Ref.TH.3608-CERN

ON SOME MESON-BARYON THREE-POINT FUNCTION SUM RULES IN QCD *)

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A B S T R A C T

We discuss the applications of the Laplace transform sum rules to the three-point functions involving two baryons and one meson. Evaluating the QCD part of these three-point functions in the symmetrical subtraction configuration, where all external masses are equal and much larger than Λ_{QCD} , we obtain relations between the pseudoscalar meson-nucleon couplings ($g_{\pi NN}$, $g_{\eta NN}$), the vector meson-nucleon tensor couplings ($G_{\rho NN}^T$, $G_{\omega NN}^T$) to quark masses and quark vacuum condensates.

*) Supported in part by the Istituto Nazionale di Fisica Nucleare, Sezione di Trieste.

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1. - INTRODUCTION

Recent progress has been made in extending the application of the Laplace transform sum rules, originally proposed by SVZ ¹⁾, to the case of three-point functions of currents and of current divergences, with the aim of analyzing, in the context of QCD, the phenomenological situation concerning meson and baryon vertices. The method, whose outstanding features are, on the one hand, the improvement of the lowest state contribution to the sum rule by an exponential factor and on the other, the inclusion of non-perturbative terms into the operator product expansion, seems to be promising.

In a previous paper ²⁾ we analyzed some examples relevant to meson tri-linear vertices with satisfactory results (as far as the orders of magnitude are concerned), despite the approximations used (leading order in QCD and lowest meson contributions). The analysis was performed by working in the condition of symmetrical subtraction $p^2 = q^2 = (p+q)^2 = -Q^2$ and $Q^2 \gg \Lambda_{\text{QCD}}^2$ (see Fig. 1), in order to have an amplitude automatically free of mass and infra-red singularities ³⁾, estimating in this configuration the QCD expression of the full (not just the imaginary part) three-point function, and finally applying the familiar technique leading to the Laplace (or Borel) transform sum rules. Also, some discussion was given about the assumption that the lowest states continue to dominate the sum rule, in this more complicated situation, as in the case of two-point functions.

In this paper, we would like to extend such an analysis to the case of three-point functions involving two baryons and one meson. For simplicity, we will limit ourselves to the three-point functions which are associated with the spin $\frac{1}{2}$ nucleon coupled to pseudoscalar mesons and vector mesons. Then, we try to relate the couplings $g_{\pi NN}$, $g_{\eta_8 NN}$, $G_{\rho NN}^T$ and $G_{\omega NN}^T$ along the lines of Ref. 2), to the typical QCD parameters represented by quark masses and vacuum condensates.

2. - ON THE CHOICE OF THE NUCLEON OPERATORS

As is well known, there is some freedom in the definition of composite operators to describe nucleons in terms of quark fields. According to the analysis of Ref. 4), one can choose for the proton, without loss of generality, the following basis of lowest dimensional independent operators

$$O_1^P(x) = \epsilon_{abc} (u^a(x) \otimes d^b(x)) \gamma_5 u^c(x) , \quad (1)$$

$$O_2^P(x) = \epsilon_{abc} (\bar{u}^a(x) \mathbb{C} \gamma_5 d^b(x)) u^c(x), \quad (2)$$

where a, b, c are colour indices and $\mathbb{C}$ is the charge conjugation matrix ^{*)}. The operators with the quantum numbers of the neutron are obtained by replacing up quarks by down quarks and vice versa. Other composite operators for the nucleon can be reduced to a combination of O_1 and O_2 by a Fierz re-arrangement. One of the advantages of O_1 and O_2 is that they do not mix under renormalization, at least at the one-loop level : their renormalization group invariant analogue is

$$\sqrt{m_R} (O_{1,2}(x))_R = \sqrt{m_B} (O_{1,2}(x))_B, \quad (3)$$

where m is any quark mass and B, R mean bare and renormalized, respectively. We shall work in the following with the general combination

$$O(x) = O_1(x) + t O_2(x) \quad (4)$$

with t a real parameter, not known a priori. The values of t which emerge from consideration of two-point baryon correlation functions are not so uniquely determined. In Ref. 5), for example, the value $t \approx -1$ is suggested whereas $t \approx -0.2$ was obtained in Ref. 6) ^{**)} . Also, there seems to be no fundamental reason to expect, in the case of three-point functions, the same value of t indicated by the analysis of two-point functions ^{***)} . We will thus leave in what follows t as a free parameter.

