Ref.TH.3630-CERN

CP VIOLATION WITH MAJORANA NEUTRINOS

A. Barroso^{*)} and J. Maalampi^{**)}

CERN - Geneva

A B S T R A C T

In the framework of the standard two-generation $SU(2) \times U(1)$ model we analyze the general case of CP violation with Majorana neutrinos. We show that the CP violating phase can be either explicit in the weak charged current or hidden in the self-conjugation condition of the neutrinos. Besides processes which involve neutrino propagators, e.g. double beta decay, CP violation can also be observed in neutrino radiative decays.

*) On leave from Centro de Fisica Nuclear, Universidade de Lisboa, Portugal.

**) On leave from Department of High Energy Physics, University of Helsinki, Finland.

In a large class of grand unified models [see, e.g., Ref. 1)], neutrinos quite naturally become massive. This and the present experimental indication²⁾ of a non-zero mass of the electron neutrino renewed the interest in the general problem of neutrino masses.

Once neutrinos acquire mass, the question of a possible CP violation in the lepton sector becomes a very interesting one, especially if the neutrinos are Majorana particles. In fact, it was previously recognized^{3),4)} that, in the case of Majorana neutrinos even with only two generations of fermions, there will be a CP violating phase in the leptonic analogue of the Cabibbo⁵⁾ - Kobayashi-Maskawa⁶⁾ matrix.

Following these studies, our aim in this paper is to analyze in detail the general case of CP violation with Majorana neutrinos. To keep the algebraic manipulations to a minimum while emphasizing the physical consequences we shall consider only a two-generation version of the standard SU(2)×U(1) Glashow-Salam-Weinberg model. We will show that, depending on the form of the complex mass matrix, there is either a single CP violating phase or no phase at all. We present two alternative ways to parametrize the model, one where the CP non-conservation is explicit in the weak current and another one where the phase is hidden in the definitions of the neutrino field operators.

In the basis where e and μ are already the mass eigenstates, the weak left-handed doublets are

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \text{and} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L ,$$

and there are no right-handed partners of the neutrinos. The over-all phase of each doublet is arbitrary, and this freedom will be used later. To be as general as possible, we shall not specify the Higgs content of the model. It is sufficient to assume that the Higgs mechanism generates in the Lagrangian the mass term,

$$-\mathcal{L}_m = (\nu_e^T \ \nu_\mu^T)_L C M \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}_L + \text{h.c.} , \quad (1)$$

where C is the charge conjugation matrix ($-C = C^T = C^\dagger = C^{-1}$) and M is a complex 2×2 matrix. Due to the Fermi statistics of the fields, M must be a symmetric matrix which in general can be written

$$M = \begin{pmatrix} m_{11} e^{i\gamma_1} & m_{12} e^{i\gamma_{12}} \\ m_{12} e^{i\gamma_{12}} & m_{22} e^{i\gamma_2} \end{pmatrix} , \quad (2)$$

with m_{ij} real and positive.

Notice that the complex phases in M could come either from an explicit CP violation in the Higgs sector or from a spontaneous CP violation. In the latter case, the general form of M (with $\gamma_{12} \neq \gamma_1 \neq \gamma_2 \neq 0$) is obtained only in models with no natural flavour conservation in the leptonic sector⁷⁾.

It is straightforward to show that the unitary matrix

$$U = \begin{pmatrix} \cos\theta e^{-i\gamma_1/2} & -\sin\theta e^{-i(\gamma_1/2 - \alpha)} \\ \sin\theta e^{-i(\gamma_2/2 + \alpha)} & \cos\theta e^{-i\gamma_2/2} \end{pmatrix}, \quad (3)$$

diagonalizes M if α and θ satisfy the condition

$$2m_{12} \cos(2\theta) e^{i\beta} = \sin(2\theta) (m_{11} e^{i\alpha} - m_{22} e^{-i\alpha}) \quad (4a)$$

with

$$\beta = \gamma_{12} - (\gamma_1 + \gamma_2)/2. \quad (4b)$$

In other words, given M , the solution of Eq.(4) gives a unitary matrix U such that

$$U^T M U = \begin{pmatrix} \hat{m}_1 & 0 \\ 0 & \hat{m}_2 \end{pmatrix}. \quad (5)$$

The neutrino states that diagonalize the mass Lagrangian (1) are

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}_L = U^\dagger \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}_L. \quad (6)$$

But these are not yet the physical eigenstates since $\hat{m}_1$ and $\hat{m}_2$ are complex.

