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NUMERICAL EVIDENCE FOR A BARRIER AT THE GRIBOV HORIZON^{*)}

E. Seiler

Max-Planck Institut für Physik und Astrophysik
München, F.R.G.

I. O. Stamatescu

CERN -- Geneva

and

D. Zwanziger

University of New York
New York NY 10003, U.S.A

ABSTRACT

We demonstrate numerically that in stochastically quantized Euclidean Yang-Mills theory with a non-conservative gauge fixing force, the Gribov horizon represents a barrier that keeps the system inside a bounded region of field space.

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1. - INTRODUCTION

In a series of papers, stochastic quantization of Euclidean Yang-Mills theory was studied¹⁾⁻³⁾. A central point is the replacement of the Faddeev-Popov procedure by a non-conservative but local gauge fixing force that does not require the introduction of ghost fields. In Ref. 3), it is argued that in the continuum, the system in the limit of an infinitely strong gauge fixing force will be confined to a convex region in field space which is bounded in each direction.

In Ref. 4), a lattice version of the system (with finite gauge fixing force) was studied. The non-conservative character of the gauge fixing force K_{GF} leads to the presence of steady currents in the equilibrium measure, as is pointed out in that work.

Furthermore, the gauge fixing force K_{GF} shows some peculiar features related to the Gribov ambiguity: while it vanishes everywhere on the gauge fixing surface given by the Landau condition $\partial \cdot A = 0$ or by the background gauge condition $\partial_{\bar{A}}(A - \bar{A}) = 0$, it is attracted to that surface only within a bounded convex subregion Ω ; the boundary of Ω is the Gribov horizon⁵⁾. Outside the Gribov horizon, K_{GF} will have "unstable modes", i.e., there will be directions along which K_{GF} is repelled from the gauge fixing surface. See Ref. 4) for details.

On the other hand, it was shown in Ref. 4) that K_{GF} is always a restoring force in the sense that it tends to reduce the size of the gauge field A . More precisely, it was shown in Ref. 4) that under the motion $\dot{A} = K_{GF}(A)$, we have the relation

$$\frac{d}{dt} |A - \bar{A}|^2 = -2\alpha |\partial_{\bar{A}} \cdot (A - \bar{A})|^2 \quad (1)$$

The interplay of these features leads to a peculiar behaviour of the system once it manages to diffuse along the gauge fixing surface beyond the Gribov horizon. The gauge fixing force K_{GF} will drive it away from the gauge fixing surface, but at the same time back towards $\bar{A}$, so after completing a kind of loop, the system will end up again essentially inside the Gribov horizon close to the gauge fixing surface. "Instability stabilizes". These "loops" are closely related to the existence of a steady current. This mechanism is illustrated in Fig. 2 of Ref. 4).

In this note, we study these effects numerically and confirm the picture outlined above. The effects are most clearly seen if we drop completely the classical force due to the Yang-Mills action (i.e., if we go to infinite coupling). It is a remarkable fact that the system remains stable in this limit; this may be called "self-compactification".

Let us now briefly recall the main features of the model. The configuration space (field space) is described by variables $A_{\mu}^a(m)$ living on the links (m, μ) of a four-dimensional simple cubic lattice with periodic boundary conditions; a is a colour index. Stochastic quantization introduces a time-dependent probability density $P(A, t)$ obeying the diffusion equation

$$\dot{P}(A, t) = - \vec{\nabla} \cdot \vec{J} \quad (2)$$

where the current $\vec{J}$ is given by

$$\vec{J} = -\frac{1}{\hbar} \vec{\nabla} P + \vec{K} P \quad (3)$$

and $\vec{K}$ is the drift force consisting of two parts:

$$\vec{K} = \alpha \vec{K}_{GF} + \beta \vec{K}_{cl.} \quad (4)$$

$$K_{GF, \mu}^a(m) = D_{\mu}^{ab}(A) (\partial_{\bar{A}} \cdot (A - \bar{A}))^b(m) \quad (5)$$

$$K_{cl., \mu}^a(m) = - \frac{\partial S}{\partial A_{\mu}^a(m)} \quad (6)$$

where S is the lattice version of the classical Yang-Mills action [see Ref. 4]. As remarked, we will mostly set $\beta = 0$ in this note; since we also found essentially no effect due to the background field $\bar{A}$, we have also put $\bar{A} = 0$ to save computing time.

To study the system numerically, we discretize both time and field space. All fields $A_{\mu}^a(m)$ will be integer multiples of a basic unit η , called step size.

The diffusion equations (2), (3) are then simulated by a random walk in field space; the transition probability from a configuration to a neighbouring one [changing one of the components $A_\mu^a(m)$ by a unit η in $\pm$ direction] is given by $\frac{1}{2}(1 \pm \frac{1}{2}\eta K_\mu^a(m))$ for small ηK - for large ηK some modification is necessary in order to avoid negative probabilities. Details may be found in Ref. 6).

2. - STABILITY OF THE SYSTEM STARTED AT $A = 0$

We started the system "cold", i.e., at $A \equiv 0$ on a 6^4 lattice with step size $\eta = \frac{1}{8}$, $\alpha = 1$, $\beta = 0.0001$; another run was done on a 4^4 lattice with $\eta = \frac{1}{4}$, $\alpha = 0.15$, $\beta = 0.0$. The results are presented in Figs. 1 and 2.