3. - PSEUDOSCALAR MESON COUPLINGS TO NUCLEONS

In this case, the relevant three-point function is :

$$T(p, q) = \int d^4x d^4y e^{i(p \cdot x + q \cdot y)} \langle 0 | T O(x) \bar{J}_p(0) \bar{O}(y) | 0 \rangle \quad (5)$$

^{*)} We shall work with the following properties of the $\mathbb{C}$ matrix :
 $\mathbb{C} \gamma_\mu \mathbb{C}^{-1} = -\gamma_\mu^T$ and $\mathbb{C}^2 = -1$.

^{**)} Some criticisms on the present existing choices of the nucleon operators are discussed in Ref. 4).

^{***)} Of course, the ideal situation should be that the physical parameters derived theoretically do not depend (or at least have a little dependence) on t .

where, for $P = \eta, \pi$:

$$J_\pi = im : \bar{u} \gamma_5 u - \bar{d} \gamma_5 d : , \quad (6a)$$

$$J_\eta = im : \bar{u} \gamma_5 u + \bar{d} \gamma_5 d : - 2im_s : \bar{s} \gamma_5 s : . \quad (6b)$$

In Eq. (6), $m \equiv m_u = m_d$ and m_s are the quark masses of the QCD Lagrangian. The pseudoscalar current is normalized such that :

$$\langle 0 | J_P | P \rangle = m_P^2 f_P , \quad (7)$$

with $f_\pi \approx 93$ MeV and $f_\eta = \sqrt{3} f_\pi$ in the SU(3) limit.

The leading QCD contribution to Eq. (5) is represented in Fig. 2. In the symmetrical subtraction configuration $p^2 = q^2 = -2pq = -Q^2$, $Q^2 \gg \Lambda_{\text{QCD}}^2$ one finds in the chiral limit (we are using the dimensional regularization scheme in $n \equiv 4-\epsilon$ space-time dimensions ^{*)}),

$$T(p, q) = i \gamma_5 (\hat{p} + \hat{q}) \frac{1}{2} m \langle \bar{\psi} \psi \rangle \Gamma(\epsilon/2) \left(\frac{Q^2}{2} \right)^{-\epsilon/2} \left(-\frac{1}{16\pi^2} \right) \cdot \left[(6 \mp 2) + (9 \pm 1)t + (3 \pm 1)t^2 \right] \quad (8)$$

for the currents in Eq. (6) [the upper signs in Eq. (8) correspond to the pion current and the lower one to the η current], where $\langle \bar{\psi} \psi \rangle \equiv \langle u\bar{u} \rangle = \langle d\bar{d} \rangle$ is the quark vacuum condensate. From simple dimensional arguments, it is easy to see that Fig. 2 really represents the leading QCD contribution to Eq. (5). The gluon condensate contribution would be suppressed by the smallness of the light quark mass as is also the case for the quark mass corrections. The $\langle \bar{s}s \rangle$ component of the meson cannot play a rôle in our leading order analysis and to a first order approximation, we neglect the effect of the η - η' mixing. The phenomenological information on $T(p, q)$ can be expressed by retaining only the nucleons and meson poles, as :

$$T(p, q) = i \gamma_5 (\hat{p} + \hat{q}) \left(\frac{m_P^2 f_P}{Q^2 + m_P^2} \right) \frac{\lambda_N^2}{(Q^2 + M_N^2)^2} g_{PN\bar{N}} \cdot M_N , \quad (9)$$

^{*)} For a review, see, e.g., Ref. 7). We have used, in n dimensions, the fact that $(\gamma_5)^2 = 1$ and $\gamma_\mu \gamma_5 = -\gamma_5 \gamma_\mu$.

