ARE THERE MULTIQUARK INTERACTIONS?

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ABSTRACT

The simultaneous interaction of two (or more) pairs of quarks/gluons leads at collider energies to pair-wise p_T -balanced 4- (or more) jet events. We determine their cross-section, which is at an appreciable level as compared to the 2-jet cross-section if the total transverse energy is not very large, and we analyze their kinematical characteristics. We expand on the implications and point to further possibilities for the search for such multiquark interactions.

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The production of pairs of hadron jets has recently been established at the CERN $p\bar{p}$ -collider¹⁾. Their cross-section follows the predictions of the parton model involving QCD corrections, and there is little doubt that a detailed analysis of the kinematical distributions will add further evidence in favour of perturbative QCD. Moreover, these assumptions suffice to determine the $q_1 + q_2 \rightarrow n$ -jet cross-sections.

The assumption of a conglomerate of quarks/gluons in the nucleon together with the hard constituent interactions being determined by perturbative QCD also allows for the simultaneous interaction of two (or more) pairs of quarks/gluons which, in lowest order perturbative QCD, will lead to : $(q_1 q_3) + (q_2 q_4) \rightarrow 4$ jets as shown in Fig. 1a. These types of processes have earlier been considered in Ref. 2a) for the normalization of the multiple-scattering cross-sections and in Ref. 2b) for a cross-section estimate of the double Drell-Yan process. In the context of the large- p_T processes, some qualitative features were pointed out in Ref. 2c), whereas the present paper, by analyzing in detail the large- p_T 4-jet process, presents the first quantitative results.

The differential cross-section for the 4-jet process shown in Fig. 1 is given by

$$d\sigma = \sum_{q/g} \frac{d\sigma_{12}}{\pi R^2} V(x_1, x_3) V(x_2, x_4) \quad (1)$$

The $d\sigma_{ij}$ stands for the 2-jet cross-section. Choosing as kinematical variables the jet rapidities y_i and the transverse momenta $q_{t_i} \equiv q_i$,

$$d\sigma_{12} \equiv \frac{d\sigma}{dq_1 dy_1 dy_2} = (2q_1) x_1 x_2 \frac{\hat{d}\sigma}{d\hat{t}_1} \quad (2)$$

The $\hat{d}\sigma$ stands for the constituent cross-section with the subvariables $\hat{s}_1, \hat{t}_1, \hat{u}_1$, and analogously for the subprocess (34). Thus for each of the two subprocesses the familiar 2-jet kinematics³⁾ applies separately.

The $V(x_1, x_3)$ represent the momentum distributions of two constituents. We use the simple ansatz

$$V(x_1, x_3) = u(x_1, Q_1^2) u(x_3, Q_1^2) (1-x_1-x_3) \quad (3)$$

where the $u(x_1, Q_1^2)$ are the single quark/gluon momentum distributions involving (leading-log) QCD corrections⁴⁾. We have explicitly verified that this form constitutes an excellent approximation to the generalized Kutti-Weisskopf model (KWG)⁵⁾. This model assumes the infinity of quarks in

the nucleon to be uncorrelated apart from a strict observation of momentum conservation ; the confinement effects on each constituent type are described by the "primitive structure functions". This attempt at the multi-quark distributions has been successfully applied to the soft-hadronic processes^{6,7)}. The scale Q^2 in the single quark distributions is chosen as for the large- p_T 2-jet processes : $Q^2 = 2 \hat{s} \hat{t} \hat{u} / (\hat{s}^2 + \hat{t}^2 + \hat{u}^2)$.

The factor πR^2 in the denominator of Eq. (1) represents the total hadronic cross-section which is the estimate of the size of a hadron. The reason for this factor is as follows : given that a pair of constituents interacts, the probability that there is a second interaction process is proportional to the flux factor of the accompanying quarks ; these are confined to the hadrons and their flux is thus inversely proportional to the area of a hadron. A more accurate estimate requires more detailed information on the hadron structure. In our subsequent analysis we choose : $\pi R^2 = 100 \text{ GeV}^{-2}$.