Both runs show eventual stabilization after a slow and long heating period. Even after equilibrium has been reached, low frequency fluctuations are clearly visible; they reflect the presence of a steady current in equilibrium. Presumably what is happening is this: once in a while, the system diffuses a little bit outside the Gribov horizon; then it picks up an unstable mode of the gauge force, driving it away from the surface $\partial \cdot A = 0$. On the other hand, as is shown by Eq. (1), non-negligible values of $\partial \cdot A$ will tend to reduce $|A|$ and bring the system back near the region Ω inside the Gribov horizon. This phenomenon will be seen in a much more dramatic way if we start the system far outside the Gribov horizon, as described in the next section.

3. - RETURN FROM OUTSIDE THE GRIBOV HORIZON

The simplest field configurations outside the Gribov horizon are given by sufficiently large constant colour vectors

$$A_\mu^a(m) = c \delta_{a3} \delta_{\mu 4} \quad (7)$$

[see Ref. 4)].

So, by starting the system at such a configuration, we can induce a return cycle. We present three examples.

The first one (Fig. 3) was done in the presence of the classical force but at $\alpha > \beta$ (at $\alpha \leq \beta$ the cycles can hardly be seen). We chose $\beta = 0.25$, $\alpha = 1$, $\eta = \frac{1}{8}$ and a starting configuration

$$\vec{A}_\mu^a = 36\eta \delta_{\mu 4} \delta_{a3} = 4.5 \delta_{\mu 4} \delta_{a3}$$

on a 4^4 lattice.

Figures 4 and 5 show runs done with $\beta = 0$ on 6^4 lattices; in Fig. 4, $\alpha = 0.15$, $\eta = \frac{1}{6}$, $A_u^a = 108\eta \delta_{\mu_4 a_3} = 18\delta_{\mu_4 a_3}$, whereas in Fig. 5, $\alpha = 5$, $\eta = \frac{1}{20}$, $A_u^a = 144\eta \delta_{\mu_4 a_3} = 7.2\delta_{\mu_4 a_3}$. The return cycles are very clearly visible; the approximate validity of Eq. (1) can also be seen. Note that $|A|$ always reaches its maximum before $|\partial \cdot A|$. Of course, we cannot expect Eq. (1) to hold exactly because it neglects the fluctuating force and comes from a purely classical treatment.

4. - CONCLUSIONS

The numerical simulations confirm the intuitive picture developed in Ref. 4) and here. It is very striking that the system "self-compactifies" and allows the limit $\beta \rightarrow 0$ without showing any instability.

Of course, this "self-compactification" should be understood in an average sense. If α is not infinite, arbitrary large values of A are possible, but their probability decreases rapidly with $|A|$. In equilibrium, the system will apparently mostly drift within the Gribov horizon, performing small cycles; only occasionally will it venture out further, performing a larger cycle. From the numerical point of view, these cycles are, of course, a nuisance, since they mean slow convergence. So it is gratifying that for $\beta \geq \alpha$ they seem to be already pretty much suppressed [see Ref. 4)]. However, we think it is remarkable that we can "see" the Gribov horizon in the way shown here.

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FIGURE CAPTIONS

- Fig. 1 : Action $(\times \frac{1}{100})$: crosses, $A_\mu^2(\times \frac{1}{10})$: circles and $(D_\mu A_\mu)^2(\times \frac{1}{10})$: triangles. Each point represents an average over the 50 sweeps (respectively 10, in the box) between it and the preceding point. Here $\beta = 0.0001$, $\alpha = 1$ and the step size $\eta = \frac{1}{8}$ on a 6^4 lattice. All the averages are given per link.
- Fig. 2 : Action $(\times \frac{1}{1000})$, (left-hand scale), $A_\mu^2 \times (\frac{1}{100})$ and $(D_\mu A_\mu)^2(\times \frac{1}{100})$, (right-hand scale). Averages over 50 sweeps; $\beta = 0$, $\alpha = 0.15$ and $\eta = \frac{1}{4}$ on a 4^4 lattice. Same symbols as before.
- Fig. 3 : Action, $A_\mu^2(\times \frac{1}{5})$ and $(D_\mu A_\mu)^2(\times \frac{1}{2})$. Averages over 25 sweeps; $\beta = 0.25$, $\alpha = 1$ and $\eta = \frac{1}{8}$ on a 4^4 lattice started outside the Gribov horizon. Same symbols as before.
- Fig. 4 : Action $(\times \frac{1}{1000})$, $A_\mu^2(\times \frac{1}{100})$ and $(D_\mu A_\mu)^2(\times \frac{1}{200})$. Averages over 25 sweeps; $\beta = 0$, $\alpha = 0.15$ and $\eta = \frac{1}{6}$ on a 6^4 lattice started outside the Gribov horizon. Same symbols as before.
- Fig. 5 : Action $(\times \frac{1}{100})$, $A_\mu^2(\times \frac{1}{10})$ and $(D_\mu A_\mu)^2(\times \frac{1}{50})$. Averages over 25 sweeps; $\beta = 0$, $\alpha = 5$, $\eta = \frac{1}{20}$ on a 6^4 lattice started outside the Gribov horizon. Same symbols as before.

Fig.1

Fig. 2

Fig.3

Fig. 4

Fig. 5

Iterations