where λ_N is the coupling of the nucleon to its current :

$$\langle 0 | O(x) | N \rangle = \lambda_N u_n(p) . \quad (10)$$

$O(x)$ is the operator defined in Eq. (4), $|N\rangle$ is the nucleon state and $u_n(p)$ is the Dirac spinor. In fact, λ_N is known from the nucleon two-point function sum rule as ^{4),6),*)} :

$$\lambda_N^2 \approx \frac{1}{(2\pi)^4} e^{M_N^2/M^2} (M^2)^3 \frac{1}{64} (5+2t+5t^2) \quad (11)$$

or

$$\lambda_N^2 \approx \frac{1}{(2\pi)^4} e^{M_N^2/M^2} (M^2)^2 \frac{\pi^2}{4} \frac{\langle \bar{u}u \rangle}{M_N} (5+2t-7t^2) . \quad (12)$$

Multiplying Eqs. (8) and (9) by Q^2 much larger than m_p , and applying to them the Laplace transform operator :

$$\hat{L} \equiv (-1)^N (Q^2)^N \frac{1}{(N-1)!} \frac{\partial^N}{(\partial Q^2)^N} , \quad (13)$$

it becomes easy to get the following leading order constraints :

$$\int_P m_p^2 \lambda_N^2 M_N g_{\pi N \bar{N}} \frac{1}{(M^2)^3} e^{-M_N^2/M^2} \approx \frac{1}{32\pi^2} m \langle \bar{u}u \rangle \cdot \left\{ (6 \mp 2) + (9 \pm 1)t + (3 \pm 1)t^2 \right\} . \quad (14)$$

Using the well-known PCAC relation

$$m \langle \bar{\psi} \psi \rangle \approx - \frac{1}{2} m_\pi^2 f_\pi^2 \quad (15)$$

and, for example, the expression for λ_N^2 in Eq. (11), we deduce

$$g_{\pi N \bar{N}} \approx \left(\frac{f_\pi}{M_N} \right) (16\pi^2) \left(\frac{8+8t+2t^2}{5+2t+5t^2} \right) . \quad (16)$$

*) We work with only the leading expression of λ_N^2 for consistency.

Notice that because of this freedom of parametrizing λ_N^2 according to Eqs. (11) and (12), it may be misleading to compare ⁸⁾ the analytic form of $g_{\pi NN}$ obtained from the sum rule to the one which would be obtained from the low energy behaviour of the pion-nucleon amplitude, the well-known Goldberger-Treiman relation.

Using the average $g_{\pi NN} \approx 13.3$ ⁹⁾ and $M_N \approx 939$ MeV, Eq. (16) allows us to find the value of t which we shall use in the subsequent analysis :

$$t \approx -0.51 \quad \text{or} \quad t \approx 3.38 . \quad (17)$$

If instead, we use the expression of λ_N^2 in Eq. (12), we obtain, from Eq. (14), an interesting constraint between the running renormalization group invariant improved quark mass ⁷⁾ and the pion-nucleon coupling :

$$\bar{m}_u(M^2) \approx \left(g_{\pi N \bar{N}} \frac{f_\pi}{2} \right) \left(\frac{m_\pi^2}{M^2} \right) \left(\frac{5 + 2t - 7t^2}{8 + 8t + 2t^2} \right) . \quad (18)$$

This relation shows that $\bar{m}_u$ vanishes, in the chiral limit, like m_π^2 , as usually expected. Evaluating for example the running mass at $M \approx 1$ GeV where we hope that the unknown corrections to the leading term are small, we get :

$$\bar{m}_u(1 \text{ GeV}) \approx 12.3 \text{ MeV} \quad (19)$$

The corresponding value of the invariant quark mass, defined to one loop is :

$$\hat{m}_u \equiv \bar{m}_u(M^2) \left(\log M/\Lambda \right)^{4/9} \approx 16.3 \text{ MeV} . \quad (20)$$

for $\Lambda \approx 150$ MeV. The result is perfectly consistent with the bound obtained in Refs. 10). An improvement of our result in Eq. (18) needs the evaluation of radiative, quark mass and non-perturbative corrections as well as the inclusion of the contribution of excitations of the higher states.