The sum $\sum_{q/g}$ is taken over all quark/gluon combinations. For each of the two $2 \rightarrow 2$ subprocesses there are eight different reactions :

$$\begin{array}{ll}
 1 : & q_1 q_2 \rightarrow q_1 q_2 \\
 2 : & q_1 q_1 \rightarrow q_1 q_1 \\
 3 : & q_1 \bar{q}_1 \rightarrow q_2 \bar{q}_2 \\
 4 : & q_1 \bar{q}_1 \rightarrow q_1 \bar{q}_1 \\
 5 : & q \bar{q} \rightarrow gg \\
 6 : & gg \rightarrow q \bar{q} \\
 7 : & qg \rightarrow qg \\
 8 : & gg \rightarrow gg
 \end{array} \quad (4)$$

with their cross-sections (in lowest order QCD) listed in Ref. 8. The significant contributions in hadronic 2-jet production come from the subprocesses 1,4,7,8 in the above list. We therefore limit our subsequent 4-jet analysis to this set.

We have carried out a numerical analysis of the 4-jet differential cross-section : $d\sigma/(dq_1 dy_1 dy_2)/(dq_2 dy_3 dy_4)$, and we have also integrated over its transverse momenta q_1 and q_2 . The results are presented in Figs. 2-4 where the size of the analogous 2-jet cross-section is also indicated.

We now proceed to a comparison of the cross-section size of the 2-jet and 4-jet processes. We therefore integrate over the transverse momenta of the 4-jet differential cross-section by varying at the same time the lower integration boundaries

$$q_1 \geq q_{1,\min} \equiv x Q_T, \quad q_2 \geq q_{2,\min} \equiv (1-x) Q_T \quad (5)$$

such that their sum : $Q_T = q_{1,\min} + q_{2,\min}$ remains fixed at the chosen

value for Q_T . All rapidities satisfy $y_i = 0$. The results are shown in Fig. 2. The Q_T values are indicated at the bottom of each curve. The fraction x shows whether the boundaries are equal ($x = 0.5$), or whether one boundary ($q_{1,\min}$ say) is large and the other ($q_{2,\min}$ consequently) is small ($x \sim 0.8$). Our results indicate that two jet-pairs, one with a large and the other with a small transverse momentum, are preferred. Furthermore, the smaller the cut-off values, the larger the cross-section. We have similarly indicated the 2-jet cross-section integrated over the transverse momentum with $q \geq Q_T$, at $y_i = 0$. The result is shown in Fig. 2, on the right-hand side. It becomes immediately obvious that the 4-jet cross-section, with unequal transverse momentum cut-offs, can reach values which are comparable to the corresponding 2-jet cross-section. This however becomes less likely the bigger Q_T . Comparing the 4-jet cross-section at equal cut-off values ($x = 0.5$) with the corresponding 2-jet cross-section, we immediately notice the growing predominance of the 2-jet over the 4-jet cross-section as the transverse energy grows. This result is in agreement with the experimental findings^{1,9)} that the mean values of the fractions $h_1 \equiv E_T^1 / \Sigma E_T$ and $h_2 \equiv (E_T^1 + E_T^2) / \Sigma E_T$, where E_T^1 (E_T^2) are the largest (second largest) transverse energy of the clusters in an event, tend towards 0.5 and 1.0 as one would expect for complete 2-jet dominance. From the experimental results presented at the Rome Workshop⁹⁾ we derive : at $\Sigma E_T = 50$ (100) GeV, $h_1 \approx 0.3$ (0.5), $h_2 \approx 0.45$ (0.8). Thus above $\Sigma E_T = 60$ GeV the number of 2-jet events will be predominant. Comparing in Fig. 2 with the corresponding curve (labelled 60 GeV) with the factors due to rapidity integration in the UA2-range taken into account, the number of 4-jet events is significantly below the number of 2-jet events even near $x \approx 0$ and 1.

