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CHARGE DEPENDENCE OF NUCLEAR FORCES

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A B S T R A C T

The observed charge dependence in the singlet NN scattering lengths ($\bar{a}-a_{np} = 5.5$ fm) is explained in terms of one- and two-pion exchange mechanisms as well as $(\gamma\pi)$ exchanges.

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1. - INTRODUCTION

The problem of understanding the difference between proton-proton (pp) [or neutron-neutron (nn)] and neutron-proton (np) singlet scattering at threshold has perplexed physicists for over forty years ^{1),2)}. For example, the difference of the $^1S_0$ scattering lengths $[a_{pp} - a_{np} = (-17.9 \text{ fm}) - (-23.748 \text{ fm}) = 5.8 \text{ fm}]$ ³⁾ indicates that the (np) force is slightly more attractive than the (pp) force in this state, once the straightforward electromagnetic effects have been removed. Despite considerable recent theoretical efforts ⁴⁾⁻⁷⁾ only about half of this discrepancy has been explained, the bulk coming from the $(\pi^\pm, \pi^0)$ mass difference in one-pion exchange (OPEP). This lack of success is disturbing in light of the enormous progress in understanding the NN force over the last twenty years. It is perfectly natural to expect that the entire discrepancy should be accounted for as reasonable electromagnetic and charge-dependent corrections to meson propagators and couplings. Still this goal could not be obtained until now. In the present letter, we examine a variety of charged-dependent effects ^{*)} in OPEP, TPEP (two-pion exchange potential) and $(\pi\gamma)$ exchange. We find that the entire discrepancy can be explained with significant contributions from all of these as summarized in the Table. To assess the uncertainty associated with the lack of knowledge about the NN system at short distances, the origin of the contributions in co-ordinate space is carefully examined.

2. - THE PERTURBATION EXPRESSION FOR Δa

Insofar as the $^1S_0$ scattering length difference originates in a potential energy difference $(\bar{V} - V_{np})$, the difference in the scattering lengths is given by the following exact relation (the np mass difference is systematically neglected since it contributes negligibly to charge dependence primarily breaking charge symmetry) :

$$\bar{a} - a_{np} \equiv \Delta a = \bar{a} a_{np} M \int_0^\infty (\bar{V} - V_{np}) \bar{u} u_{np} dr. \quad (1)$$

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^{*)} We define the measure of charge dependence as the difference between the average of the nn and pp scattering lengths $a = \frac{1}{2}(a_{nn} + a_{pp})$ and the np one. This procedure removes charge symmetry violations which cause a_{nn} and a_{pp} to differ.

Here the barred quantities $\bar{u}(r)$ and $u_{np}(r)$ are the zero energy $^1S_0$ wave functions normalized so that $u(r) \xrightarrow{r \rightarrow \infty} (1-r/a)$ and M is the average of the nucleon masses.

The large factor $\bar{a} a_{np}$ on the right-hand side of Eq. (1) has the consequence that a very small change in the potential energy easily leads to a large change in the scattering length. This reflects that the perturbative expansion concerns the inverse scattering length. Since we derive various contributions to the potential energy difference $\bar{V} - V_{np}$ ($\equiv \Delta V$), it is extremely convenient to fix the scattering lengths a_{np} and $\bar{a}$ in the product at their experimental values ³⁾

$$a_{np}^{exp} = -23.748 \text{ fm} \quad \text{and} \quad \bar{a}^{exp} = \frac{1}{2}(-17.9 - 18.45) = -18.2 \text{ fm}.$$

The various contributions to Δa are then additive. Compared to other authors ⁵⁾⁻⁷⁾ who used a procedure equivalent to replacing $\bar{a} a_{np}$ by a_{pp}^2 , our use of Eq. (1) introduces a factor of approximately 1.3. If the perturbing potential ΔV is to result in the experimental value of Δa , the use of the product $\bar{a} a_{np}$ is the only consistent procedure.

