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SPONTANEOUS SYMMETRY BREAKING WITHOUT POTENTIAL
IN A SCALAR-TENSOR THEORY OF GRAVITY

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A B S T R A C T

Scale invariance and global internal symmetry in a new scalar-tensor theory of gravity are shown to break down spontaneously, without the employment of a potential in the Lagrangian.

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Among the fundamental interactions, only gravity still involves a dimensionful coupling. The idea has been suggested by many authors independently¹⁾ that Newton's constant G_N should be understood within a new field theoretical framework, in analogy with the developments involved in the unification of the weak and electromagnetic interactions, as they emerged from the old theory with Fermi's dimensionful coupling constant. The hope is that such a development will not only lead to a consistent theory of gravity, but that it will also resolve a number of related issues, like the horizon problem, the initial singularity, the missing-mass problem and so on^{2),3)}, and eventually lead to the unification of gravity with the other fundamental interactions.

The requirement of scale invariance in the Einstein-Hilbert Lagrangian ($g_{\alpha\beta} = \text{diag}(-1,1,1,1)$ in an orthonormal frame $\{\omega^\alpha\}$ and $g = -\det g_{\alpha\beta}$) :

$$\mathcal{L} = -\sqrt{g} (16\pi G)^{-1} R \quad (1)$$

implies that G is not a fundamental constant, but rather a function of some field, which we assume to be a scalar ϕ^i , valued in, say, a real N -dimensional representation ($i = 1, \dots, N$) of some internal symmetry group, with inner product

$$\varphi^2 = \varphi_i \varphi^i. \quad (2)$$

The symmetry group and the particular representation for ϕ are generally a matter of choice or convenience. However, of crucial importance for us here is the form of the relation

$$G = G(\varphi^i, \varphi^i_{;\alpha}, \dots), \quad (3)$$

where $\phi^i_{;\alpha} = \phi^i_{;\alpha} = \partial_\alpha \phi^i$ (the semicolon denotes the covariant space-time derivative). Such considerations were originally involved in the scalar-tensor theories of gravity like the Brans-Dicke theory⁴⁾, in which $G^{-1} \sim \phi$, with ϕ a true scalar. The main motivation for these was the incorporation of Mach's Principle into general relativity. In the modern approach the motivation is the one mentioned earlier, with Eq.(3) having the form $G^{-1} \sim \phi^2$. Unfortunately, in all cases (or, in general, if G is any function of just ϕ itself), not only additional terms are required in the Lagrangian, apparently giving rise to an unwanted cosmological constant, but, more importantly, the introduction of the scalar field does not seem to justify the original motivations: the difficulties involved range from failure to incorporate Mach's Principle and apparent difficulties in solar-system tests, to having masses for intermediate vector bosons of the order of the Planck mass.

However, such difficulties seem to be avoided if, instead of relating G to the scalar field itself, we relate it to its gradient, by adopting the choice^{*)}

$$(16\pi G)^{-1} = -\varphi_i^\alpha \varphi_{,\alpha}^i . \quad (4)$$

It has already been discussed elsewhere how a relation like Eq.(4) may successfully incorporate Mach's Principle into general relativity, without upsetting its solar-system predictions^{3),5)}. On the other hand, the same relation is implied by the following considerations. Recently, a unified theory for the gravitational and electromagnetic interactions plus matter fields was proposed, which incorporates the EM field and, possibly, when maximally extended, the electroweak field, in the torsion of a Riemann-Cartan space-time⁶⁾. The total Lagrangian is given there also by Eq.(1), but with R understood as the corresponding full curvature scalar, ϕ is associated with a Higgs field and the identification (4) follows from the requirement of having the contribution of the EM field (or, eventually, the EW field) emerge automatically as part of that single Lagrangian. The presently suggested theory, described by Eqs.(1) and (4), is the Riemannian limit thereof when the torsion vanishes, that is, when the other fields are switched off and the gauge symmetry (now involving only the ϕ field) becomes rigid. Here, Eq.(1) will be considered in its own right, as a scale invariant Lagrangian, which is also invariant under global internal symmetry transformations. We will see that both these symmetries are spontaneously broken.

Let us consider the source-free field equations, which follow from Eq.(1) by varying with respect to ϕ^i :

$$(R \varphi_{,\alpha}^i);^\alpha = 0 \quad (5)$$

and with respect to $g^{\alpha\beta}$:

$$R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} - 8\pi G T_{\alpha\beta} = 0 , \quad (6)$$

^{*)} Any numerical factor in Eq.(4) can be absorbed in ϕ but the overall sign is important: from arguments involving the limiting case of a homogeneous and isotropic space-time, ϕ is a time function and thus $\phi_{,\alpha}$ a timelike vector. Also, we have already outlined elsewhere [Ref. 6)] one possibility, by which the dimensionless field ϕ may eventually acquire the correct dimensionality, through a rescaling involving R .

