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FINITE TEMPERATURE EFFECTS IN PRIMORDIAL INFLATION

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A B S T R A C T

We present a detailed study of a recently proposed model for primordial inflation based on a $N = 1$ locally supersymmetric potential. For a large class of parameters with which all cosmological constraints are satisfied, the temperature corrections can be neglected during the inflation period. At higher temperatures, the minimum is not at the origin, but very close to it.

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The possibility that the Universe has passed through a period of exponential expansion¹⁾ is certainly a very exciting one. In particular, the new inflationary Universe scenario seems to be able to solve not only several mysteries of modern cosmology, such as the horizon, flatness, homogeneity and isotropy problems, as well as the spectrum of density perturbations adequate for producing galaxies, but also questions which have been raised by Grand Unified Theories, ordinary and supersymmetric, such as the existence of a large number of primordial monopoles and/or gravitinos, the existence of domain walls, etc.

The new inflationary Universe scenario²⁾ cures the "graceful exit" difficulty left unsolved in Guth's original version¹⁾. By considering the whole visible Universe contained inside a single bubble, the necessity of completing the transition no longer exists. The inflationary period begins after a certain supercooled phase transition and bubble formation have started. The associated Higgs field (the inflaton) vacuum expectation value is moving inevitably towards the new global minimum of the scalar potential. This phase transition has frequently been taken²⁾⁻⁴⁾ to be the one responsible for the breaking of $SU(5)$ into $SU(3) \times SU(2) \times U(1)$, with the inflaton in the adjoint representation, and a Coleman-Weinberg⁵⁾ (C-W) potential. Even if the qualitative features are produced easily, the numbers do not fit. It appears quite clear by now that a standard C-W- $SU(5)$ phase transition cannot produce an acceptable inflationary scenario^{6),7)}.

The search for suitable mechanisms led us to consider supersymmetric versions of the C-W potential^{6),8)}, and, more recently^{9),10)}, a potential dictated by $N = 1$ supergravity¹¹⁾. In the first case, the ϕ^4 coupling can be made independent of the gauge coupling (while in the ordinary version sufficient inflation only occurs when $g^2 \lesssim 10^{-2}$) and the potentials are considerably flatter than in standard $SU(5)$. Inflation at the GUT scale still possesses possible but unnatural fine-tunings of several parameters. However, these fine-tunings begin to disappear (all the couplings and their ratios are bigger than 10^{-3}) as the transition scale approaches the Planck scale. Because the scale of the transition driving the inflation is close to the Planck scale, it can no longer coincide with the GUT breaking scale. Thus, the inflation so produced has been called primordial and the inflaton is a new Higgs boson, singlet under the GUT group⁸⁾⁻¹⁰⁾. In the case of primordial inflation, the monopole problem is not solved by the inflation (the number of monopoles can still be small enough if the $SU(5)$ transition takes place at $T \leq 10^{10}$ GeV). Being so close to the Planck scale, possible gravitational effects would not be completely suppressed by the presence of inverse powers of the Planck mass M_P . All terms in the potential possibly induced by radiative correction are automatically taken into account by writing the most general potential compatible with the symmetry of the theory, including the non-renormalizable terms¹²⁾.

Here we reconsider the idea presented in Refs. 9) and 10) in an attempt to treat some delicate aspects carefully. Since the inflaton is a complex field, the potential must be studied in the complex plane, and only when the line of minima follows the real axis can one consider the potential as a function of a real field. Moreover, finite temperature effects must be considered in order to be sure that the picture is consistent, even if they can be neglected for the inflationary period itself. Following closely the notation of Ref. 9), we write the superpotential $f(\phi)$ and the potential $V(\phi, \phi^*)$ at temperature $T = 0$ as functions of the complex inflaton scalar field ϕ :

$$f(\varphi) = \mu^3 \left[\lambda + \sum_{n=0}^m \frac{\lambda_n}{(n+1)} \left(\frac{\varphi}{M} \right)^{n+1} \right] \quad (1)$$