While the distinction between a_{np} and $\bar{a}$ is important, the neglect of charge dependence in the product of the wave functions $u_{np} \bar{u}$ in Eq. (1) has a negligible effect. This is because the error introduced in the integrand by using this perturbation expansion is of second order in ΔV . We therefore simply use phenomenological $T = 1$ wave functions $u(r)$ for both $u_{np}(r)$ and $\bar{u}(r)$. Furthermore it turns out that the main contributions to the integrand come from the region dominated by the asymptotic normalization. Our results are thus the same whether the Reid soft core ⁸⁾ or Paris potential ⁹⁾ is used to generate $u(r)$.

With these very minor approximations, our basic equation is therefore

$$\Delta a = \bar{a}^{exp} a_{np}^{exp} M \int_0^\infty dr \Delta V(r) u(r)^2. \quad (2)$$

We now evaluate the various terms in ΔV , beginning with the simplest and best known ones.

3. - π MASS DIFFERENCES IN OPEP

The relatively large mass difference between the charged and neutral pion is a substantial source of charge dependence ¹⁰⁾. The OPEP contribution $\Delta V^{\text{OPEP}} = 2[V_{\pi^0}^{\text{OPEP}} - V_{\pi^{\pm}}^{\text{OPEP}}]$, where $\pi^{c,o}$ refers to charged or neutral exchanged pions. For a pseudoscalar (γ_5) πN interaction the contribution is :

$$\Delta V_{\text{PS}}^{\text{OPEP}} = -2f^2 \frac{e^{-\mu_0 r}}{r} \left[1 - \frac{\mu_c^2}{\mu_0^2} e^{-(\mu_c - \mu_0)r} \right], \quad (3)$$

where $f^2 \approx 0.08$ and $\mu_{c,o}$ is the mass of charged and neutral pions. The corresponding integrand in Eq. (2) is dominated by the positive contributions inside 2.8 fm as seen in Fig. 1. Assuming no further charge dependence of the πN coupling constants, the pseudoscalar OPEP gives a contribution to Δa of 3.5 fm.

An alternative version of OPEP might be obtained with a pseudovector coupling with strength inversely proportional to the pion mass $f/\mu_{c,o}$. This is known ¹⁰⁾ to give a large and negative contribution to Δa . However, such a procedure introduces a very strong charge dependence in the πNN coupling constants, larger than the experimental limits. It should be noted, however, that there is no reason to scale the PV coupling strength with the pion mass since, for example, the pion decay constant $f_\pi \approx 93$ MeV provides a natural charge-independent mass scale. Using such a charge-independent coupling, the previous result (3) is obtained also from PV coupling.

4. - MASS DIFFERENCES IN 2π EXCHANGE

The substantial effects produced by the π mass difference in OPEP initially make one suspect important mass difference effects also in the 2π exchange potential (TPEP). Such an expectation is seemingly natural, since the OPEP contributions to the $^1S_0$ interaction are basically much weaker than the TPEP contributions : in fact, the latter are dominant even at 2.5 fm. However, in the perspective that the charge-independent potential can be analyzed in terms of the exchanged isospin as an isoscalar and isovector potential

$$V = V_0 + \vec{\tau}_1 \cdot \vec{\tau}_2 V_1, \quad (4)$$

any charge-dependence originating in mass differences (or coupling constants) occurs from a modification of the isovector potential, while the isoscalar potential is unchanged. The charge-dependence from TPEP is thus much weaker than might first be expected. The reason for the weakness of the isovector contributions is seen by examining the dominant TPEP contributions ¹¹⁾ in the $^1S_0$ channel. These are due to the so-called "fourth order" crossed and uncrossed box diagram exchanges (Fig. 2), which are calculated relatively straightforwardly. The two contributions have similar strength but with an isospin factor $(3+2\tau_1 \cdot \tau_2)$ for the former and $(3-2\tau_1 \cdot \tau_2)$ for the latter ¹²⁾. Consequently, this gives severe cancellations so that the isovector TPEP is about seven times weaker at the peak of the integrand than the isoscalar one. This is because the TPEP isovector interaction requires the two pions to be in a relative P state which kinematically suppresses the low mass contributions. [There are of course additional isovector exchange contributions from P wave $\pi\pi$ exchange (ρ exchange), but little charge dependence results from this as discussed below.]

