3}{D}[D^2(D-1)-4D-16]\sigma \\ a_4 &= -D(D-1)+12 - D(D-1)\rho - [D(D-1)-12]\sigma \end{aligned} \quad (11)$$

and

$$\begin{aligned} \sigma &= -D(D-1)\alpha / \rho \\ \rho &= -(D-4)\beta / \rho \end{aligned} \quad (12)$$

The Hubble constant H is the characteristic scale for the exponential expansion. Compared with the Planck scale, it is given by

$$\frac{H^2}{M_p^2} = \frac{1}{384\pi} \frac{D(D-1)}{(D-1)\delta + 2\gamma} (1-z)^2 (1+\sigma z)^{-1} \quad (13)$$

^{*}) De Sitter solutions in four-dimensional gravity theories including higher derivative invariants have been discussed by Starobinski ¹⁰).

Note that for not too small values of $(D-1)\beta+2\gamma$ this ratio may be considerably smaller than 1. If z is also small, all relevant mass scales of such a de Sitter solution are well below the Planck mass ! Note that for

$$\gamma = \frac{1}{2} \left(\frac{12}{D} - 3 \right) \left(1 - \frac{12}{D(D-1)} \right)^{-1} \beta \quad (14)$$

one has $a_4 = 0$ and therefore $z=0$ is a solution of (10). We will come back to this interesting case when we consider values of γ near the value (14).

4. PRIMORDIAL INFLATION

For a given set of parameters α, β, γ we may have up to three de Sitter solutions corresponding to the solutions of Eq. (10). However, there is no special reason why the $D+4$ dimensional universe should have started in any of the de Sitter solutions. Imagine, however, that $S(t)$ and $R_3(t)$ are both roughly of order M_P^{-1} and evolve more or less randomly in phase space. Different regions of space-time may have different values of S and R_3 . Assume now that at some time one space-time region comes close in phase space to one of the above de Sitter solutions. If this region stays long enough in the vicinity of such a solution the usual three space dimensions expand exponentially during this time and the whole observed universe could have evolved from such a region. The solution of the cosmological puzzles in the inflationary scenario requires that the time t_c during which the universe is in a phase of exponential expansion exceeds by a factor 60 or more the inverse Hubble constant (8) :

$$t_c \gtrsim 60 H^{-1} \quad (15)$$

In order to estimate t_c we consider solutions of the field equations (2) which describe small deviations from the de Sitter solutions. We define

$$\begin{aligned} s(t) &= \ln(S(t)/S_H) \\ v(t) &= \ln(R_3(t)/\bar{R}_3) - u(t) \end{aligned} \quad (16)$$

where $u(t)$ corresponds to a de Sitter solution with $\dot{u}(t)^2 = H^2 - \bar{R}_3^{-2} \exp(-2u(t))$. Since $u(t)$ grows with a rate given by H , we may neglect all factors $\exp(-u(t))$ after a relatively short time in the de Sitter phase. Since we

are interested in small deviations from the de Sitter solution, we first linearize the field equations in s and v . Furthermore, we are interested in solutions with a slow time dependence and we may therefore neglect the higher time derivatives appearing in the linearized field equations. Since $\dot{v} \sim Hs$, a consistent approximation to the field equations near a de Sitter solution is given as follows :

$$\begin{aligned} c_1 s + c_2 \dot{s} + c_3 \dot{v} + c_4 \ddot{v} &= 0 \quad (\text{from } \hat{T}_{00} = 0) \\ c_1 s + (c_2 - b_2) \dot{s} + c_3 \dot{v} + (c_4 + b_1) \ddot{v} &= 0 \quad (\text{from } \hat{T}_{ij} = 0) \\ d_1 s + d_2 \dot{s} + d_3 \dot{v} + d_4 \ddot{v} &= 0 \quad (\text{from } \hat{T}_{\alpha\beta} = 0) \end{aligned} \quad (17)$$

The constants b_i , c_i and d_i are calculated as functions of the parameters α , β , γ and δ .

This system of equations has a solution ^{*)}

$$\begin{aligned} s &= s_0 \exp \omega t \\ v &= \frac{b_2}{b_1} s_0 \exp \omega t \end{aligned} \quad (18)$$

which is within the validity of our approximations, provided $\omega \equiv t_c^{-1} \ll H$. In this case, it approximates the evolution towards or away from a de Sitter solution, depending on the sign of ω . A sufficient inflation requires $\omega/H \lesssim 1/60$. One finds

$$\frac{\omega}{H} = \frac{4z(1+\sigma)}{D} \frac{(1+\sigma z)^2}{(1-z)^2} \frac{\tilde{A}}{\tilde{B}} \quad (19)$$

with

*) Two linear combinations of the three equations (17) are fulfilled by the construction of the solution. The third will then give a restriction on the parameters α , β , γ and δ for which the zero temperature approximation is valid for the solutions near the de Sitter solutions. Weakening this restriction, forces us to include temperature effects in our discussion. This makes the calculations somewhat longer, but we do not expect that our answer depends crucially on the zero temperature approximation.

$$\tilde{A} = (1 + \sigma z)(1 - z) - \frac{z}{D} [1 + 2(1 + \sigma)z + \sigma z^2] \quad (20)$$

and $\tilde{B}$ a lengthy function of σ , and z typically of the order of unity. The critical quantity ω/H is small for small z , large D , small $\tilde{A}/\tilde{B}$ and σ near -1 . Without fine tuning of parameters, it seems possible to obtain $\omega/H \lesssim 1/60$. Small values of z [see Eq. (14)] are especially interesting, since in this case all length scales are large compared to the Planck length during the de Sitter phase.

To conclude, our results suggest that sufficient inflation can occur in theories of higher-dimensional gravity. More work is needed to establish a complete scenario of the transition to the Friedmann universe. The mechanism for the generation of entropy (heating the universe after the expansion) remains to be clarified. The scalar corresponding to S is a singlet of the gauge group and therefore decouples from the massless gauge bosons (and the chiral fermions in a more realistic model). However, in a more elaborate model some non-singlet scalars must have masses many orders of magnitude below the Planck mass in order to induce spontaneous symmetry breaking down to the $SU(3)_c \times U(1)_{em}$ gauge group. Provided the couplings of these scalars to the singlet are not too small, they will be copiously produced by the oscillations of S around $\langle S \rangle$. Their decay into gauge bosons and fermions will then quickly establish thermodynamic equilibrium. Other questions concern the generation of primordial density fluctuations and the possibility that a GUT phase transition may occur after the transition to the Friedmann universe.

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