$$V(\varphi, \varphi^*) = e^{\frac{\varphi \varphi^*}{M^2}} \left[|f_\varphi|^2 - \frac{3}{M^2} |f|^2 \right] \quad (2)$$

where $M = M_p / \sqrt{8\pi} = 2.4 \times 10^{18} \text{ GeV}$, m is arbitrary and may be infinite, the couplings λ_i are all dimensionless, and μ is an as yet unspecified mass parameter. The generalized derivative $f_\phi(\phi, \phi^*)$ is

$$f_\varphi(\varphi, \varphi^*) = \frac{\partial f(\varphi)}{\partial \varphi} + \frac{\varphi^*}{M^2} f(\varphi) \quad (3)$$

From now on, we will use units in which $M = 1$.

Analysis of the potential at $T = 0$.

The conditions we will impose on the potential at $T = 0$ will be those necessary to produce a "phenomenologically" acceptable inflation. This is worthwhile only because, as we will see in the next section, at the temperature T_H , (the Hawking temperature)¹³⁾ at which the phase transition takes place, the finite temperature corrections are negligible for certain large classes of parameters. We ask for a metastable minimum at $\phi = 0$, and a global minimum at a value of $\phi \approx 1$, with a suitable barrier between them. We will also demand higher barriers in all other directions except the real axis in order to ensure that the Universe starting at high T at a minimum near to $\phi = 0$ will end up with a certain high probability in the desired minimum near to $\phi = \phi^* = 1$ *). We choose this last minimum to preserve supersymmetry, thus $f_\phi(\phi = \phi^* = 1) = 0$. In addition we require a vanishing cosmological constant at the global minimum $V(\phi = \phi^* = 1) = 0$, which,

*) The real requirement is employed solely to simplify the analysis.

together with the previous requirements, imposes $f(\phi = 1) = 0$. We use $\phi = \phi^* = 1$, even if we know that we should not exactly arrive at this point, at which our effective theory loses its validity. The choice of this minimum is the result of a compromise, and could be slightly changed without essentially modifying the following analysis, which relies mainly on the behaviour of the potential for small values of ϕ .

Being a supersymmetric minimum, the curvature of the potential is

$$\left. \frac{\partial^2 V}{\partial \varphi^* \partial \varphi} \right|_{\varphi = \varphi^* = 1} = e^1 \left| f''(\varphi = 1) \right|^2 > 0 \quad (4)$$

since $(\partial^2 V / \partial \phi \partial \phi) |_{\phi = \phi^* = 1} = 0$. Thus, $f''(\phi = 1)$ is the inflaton mass M_ϕ . It has been estimated in Ref. 10) to be $0(10^{13})$ GeV through three independent arguments. To solve the gravitino problem, we need $M_\phi \lesssim 10^{15}$ GeV, while baryon generation requires $M_\phi \gtrsim 10^{13}$ GeV, and finally the density perturbations are too large unless $M_\phi \lesssim 0(10^{13})$ GeV. Let us concentrate in the region near to the origin, where only the first few terms of the series expansion of the superpotential f dominate. Expanding also the exponential of Eq.(2), and going up to the fourth order in the fields, one gets

$$\begin{aligned} V(\varphi, \varphi^*) = & (\lambda_0^2 - 3\lambda_1^2) + (\varphi + \varphi^*)(\lambda_0\lambda_1 - 2\lambda_1\lambda_0) + \varphi\varphi^*(\lambda_1^2 - 2\lambda_2^2) + \\ & + (\varphi^{*2} + \varphi^2)(\lambda_0\lambda_2 - \frac{1}{2}\lambda_1\lambda_1) + (\varphi^{*3} + \varphi^3)\lambda_0\lambda_3 + \\ & + \varphi\varphi^*(\varphi^* + \varphi)(\lambda_1\lambda_2 + \lambda_1\lambda_0) + \\ & + (\varphi\varphi^*)^2(\frac{\lambda_0^2}{2} + \lambda_1^2 + \frac{\lambda_2^2}{2}) + (\varphi^{*4} + \varphi^4)(\lambda_0\lambda_4 + \frac{\lambda_3\lambda_1}{4}) + \\ & + (\varphi\varphi^*)(\varphi^2 + \varphi^{*2})(\frac{4}{3}\lambda_0\lambda_2 + \lambda_1\lambda_3) + \\ & + \varphi^2\varphi^{*2}(\frac{\lambda_1^2}{4} + \lambda_2^2) \end{aligned} \quad (5)$$