The TPEP box contributions and their charge dependence were derived by Partovi and Lomon (PL) ¹²⁾ and can be expressed as $\Delta V_{\Delta\mu}^{\text{TPEP}}$. They find [PL, Eq.(4.39)] the result in the form

$$\Delta V_{\Delta\mu}^{\text{TPEP}} = 4 \left\{ [B(I) - B(M)] - [A(I) - A(M)] \right\}. \quad (5)$$

Here the terms A and B refer to the uncrossed and crossed contributions in Fig. 2 [For more details, see Fig. 1 of Ref. 12)]. The use of the notation A and B allows the use of one single pion mass in a given diagram. The value of that mass depends on the index, however [index I uses μ_c , index M uses $[\frac{1}{2}(\mu_c + \mu_o)]^{\frac{1}{2}} \approx \frac{1}{2}(\mu_c + \mu_o)$]. The charge-dependent force (5) therefore consists of two terms each of which is proportional to the not so small mass ratio $(\mu_c - \mu_o)/\mu_c$. This contrasts with the result of Coon and Scadron ⁶⁾ who ignored the term of Eq. (5).

On the other hand, the contribution $\Delta V_{\Delta\mu}^{\text{TPE}}$ is reduced due to the same cancellation between A and B which causes the isovector TPEP to be so small.

The equations for A and B are given in PL, Eqs. (4.29)-(4.31), (4.38), and are not repeated. As in PL, no charge dependence of the πN coupling constants is included at this point. The integrand of this contribution according to Eq. (2) is displayed in Fig. 1. It peaks near 1 fm in co-ordinate space with $\frac{2}{3}$ of its contribution coming from larger r. Evaluation of the integral gives a contribution to Δa of 0.88 fm. This is the first estimate of this significant term.

5. - CHARGE DEPENDENCE OF THE πN COUPLING CONSTANTS

Even in the absence of charge dependence of strong interactions, electromagnetic corrections (e.g., Figs. 2C and 2D) cause the charged coupling constants ($f_{\pi^- p}$ for the vertex $p + \pi^- \rightarrow n$ and $f_{\pi^+ n}$ for $n + \pi^+ \rightarrow p$) to differ from the neutral couplings ($f_{\pi^0 p}$ and $f_{\pi^0 n}$). Logical consistency requires that the modified couplings be used both in OPEP and TPEP. Define to leading order

$$\begin{aligned} f_{\pi^0 p} &= \left(1 - \frac{\delta_0}{2}\right) f, \\ f_{\pi^0 n} &= \left(1 + \frac{\delta_0}{2}\right) f, \\ f_c &\equiv \frac{1}{2} (f_{\pi^- p} + f_{\pi^+ n}) = (1 + \delta_c) f. \end{aligned} \quad (6)$$

The resulting charge dependence of OPEP is

$$\Delta V_c^{\text{OPEP}} = 4 \delta_c f^2 \frac{e^{-\mu r}}{r}. \quad (7)$$

The appropriate modification of the PL TPEP interaction is easily made by replacing the charge-independent coupling constants of PL by those of Eq. (6). The resulting charge dependence is

$$\Delta V_c^{\text{TPEP}} = 8 \delta_c [B(1) - A(1)]. \quad (8)$$

The only published computation of all the electromagnetically corrected coupling constants needed in OPEP and TPEP is that of Morrison¹³⁾. Of the various approaches he employs we consider the Feynman graph approach the most reliable one. Morrison's results are¹³⁾ (with his notations indicated)

$$\begin{aligned} f_{\pi^- p} &\approx f_{\pi^+ n}, \\ \delta_0 &\equiv -\Delta_0 = 0.0023, \\ \delta_c &\equiv \frac{1}{2} (\Delta_+ + \Delta_-) = -0.0058. \end{aligned} \quad (9)$$

Henley and Morrison¹⁰⁾ studied the effects of charge-dependent coupling constants in OPEP and TPEP, but used a static version of TPEP. However, the static approximation in TPEP is very bad¹²⁾, and it is necessary to use the modern versions.