For a numerical comparison we must consider the integrated cross-sections. We therefore integrate the 4-jet (2-jet) differential cross-sections over the transverse momenta : $q_{1,\min} \leq q_1$, $q_{2,\min} \leq q_2$ ($q \geq q_{1,\min} + q_{2,\min}$) with the actual values of the transverse momentum cut-offs $q_{i,\min}$ fixed by Eq. (5). The rapidity dependence and the strength of the rapidity correlations can be inferred from Fig. 4. We notice in particular that the cross-sections show moderate variation within the typical range of the experimental detectors : $-1 \leq y_i \leq +1$, which we choose for a comparison of the cross-sections. The integrated 4-jet(2-jet)

cross-section is 16 (4) times the corresponding rapidity cross-section at $y_i = 0$. The 4-jet/2-jet cross-section ratio is therefore given by

$$R \equiv \frac{\sigma_{4\text{-jet}}}{\sigma_{2\text{-jet}}} \bigg|_{-1 \leq y_i \leq +1} \approx 4 \frac{d\sigma_{4\text{-jet}}}{d\sigma_{2\text{-jet}}} \bigg|_{y_i = 0}$$

where the $d\sigma$ are read from Fig. 2. Typical findings are

$$R_{10+10} = \frac{1}{15}, R_{15+15} = \frac{1}{125}, R_{20+20} = \frac{1}{570}, R_{5+25} = \frac{1}{12}$$

where the R indices indicate the values for $q_{1,\min}$ and $q_{2,\min}$ in GeV units.

Since any uncertainty in the 2-jet cross-section will manifest itself in amplified form in the 4-jet cross-section, we consider it as important to keep clearly in mind the assumptions and approximations :

- i) the 2-constituent momentum distributions assume almost uncorrelated quarks/gluons. Leaving aside all QCD corrections, the simple ansatz (3) lies a factor 1.5-2 below the KWG-model but follows very well the shape of its predictions. To implement QCD corrections on the KWG-model⁵⁾ is highly non-trivial and has so far not been achieved.
- ii) the uncertainties in the QCD input momentum distributions manifest themselves in a substantially different behaviour at low x values, leading to uncertainties of a factor ~ 3 in the cross-section predictions for the 2-constituent processes. Such uncertainties could amplify in the 4-jet process although compensations can not be excluded. The QCD corrections are taken into account on the leading-log level ; all next-to-leading corrections are ignored. Such approximation might be acceptable for the phenomenology of the 2-jet processes. For the 4-jet process under consideration it could give rise to substantial errors. At large enough momentum transfer, however, the particular choice of the scale Q^2 as well as the next-to-leading corrections, are of minor importance.
- iii) there is a significant uncertainty in the particular choice of the interaction region πR^2 as a means for the constituent-flux. If the actual interaction radius should indeed be smaller, the 4-jet cross-section would be bigger.
- iv) the constituent processes 2,3,5,6 in (4) were ignored.
- v) the influence of the two constituent initiated order- α_s^4 4-jet process,

which is considered as a higher order QCD correction to the leading 2-jet process, is unknown.

In the following we enter more into the details of the transverse momentum and rapidity dependences and, towards the end, comment on the influence of this mechanism in other processes.