We need a minimum at the origin, so that $(\partial V / \partial \phi) |_{\phi=0} = 0$ which means that $\lambda = \lambda_1/2$, with a positive cosmological constant Λ ,

$$V(\varphi=0) = \Lambda = \mu^6 \left(\lambda_0^2 - \frac{3}{4} \lambda_1^2 \right) \quad (6)$$

It is, therefore, a non-supersymmetric point. Both eigenvalues of the second derivative of the potential must be positive at the origin; moreover, we will ask for the curvature on the real axis to be the smallest one, as an indication that the barrier will be smaller in that direction; that means

$$0 < 2\lambda_0\lambda_2 < \frac{\lambda_1^2}{2} - 2\lambda_0\lambda_2 \quad (7)$$

We write down the conditions on $V(\phi = \phi^*)$ to solve the cosmological requirements for inflation. We keep in mind that only if the real axis is the preferred way is this a correct procedure. We have numerically analyzed the potential on the complex plane for various sets of the parameters and, as can be seen for a particular choice in Fig. 1, it is possible to fulfil this requirement. On the real axis, the potential is

$$V(\varphi = \varphi^*) = \mu^6 \left[\delta + \gamma \varphi^2 - \beta \varphi^3 + \alpha \varphi^4 + \dots \right] \quad (8)$$

where, for $\lambda = \lambda_1/2$,

$$\delta = \lambda_0^2 - \frac{3}{4} \lambda_1^2 \quad (9)$$

$$\gamma = 2 \lambda_0 \lambda_2 \quad (10)$$

$$-\beta = 2 \left(\lambda_0 \lambda_3 + \lambda_1 \lambda_2 + \frac{\lambda_1 \lambda_0}{2} \right) \quad (11)$$

$$\alpha = \frac{\lambda_0^2}{2} + \frac{9}{8} \lambda_1^2 + \lambda_2^2 + \frac{8}{3} \lambda_0 \lambda_2 + \frac{9}{4} \lambda_1 \lambda_3 + 2 \lambda_0 \lambda_4 \quad (12)$$

In order to fix ideas with concrete numbers, we will take the following set of parameters as an example : $\mu = 10^{-2}$, $\lambda_0 = 0.4$, $\lambda_1 = 0.05$, $\lambda_2 = 0.001$, $\lambda_3 = -0.1$, $\lambda_4 = 0.01$. The vacuum expectation value of the field ϕ must be constrained, to remain near to the origin long enough to allow the temperature to lower to $T_H = H/2\pi \sim 2/3 \times 10^{-7}$, with H the Hubble parameter :

$$H^2 = \frac{1}{3} \mu^6 \delta \sim 5 \times 10^{-14} \quad (13)$$

before the transition begins. The reason is that otherwise, thermal fluctuations could carry the field to the global minimum and the amount of inflation obtained could not be determined. Therefore, the action governing the transition must be large, corresponding to a large barrier between the two minima.

The position of the maximum near to zero, calculated by taking V only up to the fourth order in ϕ , is

$$\varphi_{max.} \sim \frac{2\gamma}{3\beta} \sim 0.01 \quad (14)$$

in the approximation $32/9 \alpha\gamma/\beta^2 \sim 0.07 \ll 1$ (there is also another minimum predicted at $\phi \sim 3/4 \beta/\alpha \sim 0.5$ which must fall outside the validity of this approximation, which can very roughly be estimated as $\phi < \alpha \sim 0.08$). The value of V at this maximum, $V_{\max}$, is