The characteristic feature of the OPEP and TPEP contributions is that they have opposite signs and lead to a substantial systematic cancellation. The integrands of the two contributions are shown in Fig. 1 using Morrison's coupling

constants. One notes that the TPEP contributions in part come from distances below 1 fm and thus are more model-dependent. The numerical contributions are $\Delta a_{cc}^{OPEP} = -1.6$ fm and $\Delta a_c^{TPEP} = +1.6$ fm, so that the net effect of the electromagnetic charge dependence is nearly zero. This cancellation between OPEP and TPEP terms renders Δa insensitive to the value of δ_c . Hence Δa is a poor source of information regarding the charge dependence of the πNN coupling constants.

6. - PHOTON-PION EXCHANGE POTENTIAL

An additional source of charge dependence arises from the replacement of any one of the exchanged pions of Fig. 2A or Fig. 2B by a photon. It is reasonable to expect this term to be of order α times the (very large) TPE interaction, so that a significant contribution to Δa may result. There are two calculations in the literature ^{5),7)} with different approaches and different conclusions. Chemtob ⁵⁾ uses a dispersive approach based on pseudovector πN coupling and he employs the static approximation. He finds that the $\gamma\pi$ contribution to Δa is 0.8 fm. However, he used a pp scattering length of -15.1 fm, so that his result is to be scaled, according to Eq. (2), by $(23.75 \times 18.2)/15.1 = 1.88$. The correct result is then 1.5 fm. Banerjee ⁷⁾ uses Feynman perturbation theory, includes only box diagrams and avoids the static approximation. He obtains a contribution of 0.51 fm. The appropriate scaling leads to the result 0.75 fm. Chemtob's use of the static approximation is expected to over-estimate the effect ⁵⁾, and we note that the corresponding approximation for TPE is poor ¹²⁾. The difference between the results of the two approaches is disturbing but, fortunately, the values 0.75 fm and 1.5 fm represent reasonable limits for the $\gamma\pi$ contribution. At present we simply choose to use the average value which gives the estimate $\Delta a^{\gamma\pi} = 1.1 \pm 0.4$ fm.

7. - SHORT-RANGE EFFECTS

It is always a problem using OPEP or TPEP or other model potentials, to know whether they should be employed at small r . Consider, for example, the dominant contribution to the charge dependence of 3.5 fm from the π mass dependence of OPEP. Its integrand peaks at 1 fm with about 30% of the contribution coming from inside that region (see Fig. 1). Nucleons with a radius of about 1 fm overlap in a considerable part of the integrands indicating that finite size corrections may be necessary. This possibility is of even greater concern for the coupling constant term in TPEP which peaks at 0.7 fm.

A rough estimate of the effects of overlapping nucleons on the OPEP term can be obtained using a six-quark picture^{14),15)} as follows. Divide the co-ordinate space into regions. The ordinary potential model will be employed for $r > r_0$, while for $r < r_0$ the interactions are approximated as those between six quarks confined to a single spherical bag. The two regions are related by "probability conservation" so that the six-quark "probability" P_{6q} is given by¹⁵⁾

$$P_{6q} = \int_0^{r_0} u^2 dr. \quad (10)$$

By removing the OPEP contributions inside r_0 one loses a contribution of about 0.8 fm to Δa for $r_0 = 0.8$ fm (see Fig. 1). This difference is largely compensated by charge-dependent effects arising from pion exchange (with different masses) between quarks (charge-dependent quark effects from other mass differences and Coulomb interactions have a very small effect on Δa ¹⁶⁾). The six-quark contribution Δa_{6q} may be expressed in close analogy to Eq. (2) as