Let us now address the problem of CP violation. We want to see what happens to the phases in $\hat{m}_1$ and $\hat{m}_2$ as well as in the matrix U . It is convenient to consider two cases separately.

Case 1 : $\beta = 0$.

If $\beta = 0$, Eqs.(4) imply $\alpha = 0$ and $\tan(2\theta) = 2m_{12}/(m_{11} - m_{22})$. Furthermore, it is easy to see that both eigenvalues are real and given by the expressions :

$$\hat{m}_1 = m_{11} c^2 + m_{22} s^2 + 2m_{12} sc,$$

$$\hat{m}_2 = m_{11}s^2 + m_{22}c^2 - 2m_{12}sc, \quad (7)$$

where $c(s)$ is an abbreviation for $\cos\theta(\sin\theta)$. Since $\alpha = 0$, the remaining phases in U can be absorbed in the electron and muon fields, i.e.,

$$\begin{aligned} \exp(i\gamma_1/2)e &= e' \\ \exp(i\gamma_2/2)\mu &= \mu' \end{aligned} \quad (8)$$

On the other hand, the physical Majorana neutrino fields are

$$\chi_i = \nu_{iL} \pm \nu_{iR}^c, \quad i = 1, 2, \quad (9)$$

which obey the self-conjugate condition

$$\chi_i^c = \pm \chi_i. \quad (10)$$

Then the mass Lagrangian becomes

$$- \mathcal{L}_m = \sum_{i=1,2} |\hat{m}_i| \bar{\chi}_i \chi_i. \quad (11)$$

The plus or minus sign in Eqs.(9) and (10) should be taken for positive or negative $\hat{m}_i$ respectively⁸⁾. The weak charged current is

$$J^\alpha = (\bar{e}' \bar{\mu}')_L \gamma^\alpha \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}_L. \quad (12)$$

Hence, it is clear that the theory does not involve any CP violating phase.

Case 2 : $\beta \neq 0$

Now both eigenvalues $\hat{m}_i$ are complex numbers and in terms of the mass matrix parameters, they are either

$$\begin{aligned} \hat{m}_1 &= m_{11} + m_{12} e^{i\beta}, \\ \hat{m}_2 &= m_{11} - m_{12} e^{i\beta}, \end{aligned} \quad (13)$$

if $m_{11} = m_{22}$ (which in turn implies $\alpha = 0$ and $\theta = \pi/4$), or

$$\begin{aligned} \hat{m}_1 &= m_{11} \cos^2\theta / \cos(2\theta) - m_{22} \sin^2\theta e^{-2i\alpha} / \cos(2\theta), \\ \hat{m}_2 &= m_{22} \cos^2\theta / \cos(2\theta) - m_{11} \sin^2\theta e^{2i\alpha} / \cos(2\theta), \end{aligned} \quad (14)$$

if $m_{11} \neq m_{22}$ ($\alpha \neq 0$ and $\theta \neq \pi/4$).

Once we have obtained Eqs.(13) and (14), we can write

$$\hat{m}_i = m_i e^{i\theta_i} \quad (15)$$

where $m_i = |\hat{m}_i|$. Then Eq.(5) becomes

$$U^T M U = \Theta \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \Theta, \quad (16)$$

with

$$\Theta = \begin{pmatrix} e^{i\theta_1/2} & 0 \\ 0 & e^{i\theta_2/2} \end{pmatrix}, \quad (17)$$

a diagonal unitary matrix of phases. Since we have shown that U and Θ can be calculated in terms of the original mass matrix, we have proved that the unitary transformation $U' = U\Theta^\dagger$ brings M to a diagonal form with real and positive eigenvalues^{*)}. These we interpret as the mass of the neutrino fields. Notice that Eqs.(13) and (14) also imply that the eigenvalues are always non-degenerate, i.e., $m_1 \neq m_2$. Given U' , the weak current is