$$V_{\max} = V(0) + \frac{4}{27} \frac{\gamma^3}{\beta^2} \mu^6 \quad (15)$$

The probability of overcoming the barrier is given by⁴⁾

$$P \sim \mu^{12} \gamma^2 e^{-B} \quad (16)$$

where the Euclidean action B for tunnelling from the local minimum at 0 to the local maximum can be approximated for $V_{\max} - V(0) \ll V(0)$ by

$$B \sim 24 \pi^2 \left(\frac{V_{\max} - V(0)}{V(0)^2} \right) \sim \frac{32}{9} \frac{\gamma^3}{\beta^2 \delta^2 \mu^6} \sim 2 \cdot 10^7 \quad (17)$$

The condition we need to satisfy is $B \gg 1$. Once the barrier has been overcome, the slope of the potential around the maximum must be small, so as to get a roll over time scale $\tau \sim 3H/2\gamma\mu^6$ long enough, that is, the final scale factor of the Universe has to be $e^{H\tau} > e^{65}$ times the original one, so that

$$H\tau \sim \frac{3H^2}{2\gamma\mu^6} \sim \frac{\delta}{2\gamma} \sim \frac{\lambda_0}{4\lambda_2} > 65 \quad (18)$$

It is 100 with our choice of parameters.

In the inflationary models a nearly scale independent spectrum of density fluctuations is predicted¹⁴⁾. This unique qualitative feature is one of the successes of these models, since this kind of spectrum was proposed by Harrison and Zel'dovich¹⁵⁾ as necessary for the explanation of the origin and organization of galaxies. Quantitatively, a very particular value, $(\delta\rho/\rho)|_H \sim 10^{-4}$, is the only one suitable for the formation of galaxies and simultaneously compatible with the predominant isotropy of the microwave background radiation. For a potential containing a cubic term, the magnitude of fluctuations as they re-enter the horizon is¹⁶⁾:

$$\frac{\delta\rho}{\rho} \sim \frac{\mu^6}{\sqrt{2\pi^3}} \frac{\beta}{H} \left[\ln\left(\frac{H}{k}\right) \right]^2 \sim -(2\lambda_3 + \lambda_1) \mu^3 \cdot 10^3 \sim 10^{-4} \quad (19)$$

Analysis of the potential at finite T.

To one-loop order, the finite temperature corrections to the effective potential can be written as¹⁷⁾

$$\Delta V_T(\varphi, \varphi^*) = \left(\frac{T^2}{8} \frac{\partial^2 V(\varphi, \varphi^*)}{\partial \varphi \partial \varphi^*} - \frac{\pi^2}{90} N T^4 \right) e^{-\frac{M(\varphi, \varphi^*)}{T}} \quad (20)$$

Here, $N = N_{\text{bosons}} + 7/8 N_{\text{fermions}}$ is the number of the degrees of freedom of ϕ . The T^4 term scales the potential equally at any point, and one can neglect it for our purposes. The presence of the factor $e^{-M(\phi, \phi^*)/T}$ acts as a cut-off, saying that the correction is only relevant for $T > M(\phi, \phi^*)$ where $M^2(\phi, \phi^*)$ are the eigenvalues of the curvature matrix at every point; in particular, at the minima they are the physical masses. Therefore at $\phi = 0$ the correction is relevant for $T^2 > \lambda_1^2/2 - 2\lambda_0\lambda_2 \sim 10^{-15}$ [see Eq.(7)], or for $T \gtrsim 10^{-7} \sim 10^{11}$ GeV, and near $\phi = \phi^* = 1$ the correction is important for $T^2 > e^1 |f''(0)| \sim M_\phi^2$; that is, $T > 10^{-5} \sim 10^{13}$ GeV. Let us recall that $T_H \sim 10^{-7} \sim 10^{11}$ GeV. For the given ranges of very high T, one has to worry about the T^2 term of the temperature correction. Unless the function $\partial^2 V / \partial \phi \partial \phi^*$ also has a minimum in coincidence with every minimum of the $T = 0$ potential, the position of the minima will change with the temperature. The shifting of the minimum at $\phi = 0$ with increasing T could destroy the picture if it were to move in the positive real direction going to the other side of the barrier. As we will see, however, the shifting is small and towards the negative axis. We are aware that Eq.(2) has been derived for renormalizable potentials, while the actual correction could differ by functions of T/M and ϕ/M , and the one-loop calculation would lose any meaning for $\phi = 1$ or $T = 1$. The use of Eq.(2) near to the origin, where the approximation goes up to the forth order in the field, is completely reliable, we think, while its validity breaks down as $|\phi| \rightarrow 1$.