$$\Delta a_{6q} = M \bar{a} a_{np} \Delta E P_{6q}, \quad (11)$$

where ΔE is the energy difference for the six-quark bags corresponding to equal nucleons and the np system. To compute ΔE we require the pion-six-quark self-energy contribution. This can be obtained from the literature from the cloudy bag model¹⁷⁾, but it is now necessary to keep track of the charged and neutral pion masses. One finds

$$\Delta E = -C \left\{ \langle \sum_{i,j} \vec{\sigma}_i \cdot \vec{\sigma}_j \tau_{i3} \tau_{j3} \rangle - \langle \sum_{i,j} \vec{\sigma}_i \cdot \vec{\sigma}_j \tau_{i3} \tau_{j3} \rangle_{np} \right\} \quad (12)$$

where C is a constant. The expectation values in Eq. (12) are taken between antisymmetrized six-quark states in the lowest s states of the MIT bag (4u, 2d for pp ; 2u, 4d for nn and 3u, 3d for np). The expression in brackets for spins is the number 564/15. The parameter C is given by

$$C = (\mu_c - \mu_0) \frac{f_{\pi qq}^2}{24\pi R^2} \frac{\partial}{\partial \mu_c} \left[\mu_c j_1(i\mu_c R) h_1^{(1)}(i\mu_c R) \right] \quad (13)$$

where R is the radius of the six-quark bag, and $f_{\pi qq}$ the pion-quark coupling constant which gives $f^2 = 0.08$ fm for the πNN coupling. In the following, we identify R with the cut-off r_0 as is done in Ref. 14).

With Eqs. (10)-(12), enough information is available to estimate Δa_{6q} . In the range $r_0 = R$ between 0.6 fm to 1.0 fm the result can be approximately parametrized as $\Delta a_{6q} \approx [0.60 + 3.0(r_0 - 0.7)]$ fm, while the cut-off loss $\Delta a_{r_0} \approx [-0.6 - 1.5(r_0 - 0.7)]$ fm. As might have been expected on the grounds of continuity, there is thus a considerable compensation of the loss in co-ordinate space from the 6q contribution, so that the total Δa is rather insensitive to the exact value of r_0 . A plausible choice is $r_0 = (0.8 \pm 0.1)$ fm consistent with the fact that theoretically computed the NN potential is little modified outside 0.8 fm. The total OPEP mass difference effect is then slightly increased to $\Delta a_{\Delta m}^{\text{OPEP}} = (3.6 \pm 0.15)$ fm. The uncertainty corresponds only to the ± 0.1 fm change in r_0 .

The next step is to consider the size of such corrections for the smaller contributions from TPE and $\gamma\pi$ potentials. A quark approach is not feasible at present since two-boson exchanges between quarks have never been computed. Instead we assume that the constraints of probability conservation apply in a similar manner to that of the OPEP term, so that the curves in Fig. 1 will go continuously to zero inside of r_0 . Using various straight line extrapolations it is easily seen that the model dependence in the integrals of Fig. 1 can hardly be more than 15%, and we use this as a rough upper limit.

The values of the various contributions with estimates of uncertainties are given in the Table. The sum of all these terms is $\Delta a = 5.6 \pm 0.5$ fm in agreement with the experimental value $\Delta a^{\text{exp}} = (5.5 \pm 0.3)$ fm. In the following, we comment upon this result.

8. - CHARGE SYMMETRY BREAKING (CSB)

Internal consistency of the approach requires that contributions to charge symmetry breaking are small. The charge symmetry breaking potential generated by the coupling constants (6) is

$$\Delta V_{\text{CSB}} = \left\{ \frac{f^2 e^{-\mu r}}{r} - 2[A^{(1)} + B^{(1)}] \right\} 2\delta_0 \quad (14)$$

for the difference $(a_{pp} - a_{nn})$, which, using Morrison's coupling constants ¹³⁾, gives +1.2 fm. The radiative contributions à la Chemtob ⁵⁾ and Banerjee ⁷⁾ give an additional contribution (-0.6 ± 0.6) fm, so that the total result is $(a_{pp} - a_{nn}) = (0.6 \pm 0.6)$ fm. Although we have made no attempt to obtain a complete description of charge symmetry breaking, our result is consistent with small CSB as required.