$$\begin{aligned} J^\alpha = & \bar{e}_L \gamma^\alpha \left\{ c \exp[-i(\frac{\gamma_1 + \theta_1}{2})] \chi_1 - s \exp[-i(\frac{\gamma_1 + \theta_2}{2} - \alpha)] \chi_2 \right\}_L \\ & + \bar{\mu}_L \gamma^\alpha \left\{ s \exp[-i(\frac{\gamma_2 + \theta_1}{2} + \alpha)] \chi_1 + c \exp[-i\frac{\gamma_2 + \theta_2}{2}] \chi_2 \right\}_L. \end{aligned} \quad (18)$$

However, as in Case 1, we still have the freedom of absorbing some of the phases by a convenient redefinition of the fields. This we shall do using two alternative approaches. In the first one, we choose the electron e' and muon μ' fields such that

$$\begin{aligned} \exp[i(\gamma_1 + \theta_1)/2] e &= e' \\ \exp[i(\gamma_2 + \theta_2)/2] \mu &= \mu' \end{aligned} \quad (19)$$

^{*)} One could also absorb the phases θ_i by generalizing what was done in the previous CP conserving case. Instead of Eq.(9), we could consider $\chi_i = \nu_{iL} + e^{-i\theta_i} \nu_{iR}^c$ as the mass eigenstates. The self-conjugation relation (10) would be $\chi_i^c = e^{i\theta_i} \chi_i$.

This implies the existence of a single phase in the Cabibbo-like matrix, namely

$$J^\alpha = (\bar{e}' \ \bar{\mu}')_L \gamma^\alpha \begin{pmatrix} c & -se^{i\delta} \\ se^{-i\delta} & c \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}_L, \quad (20)$$

with $\delta = (\theta_1 - \theta_2)/2 + \alpha$. Going back to Eqs.(13) and (14) one can prove that $\delta \neq 0$. Hence δ is a genuine CP violating phase.

In the second approach, we define the physical Majorana fields χ'_i in the following way :

$$\begin{aligned} \exp[-i(\gamma_1 + \theta_1)/2] \chi_1 &= \chi'_1, \\ \exp[-i(\frac{\gamma_1 + \theta_2}{2} - \alpha)] \chi_2 &= \chi'_2. \end{aligned} \quad (21)$$

Then, it is clear [cf. Eq.(18)] that, multiplying the second weak doublet by a common phase, we obtain a real Cabibbo mixing matrix. In other words, J^α becomes

$$J^\alpha = (\bar{e} \ \bar{\mu}')_L \gamma^\alpha \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} \chi'_1 \\ \chi'_2 \end{pmatrix}_L, \quad (22)$$

with

$$\mu' = \mu \exp\left[i\left(\frac{\gamma_2 - \gamma_1}{2} + \alpha\right)\right].$$

Despite the fact that J^α is given by Eq.(22), it is incorrect to assume that there is no CP violation in this case. What happens is that rather than having an explicit phase in the current J^α , now the same phase is hidden in the Majorana condition for the χ'_i fields. In fact, recalling that $\chi_i^c = \chi_i$, from Eq.(21) we have

$$\chi_1'^c \equiv C \bar{\chi}_1'^T = \exp[i(\gamma_1 + \theta_1)] \chi_1', \quad (23a)$$

and

$$\chi_2'^c = \exp[i(\gamma_1 + \theta_2 - 2\alpha)] \chi_2'. \quad (23b)$$

To illustrate the physical consequences of both approaches, let us consider, as an example, neutrinoless double beta decay [e.g., Ref. 9)]. The relevant Feynman diagram is shown in Fig. 1. Using Eq.(20), we can write the leptonic part of the amplitude as

$$L_1^{\alpha\beta} = \bar{e} \gamma^\alpha (1-\gamma_5) \left[c^2 \langle 0 | T(\chi_1 \bar{\chi}_1^c) | 0 \rangle + s^2 e^{2i\delta} \langle 0 | T(\chi_2 \bar{\chi}_2^c) | 0 \rangle \right] \times (1-\gamma_5) \gamma^\beta e^c, \quad (24)$$

where the Majorana condition $\chi_i^c = \chi_i$ implies

$$\langle 0 | T(\chi_i \bar{\chi}_i^c) | 0 \rangle = S_F(p) \equiv \frac{\not{p} + m_i}{p^2 - m_i^2 + i\varepsilon}. \quad (25)$$