In Fig. 3 we expose the dependence on the transverse momentum keeping all rapidities central : $y_i = 0$ ($i = 1-4$). The obvious characteristic of this type of 4-jet processes is a pair-wise balance of the transverse energy. We therefore may limit ourselves to the transverse momentum of one jet ; q_1 for the (12)-pair and q_2 for the (34)-pair. In the first set of curves we have varied q_1 keeping q_2 fixed at 15 GeV (label $q_2 = 15$ GeV). These curves illustrate the relative importance of the various combinations of the quark/gluon subprocesses. Curve 6 $\equiv$ (8-8) for instance exposes the cross section $gg \rightarrow gg$ combined with $gg \rightarrow gg$. The numbers (i-j) refer to the subprocesses listed in (4) which are supposed to take place simultaneously. At lower q_1 -values ($q_1 \lesssim 30$ GeV) the cross-section is dominated by the (gg) and (qg) combinations (8-8) and (8-7) whereas those contributions involving $q\bar{q}$ and qq scattering are small. The latter type such as e.g. (7-1) becomes influential at large q values where the cross-section however is unmeasurably small. We conclude : (gg) and (qg) scattering only are relevant. At $q_1 = 15$ GeV the cross-section size is $d\sigma \sim 10^{-35} \text{ cm}^2/\text{GeV}^2$; it decreases as $\sim (1/q_1)^{6.4}$ whereas at large q_1 -values the fall-off is stronger and no longer power-behaved. The influence of the QCD corrections on these results via the scale dependent momentum distributions and the running coupling constant in the constituent processes is substantial. They change the slope as well as the relative size of these curves. In the second set of curves the transverse momentum of both jet pairs grows simultaneously (label $q_1 = q_2$). Apart from a substantial change in the slope, the aforementioned features remain unchanged. The cross-section fall-off, at $q_1 = 15$ GeV $\sim (1/q_1)^{10.7}$, is now much stronger since the transverse momentum of both subprocesses is varied. We have similarly evaluated the fall-off of the 2-jet process which, around 15 GeV, is as $\sim (1/q_1)^5$. Comparing with our earlier finding of a $(1/q_1)^{6.4}$ decrease for the 4-jet process (at $q_2 = 15$ GeV) one notices a discrepancy which is explained by the presence of the second jet-pair. There is little sense in comparing the size of the 2- and 4-jet cross-sections at this

level since they involve different dimensions and number of rapidity derivatives.

We have integrated the 4-jet (2-jet) cross-section over the transverse momenta with $q_1, q_2 \geq 15$ GeV ($q \geq 30$ GeV) and expose in Fig. 4 their rapidity dependence.

We analyze the characteristics of the 4-jet process and later compare with the analogous 2-jet results. The size of the different contributions of the quark/gluon subprocesses (i-j) is compared in Fig. 4b. The rapidities of one jet-pair are kept fixed : $y_3 = y_4 = 0$, whereas the others are varied in the same way : $y_1 = y_2$. As a result the subenergies $\hat{s}_1, \hat{s}_2$ remain fixed and all cross-section variation comes from the momentum distributions. Again we observe the predominance of the combinations $(8-8) \gtrsim (8-7) > (7-7)$ around $y_1 = 0$. At larger $|y_1|$ the contribution due to exclusive gg-scattering decreases rapidly below those involving qg-scattering. This is a consequence of the QCD corrections which exercise a damping influence on this subprocess. All contributions containing a qq or $q\bar{q}$ subprocess are not substantial. At $y_i = 0$ ($i = 1-4$) the sum and all contributions reaches almost : $d\sigma \sim 10^{-34}$ cm². Assuming a rapidity bin-width of ~ 0.2 , and the integrated luminosity at ~ 100 nb⁻¹, one may expect $N \sim 10^{-2}$ events around $y_i = 0$. If the rapidity range is enlarged to : $-1 \leq y_i \leq +1$ with the cross-section assumed to be almost constant at its $y_i = 0$ value, one may expect ~ 160 events. The large discrepancy between the two numbers for N results from the fact that there are four rapidities involved. The analogous numbers for the 2-jet events are : $N \sim 2 \cdot 10^2$ ($2 \cdot 10^4$) for $\Delta y = 0.2$ (2.0).