9. - ADDITIONAL CONTRIBUTIONS AND UNCERTAINTIES

Prior to closing it is necessary to consider two questions :

- 1) Are there other significant contributions ?
- 2) What are the additional uncertainties in the Table and do they matter ?

The responses are the following.

- 1) Additional charge dependence may occur from the exchange of heavy isospin 1 mesons ⁶⁾, from intermediate delta isobars in two-boson exchange ¹⁸⁾ or from the effects of the nucleon mass differences ⁶⁾. However, these are small. The recent very small value of the $\rho^+ - \rho^0$ mass difference ¹⁹⁾ leads to a negligible contribution to Δa . There is no significant contribution from intermediate deltas nor in the $\gamma\pi$ ¹⁸⁾ or $\pi\pi$ ²⁰⁾ exchange. These mechanisms are thus unlikely candidates for charge-dependent effects.
- 2) It is true that each entry of our Table could be determined with better precision. Knowledge of quark effects in NN scattering is still at a very early stage. The theoretical discrepancy between the Chemtob ⁵⁾ and Banerjee ⁷⁾ results for $(\gamma\pi)$ exchange could and should be resolved. Nevertheless, the scale and sign of each of the contributions is of the correct nature to explain Δa^{exp} . It is noteworthy that the final result depends only weakly on the difference between the couplings of the charged and neutral pions, due to the strong and systematic cancellation between OPEP and TPEP contributions. This reduces uncertainties from this origin even if the cancellation were less complete. It follows that the scattering lengths shed little light on the charge dependence of the coupling. In our view, it is unlikely that further improvements will cause major changes in the sum of terms in the discrepancy.

10. - CONCLUSION

Our results resolve the long-standing discrepancy of the charge-dependent NN scattering lengths as a natural, though non-trivial, consequence of OPEP, TPEP and $(\gamma\pi)$ exchange. No anomalous interactions need be invoked. Charge dependence of the πNN coupling constants gives only a negligible contribution to Δa . Quark effects as well as heavy boson exchanges modify the effect only marginally. Our result should therefore be viewed as a non-trivial quantitative evidence for the importance of TPEP (and OPEP) at intermediate range.

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$\Delta a_{\Delta\mu}^{\text{OPE}}$	$3.6 \pm 0.15 \text{ fm}$
$\Delta a_{\Delta\mu}^{\text{TPE}}$	$0.88 \pm 0.1 \text{ fm}$
$\Delta a_{\text{cc}}^{\text{OPE}}$	$-1.6 \pm 0.2 \text{ fm}$
$\Delta a_{\text{cc}}^{\text{TPE}}$	$1.6 \pm 0.2 \text{ fm}$
$\Delta a_{\gamma\pi}$	$1.1 \pm 0.4 \text{ fm}$
Δa	$5.6 \pm 0.5 \text{ fm}$
Δa^{exp}	$5.5 \pm 0.3 \text{ fm}$

TABLE - Contributions to Δa from different sources.
For an explanation of how the various errors are estimated, see text, especially Sections 6 and 7.

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FIGURE CAPTIONS

- Fig. 1 Causes and location of charge dependence. The integrals of the curves give the different contributions to Δa in fm. (The term cc refers to the use of charge-dependent coupling constants.)
- Fig. 2 A and B are the graphs giving the TPEP terms A and B. C and D are two graphs [for the others, see Ref. 13), Fig. 1] that lead to charge-dependent and charge-symmetry breaking couplings.

- Figure 1 -

A

B

C

D

- Figure 2 -