Equation (14) also allows us to get the ratio between the π and η_8 coupling to the nucleons :

$$\frac{g_{\eta N \bar{N}}}{g_{\pi N \bar{N}}} \approx \left(\frac{m_\pi^2 f_\pi}{m_\eta^2 f_\eta} \right) \left(\frac{4 + 10t + 4t^2}{8 + 8t + 2t^2} \right) . \quad (21)$$

The values of $t = -0.51$ and 3.38 quoted in Eq. (17) give, respectively, the values 0 and 0.06 for the ratio in Eq. (21). The data ⁹⁾ indicate a value of the ratio of coupling constants of about 0.5 and differs therefore by a

factor of about 10 from the most favoured value corresponding to $t = 3.38$. We conclude that it is difficult to give within the actual approximation used here (or probably with the only valence quark model) a better prediction of the ratio given in Eq. (21) than obtained. One can remark, however, in this context, that the $\bar{s}s$ component of Eq. (6b), although not directly coupled to nucleons at the leading order, might give, in principle, an appreciable non-leading contribution owing to the factor $m_s \gg m$.

4. - TENSOR COUPLINGS OF VECTOR MESONS TO NUCLEONS

In this case, the relevant three-point function is :

$$A^\mu(p, q) = \int d^4x d^4y e^{i(p \cdot x + q \cdot y)} \langle 0 | T O(x) J^\mu(0) \bar{O}(y) | 0 \rangle \quad (22)$$

where $O(x)$ is the nucleon operator introduced in Eq. (4) and :

$$J^\mu \equiv \frac{1}{2} : \bar{u} \gamma^\mu u \pm \bar{d} \gamma^\mu d : \quad (23)$$

are the $\omega(\rho)$ meson currents. According to Lorentz and general invariance principles, we can decompose Eq. (22) as :

$$A^\mu(p, q) = F_1(p, q) \gamma^\mu - F_2(p, q) [\gamma^\mu, \hat{p} + \hat{q}] + \dots \quad (24)$$

In the following, we shall be interested in the magnetic form factor $F_2(p, q)$. From a dimensional analysis and working in the chiral limit ($m_u = m_d = 0$), the leading QCD contribution to the form factor comes from the quark condensate diagram in Fig. 2. In the configuration of symmetrical subtraction of Fig. 1, we get in $4-\epsilon$ space-time dimensions, to leading order in Q^2 :

$$F_2(Q^2) = \left(\frac{i}{16\pi^2} \right) \Gamma(\epsilon/2) \left(\frac{Q^2}{\mu^2} \right)^{-\epsilon/2} \frac{1}{4} \langle \bar{u} u \rangle \cdot \left\{ \left(-3 \mp \frac{7}{12} \right) + \left(1 \pm \frac{1}{6} \right) t + \left(2 \pm \frac{5}{12} \right) t^2 \right\} \quad (25)$$

for the currents in Eq. (23).

We parametrize the phenomenological part of the amplitude A^μ by taking to a first approximation the lowest meson contributions. Then we can write under the symmetrical configuration in Fig. 1 :

$$A^\mu(p, q) = \left(\frac{M_V^2}{2\gamma_V} \right) \frac{1}{(Q^2 + M_V^2)} \frac{\lambda_N^2}{(Q^2 + M_N^2)^2} \cdot (\vec{p} + M_N) \cdot$$

$$\cdot \left\{ \frac{1}{2} G_{VNN}^V \gamma^\mu - \frac{G_{VNN}^T}{8M_N} [\gamma^\mu, \vec{p} + \vec{q}] \right\} (-\vec{q} + M_N) \quad (26)$$

where γ_V is the coupling of the vector meson V to its current :

$$\langle 0 | J_V^\mu | V \rangle = \epsilon^\mu \left(\frac{M_V^2}{2\gamma_V} \right) \quad (27)$$

G_{VNN}^V and G_{VNN}^T are, respectively, the vector and tensor couplings of the vector mesons to the nucleons. Keeping the leading term in Q^2 it is easy to find from Eq. (26) that the only term proportional to $[\gamma^\mu, \hat{p} + \hat{q}]$ comes from G_{VNN}^T . Then, we have, after identification of the $[\gamma^\mu, \hat{p} + \hat{q}]$ leading term in Eqs. (24) and (26) *) :

$$\left(\frac{G_{VNN}^T}{M_N} \right) \left(\frac{M_V^2}{2\gamma_V} \right) \frac{\lambda_N^2}{(Q^2 + M_N^2)^2} \approx \frac{1}{16\pi^2} \Gamma(\epsilon/2) \left(\frac{Q^2}{J_2} \right)^{-\epsilon/2} \cdot \langle \bar{\psi} \psi \rangle \cdot$$

$$\cdot 2 \left\{ \left(-3 \mp \frac{7}{12} \right) + \left(1 \pm \frac{1}{6} \right) t + \left(2 \pm \frac{5}{12} \right) t^2 \right\} \quad (28)$$