where

$$\frac{1}{2} T_{\alpha\beta} = R \varphi_{i\alpha} \varphi^i_{\beta} + (16\pi G)^{-1}_{;\alpha;\beta} - (16\pi G)^{-1}_{;\gamma;\gamma} g_{\alpha\beta}, \quad (7)$$

represents the energy-momentum contribution of the scalar field. Taking the trace of Eq.(6) we find :

$$G^{-1}_{;\alpha;\alpha} = 0, \quad (8)$$

so the field equations reduce to Eqs.(5) and (8) plus the traceless part of Eq.(6) :

$$R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} - 16\pi G (R \varphi_{i\alpha} \varphi^i_{\beta} + (16\pi G)^{-1}_{;\alpha;\beta}) = 0. \quad (9)$$

Since we are looking for the ground state, we restrict ourselves to the homogeneous and isotropic solutions. Choosing the orthonormal frame $\{\omega^0 = dt, \omega^{1,2,3} = a\sigma^{1,2,3}\}$, homogeneity is guaranteed with the employment of the set $\{\sigma^1, \sigma^2, \sigma^3\}$ of 1-forms, characterizing one of three possible groups of motion^{*)}, labelled by $\varepsilon = 0, -1, +1$ for, respectively, the flat, open and closed cases. Isotropy follows from the employment of a common scale factor a , function of the cosmic time t only⁷⁾. After a straightforward calculation we obtain from Eqs.(5) and (8) respectively (with $\partial_0 = d/dt$) :

$$\partial_0 (R \partial_0 \varphi^i) + 3 R (\partial_0 \ln a) \partial_0 \varphi^i = 0, \quad (10)$$

$$\partial_0^2 G^{-1} + 3 (\partial_0 \ln a) \partial_0 G^{-1} = 0, \quad (11)$$

where

$$R = 6 \partial_0^2 \ln a + 12 (\partial_0 \ln a)^2 + 6 \varepsilon a^{-2}, \quad (12)$$

while the 00 and 11 components of Eq.(9) (the rest are redundant) give :

$$6 \partial_0^2 \ln a + 9 (\partial_0 \ln a)^2 + G \partial_0^2 G^{-1} + 3 \varepsilon a^{-2} = 0, \quad (13)$$

$$2 \partial_0^2 \ln a + 3 (\partial_0 \ln a)^2 + (\partial_0 \ln G) \partial_0 \ln a + \varepsilon a^{-2} = 0. \quad (14)$$

^{*)} One can easily determine spacial coordinates from the defining relations $\{d\sigma^{1,2,3} = 0\}$, or $\{d\sigma^1 = 0, d\sigma^2 = \sigma^1 \wedge \sigma^2, d\sigma^3 = \sigma^1 \wedge \sigma^3\}$ or $\{d\sigma^1 = \sigma^2 \wedge \sigma^3, \text{cyclically}\}$, corresponding to the so-called Bianchi -I, V or IX types.

We note that Eq.(4) gives

$$(16\pi G)^{-1} = (\partial_0 \varphi)^2, \quad (15)$$

and Eq.(14) is automatically satisfied in view of Eqs.(11) and (13). A solution to the above system is given by :

$$\varphi^i = V^i t, \quad (16)$$

$$(\partial_0 a)^2 = \frac{1}{3} G M_0 a^{-1} - \epsilon, \quad (17)$$

$$(16\pi G)^{-1} = V^i V_i, \quad (18)$$

where V_i , M_0 are arbitrary constants, both having the dimension of a mass. This solution, although exact, is not the most general one. However, there is no need to write down the latter, as it goes over to the above result at asymptotic infinity and also reduces to it when the presumably small constant responsible for the variability of G therein [unlike the case in Eq.(18)] is set equal to zero as a first approximation. One can now determine that the Hamiltonian density of the scalar field is

$$\rho_{(\varphi)} = T_{00} = (8\pi)^{-1} M_0 a^{-3} \quad (19)$$

and it is seen to be independent of the constants V_i . It also follows from Eq.(17) that the scalar field, acting as a source, has reproduced the well-known Friedmann solutions of relativistic cosmology. As is well-known, these are obtained by solving the general relativistic field equations with matter sources of energy density

$$\rho = (8\pi)^{-1} M a^{-3},$$

which is formally identical to Eq.(19). The corresponding solution is given exactly by Eq.(17), with M_0 substituted by the constant M .

In general relativity, the ground state is assumed to be the solution with $\epsilon = 0$ and $M = 0$, giving us a Minkowski space-time. In complete analogy, the ground state in the present case will be the solution with $\epsilon = 0$ and $M_0 = 0$. The second condition minimizes the Hamiltonian density [Eq.(19)] of the scalar field, which, in view of Eq.(17) is positive definite in this case. The resulting space-time is again Minkowski but the scale invariance and internal symmetry of

the Lagrangian are spontaneously broken, in view of Eqs.(16) and (18), which are still valid. Presumably, the attractive features of this mechanism will survive in a more realistic treatment involving local symmetry and, in particular, within the context of the unified theory proposed in Ref. 6).

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