Thus, Eq.(24) shows the well-known result that the amplitude is proportional to the neutrino mass. The phase δ is explicit in Eq.(24) and in principle it could be observed due to the χ_1, χ_2 interference⁴⁾.

In the alternative approach, Eq.(22) leads to

$$L_2^{\alpha\beta} = \bar{e} \gamma^\alpha (1-\gamma_5) \left[c^2 \langle 0 | T(\chi'_1 \bar{\chi}'_1{}^c) | 0 \rangle + s^2 \langle 0 | T(\chi'_2 \bar{\chi}'_2{}^c) | 0 \rangle \right] \times (1-\gamma_5) \gamma^\beta e^c, \quad (26a)$$

apparently, with no sign of CP violation. Nevertheless, the presence of the χ'_1 fields [cf. Eq.(23)] is such that, e.g.,

$$\langle 0 | T(\chi'_1 \bar{\chi}'_1{}^c) | 0 \rangle = \exp[-i(\gamma_1 + \theta_1)] S_F(p_1).$$

So, Eq.(26a) can be written

$$L_2^{\alpha\beta} = \bar{e} \gamma^\alpha (1-\gamma_5) \left[c^2 e^{-i(\gamma_1 + \theta_1)} S_F(p_1) + s^2 e^{-i(\gamma_1 + \theta_2 - 2\alpha)} S_F(p_2) \right] \times (1-\gamma_5) \gamma^\beta e^c, \quad (26b)$$

i.e., modulo an over-all unobservable phase, the second approach gives exactly the same result as the first one. In fact, the relative phase in Eq.(26b) is $\theta_1 - \theta_2 + 2\alpha = 2\delta$, and so $L_2^{\alpha\beta} = \exp[-i(\gamma_1 + \theta_1)] L_1^{\alpha\beta}$ *).

With this example, it should be quite clear that, in all processes where neutrino propagators are present, having the CP phase explicitly in J^α or hidden in the Majorana condition of the χ fields is an arbitrary choice with no physical consequences. Perhaps, what is not so obvious is that this is so even for the processes that do not involve neutrino propagators. On physical grounds this must be the case. Otherwise at least one of the two previously described

*) To show that the over-all phase is immaterial, it is enough to remember that one has the freedom to define the charge conjugation operator as $\psi^c = \eta_c C \bar{\psi}^T$, $|\eta_c| = 1$. Then, with $\eta_c = \exp[-i(\gamma_1 + \theta_1)]$ Eq.(23b) gives $\chi'_1{}^c = \chi'_1$ and, of course, with the same definition $\chi'_2{}^c = \exp[-i(\theta_1 - \theta_2 + 2\alpha)] \chi'_2$.

approaches would have been wrong. To elaborate on this point let us consider μ decay. We shall be concerned with the amplitude for the decay in the channel where both mass eigenstates are produced. Using Eq.(20) we have

$$T_1 = \frac{G_F}{\sqrt{2}} \bar{u}_e \gamma^\alpha (1-\gamma_5) \left[v_{\chi_1} \bar{u}_{\chi_2} c^2 - v_{\chi_2} \bar{u}_{\chi_1} s^2 e^{2i\delta} \right] \gamma_\alpha (1-\gamma_5) u_\mu, \quad (27)$$

where u and v denote the usual Dirac spinors. Naïvely one could conclude that Eq.(22) gives a similar result but without the factor $e^{2i\delta}$. The solution of this paradox lies in the fact that in the second approach the Feynman rules are modified because of the presence of the phases in Eq.(23). Indeed, if $\chi'^c = \eta \chi'$ with $|\eta| = 1$, the Fourier expansion of the χ' field is

$$\chi' = \int \frac{d^3k}{(2\pi)^3} \frac{m}{E} \sum_{\alpha=1,2} \left[a_\alpha(k) u^\alpha(k) e^{-ikx} + \eta^* a_\alpha^\dagger(k) v^\alpha(k) e^{ikx} \right]. \quad (28)$$