Applying to both sides of Eq. (28) the Laplace transform operators, we deduce :

$$G_{VNN}^T \left(\frac{M_V^2}{2\gamma_V} \right) \frac{\lambda_N^2 e^{-M_N^2/M^2}}{(M^2)^2} \approx \left(\frac{M_N}{16\pi^2} \right) \langle \bar{u} u \rangle 2 \left\{ \left(-3 \mp \frac{7}{12} \right) + \left(1 \pm \frac{1}{6} \right) t + \left(2 \pm \frac{5}{12} \right) t^2 \right\} \quad (29)$$

Then we deduce, for the ω/ρ ratio of tensor couplings :

$$\frac{G_{\omega NN}^T}{G_{\rho NN}^T} \approx \left(\frac{-43 + 14t + 29t^2}{-29 + 10t + 19t^2} \right) \quad (30)$$

*) To a first approximation, we have assumed $M_V \approx M_N$.

Introducing, for example, Eq. (12) into Eq. (29), we have :

$$G_{\rho NN}^T \approx \left(\frac{M_N}{M_\rho} \right)^2 \gamma_\rho \left(\frac{4}{3} \right) \left(\frac{-29 + 10t + 19t^2}{5 + 2t - 7t^2} \right). \quad (31)$$

We fix the value of t in this process by using the data on the ratio of the ω and ρ tensor couplings which is about 0.1⁹⁾. Then, from Eq. (30), we get :

$$t \approx -1 \quad \text{or} \quad t \approx -1.48. \quad (32)$$

Using these values of t in Eq. (31) and the value $\gamma_\rho \approx 2.5$, we deduce, respectively :

$$G_{\rho NN}^T \approx 24 \quad \text{or} \quad G_{\rho NN}^T \approx 0.8. \quad (33)$$

The first value of $G_{\rho NN}^T$ is favoured by the data which indicate an average value of about 30. If, instead of Eq. (32), we used the value of t with which we have fitted the pseudoscalar coupling to the nucleon [Eq. (17)], we would find, for the ratio in Eq. (30), a value of about 1 while the $G_{\rho NN}^T$ coupling would be 61 or 16 for the values of t about -0.5 and 3.88.

5. - CONCLUDING REMARKS

We have analyzed, using the Laplace transform sum rule, the coupling between one meson and a nucleon-antinucleon pair. We have been able to connect such couplings to the quark mass and to the quark vacuum condensate. We have left the mixing angle of nucleon composite operators as a free parameter. We have seen in these examples that the QCD leading approximation analysis can reproduce the order of magnitude of the experimental data of the $g_{\pi NN}$, $G_{\rho NN}^T$ and $G_{\omega NN}^T$ meson-nucleon coupling constants. However, the value of the ratio of the η and π couplings to the nucleons is difficult to explain with this leading order QCD approximation. Then, one can imagine other effects which may affect this ratio.

We have also seen that the value of t depends on the channels. The more favoured value of t within our framework is +3.88 in the pseudoscalar channel while it is -1 in the vector meson channel. The coincidence of the latter value to the one in the two-point function analysis of the nucleon current ^{4),5)} can be purely accidental. In fact, we do not find, to our knowledge, any fundamental reason for the universal value of t for various hadronic channels.

ACKNOWLEDGEMENTS

We thank Professor J.S. Bell for discussions and for carefully reading the manuscript. We are grateful to Professor J. Prentki for his interest in this work. N.P. thanks Professor G. Höhler for a correspondence.

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FIGURE CAPTIONS

Figure 1 Symmetrical subtraction configuration of the three-point function.

Figure 2 Leading order QCD contribution to the pseudoscalar and vector meson tensor couplings to the nucleon-antinucleon pairs.

⊗ indicates the nucleon operator and × the meson current insertion.

Fig. 1

Fig. 2