Fig. 4c gives insight into the correlations. We vary the conditions on y_i and expose the sum of all contributions to the 4-jet cross-section. We proceed in three steps. First, the rapidities of one jet-pair (12) or (34) are kept fixed at zero, whereas the rapidities of the second jet-pair are varied as : $y_1 = y_2, y_1 = -y_2, y_2 = 0$ (solid curves). Second, the rapidity of one member of each jet-pair is kept at the origin (we choose $y_2 = y_4 = 0$) whereas the remaining two are correlated as : $y_1 = y_3, y_1 = -y_3, y_3 = 0$ (dashed curves). Third, all rapidities are varied at the same time as : $|y_1| = |y_2| = |y_3| = |y_4|$ whereby the sign may differ as indicated in Fig. 4c (dotted curves). We notice three characteristic features : (a) curve 3 with three of the rapidities vanishing extends widest into the rapidity range and its cross-section dominates.

(b) curve 5, next in importance, corresponds to two jets, each from a different jet-pair, which move in opposite direction ; the remaining two jets have no longitudinal momentum. (c) if all rapidities are changed at the same time the cross-section decreases very rapidly.

The above features are in agreement with the characteristics found for the 2-jet process. In order to consider the same transverse energy ($q_1, q_2 \geq 15$ GeV in the 4-jet process) we here integrate over the transverse momentum region : $q \geq 30$ GeV. Fig. 4a illustrates the rapidity correlations. Changing one of the rapidities with the other fixed at zero gives the dominant contribution (curve 1). Varying both rapidities leads to a rapid cross-section decrease whereby at large rapidity values the 2-jet emission in opposite directions $y_1 = -y_2$ is suppressed (curves 3). This is understood by the fact that for $y_1 = y_2$ the momentum fractions satisfy $x_1 = x_2$ with $\hat{s} = \text{fixed}$. For $y_1 = -y_2 (> 0)$ instead x_1 becomes rapidly large and x_2 decreases to small values whereby $\hat{s}$ grows. The rapid decrease of the gluon distribution at large x is therefore at the origin of the suppression of curve 3 as compared to curve 2.

From the above we may conclude : the results presented earlier on the 4-jet cross-section can be qualitatively deduced from the product of two 2-jet cross-sections. Since we are working mostly at moderate x -value we indeed do not expect significant changes from the two-quark distributions which, by construction, assume only weak correlations among the constituents. Different assumptions could here however lead to different results.

The idea behind the reaction in Fig. 1a can be applied on a multitude of other processes¹⁰⁾. Going back to the more simple case of Fig. 1b, the simultaneous production of a pair of massive bosons $\gamma^*, W, Z, \dots$ or $\psi, T, \dots$ via the χ states comes to mind, which would analyze the short distance structure pairwise. Such processes then appear in competition with the usual two-constituent initiated graphs which are of great interest because of the cancellations imposed by gauge theories¹¹⁾. The two types of processes are in the double Drell-Yan process of comparable size^{2b)}, but it is not known whether this conclusion carries over to the production of two weak (weak plus electromagnetic) gauge bosons. Replacing one (or several) of the final state jets by a direct- γ or a $\psi, \dots$ leads to a set of so far unexplored processes (Figs. 1a, 1c).

FIGURE CAPTIONS

- 1 - The 4-jet production process under investigation.
- 2 - Dependence of the 4-jet (and 2-jet) cross-sections on the transverse momentum cut-offs $q_{i,\min}$ at $y_i = 0$.
- 3 - The transverse momentum dependence of the 4-jet (and 2-jet) production processes at $y_i = 0$.
- 4 - Rapidity dependence of the transverse momentum integrated 4-jet (and 2-jet) cross-sections : (a) rapidity correlations of the 2-jet cross-section, (b) exposure of the various contributions to the 4-jet cross-section, (c) rapidity correlations of the 4-jet cross-section.

Fig. 1a

Fig. 1b

Fig. 1c

Fig. 2

Fig. 3

Fig. 4