Then, using Eqs.(22) and (23) we obtain

$$T_2 = \frac{G_F}{\sqrt{2}} \bar{u}_e \gamma^\alpha (1-\gamma_5) \left\{ v_{\chi_1} \bar{u}_{\chi_2} c^2 \exp[-i(\chi_1 + \theta_1)] - v_{\chi_2} \bar{u}_{\chi_1} s^2 \exp[-i(\chi_1 + \theta_2 - 2\alpha)] \right\} \gamma_\alpha (1-\gamma_5) u_\mu. \quad (29)$$

Again the relative phase coincides with the one predicted by Eq.(27).

It is clear that the orthogonality of the neutrino spinors will prevent any interference between the two terms of the μ decay amplitude. So, if there are no further contributions to the amplitude, the CP violation will remain unobservable. There are, however, other processes, which do not involve neutrino propagators either, where the CP non-conservation can be measured. One example is the neutrino radiative decay^{10),11)}, $\chi_2 \rightarrow \chi_1 + \gamma$, depicted in Fig. 2. From Eq.(20) one can see that the amplitudes with eW and μW intermediate states (Fig. 2a) have the same phase. So no CP violation effect will result from the interference of these two diagrams. However, there are other diagrams which will have a different phase and will thus make the effect measurable. Indeed, the diagrams of Fig. 2b will also contribute if, for instance, the mass terms are generated by a Higgs triplet, H ¹¹⁾. It is then obvious that in the case of an explicit CP violation, there are relative phases between eW and eH amplitudes.

To summarize :

i) For a two-generation $SU(2) \times U(1)$ model, the most general Majorana mass matrix M is given by Eq.(2);

ii) If (and only if) the phases in M obey the relation $2\gamma_{12} = \gamma_1 + \gamma_2$ then there is no CP violation besides its possible presence in the Higgs sector which of course depends on the details of the model;

iii) If $2\gamma_{12} \neq \gamma_1 + \gamma_2$ there is a CP violating phase, which can be either explicit in the weak current or hidden in the Majorana condition. The two alternatives are equivalent;

iv) As a consequence of iii) CP violating effects can in principle be present even in processes which do not involve neutrino propagators, e.g., neutrino radiative decays.

ACKNOWLEDGEMENT

One of us (J.M.) thanks the Particle Physics Committee (Finland) for financial support.

REFERENCES

- 1) P. Langacker, Phys. Reports 72 (1981) 185, and references therein.
- 2) V.A. Lubimov et al., Phys. Lett. 94B (1980) 266.
- 3) J. Schechter and J.W.F. Valle, Phys. Rev. D 22 (1980) 2227.
- 4) J. Bernabéu and P. Pascual, CERN preprint TH.3393 (1982).
- 5) N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531.
- 6) M. Kobayashi and K. Maskawa, Progr. Theor. Phys. 49 (1973) 652.
- 7) G.C. Branco, Phys. Rev. Lett. 44 (1980) 504.
- 8) L. Wolfenstein, Phys. Lett. 107B (1981) 77.
- 9) M. Doi et al., Progr. Theor. Phys. 66 (1981) 1739, and references therein.
- 10) S.T. Petcov, Sov. J. Nucl. Phys. 25 (1977) 340; 25 (1977) 698(E).
- 11) P.B. Pal and L. Wolfenstein, Phys. Rev. D 25 (1982) 766;
R. Schrock, Nucl. Phys. B206 (1982) 359.

FIGURE CAPTIONS

Fig. 1 Diagram for the neutrinoless double beta decay.

Fig. 2 Diagrams contributing to the neutrino radiative decay.

Fig. 1

(a)

(b)

Fig. 2