Let us write the full expression of the correction near zero :

$$\begin{aligned} \Delta V_T(\varphi, \varphi^*) = \frac{T^2}{8} \bigg[& \frac{\lambda_1^2}{2} + (\varphi + \varphi^*) \lambda_1 (2\lambda_2 + \lambda_0) + \\ & + \varphi \varphi^* 2(\lambda_0^2 + 2\lambda_1^2 + 2\lambda_2^2) \\ & + (\varphi^2 + \varphi^{*2}) (4\lambda_0\lambda_2 + 3\lambda_1\lambda_3) \bigg] \end{aligned} \quad (21)$$

As can be immediately seen, this term has a minimum on the negative real axis, which, for values of λ_0 bigger than the other λ 's, is at

$$\varphi_{\min}(T \rightarrow \infty) \simeq -\frac{\lambda_1}{2\lambda_0} = -0.075 \quad (22)$$

in our case. We present this value only to argue that, for any finite T , the minimum of the whole potential, $V_{\text{eff}}(T; \phi, \phi^*) = V(\phi, \phi^*) + V_T(\phi, \phi^*)$ will be at intermediate values. The second order approximation to V actually breaks down much before, at around $\phi \sim 0.01$. The minimum, for $T^2 < 8\lambda_2/\lambda_0 \sim 2 \cdot 10^{-2}$, is at $\phi_{\min} \simeq -T^2\lambda_1/\lambda_2 \sim -3.5 T^2$. As we can see in Fig. 2, the minimum is practically already at $T \sim 10^{-2}$ at the origin for our choice of parameters. We see now that the value of $V_{\text{eff}}(T)$ at the minimum is always $V(0)$ with a very good approximation. Even for the value of the minimum given in Eq.(22), the T corrections are negligible at any $T < 1$.

$$\begin{aligned} V_{\text{eff}}(T) \Big|_{\min} &= \mu^6 \left[\left(\lambda_0^2 - \frac{3}{4} \lambda_1^2 + \frac{\lambda_1^2}{2\lambda_0} + \dots \right) + \frac{T^2}{8} \left(-2 \frac{\lambda_1^2 \lambda_2}{\lambda_0} + \frac{9}{4} \frac{\lambda_1^4}{\lambda_0^2} + \dots \right) \right] \\ &\simeq \mu^6 \lambda_0^2 \simeq V(\varphi = \varphi^* = 0) \end{aligned} \quad (23)$$

for $\lambda_0 \gg \lambda_1, \lambda_2, \lambda_3$.

We have proved that even for T much bigger than $T_H \sim 10^{-7}$, the temperature corrections around zero are negligible. The validity of these equations allow us to follow the system only up to, say, $T \sim 10^{-1}$. Assuming that at the minimum near to $\phi = 1$, the correction is proportional to $M_\phi^2 T^2/8$, we could estimate the critical temperature T_c at which the values of $V_{\text{eff}}(T_c; \phi, \phi^*)$ in both minima, $\phi = 0$ and $\phi = 1$, are equal. Additional conditions could be imposed on the derivatives of $f(\phi)$ in order to ensure that the minimum near to $\phi = 1$ does not move with T so that $V_{\text{eff}}(T_c; \phi = \phi^* = 1) \sim M_\phi^2 T_c^2/8$. Therefore, T_c would be $T_c^2 \sim 8\mu^6 \lambda_0^2 / M_\phi^2 \sim 10^{-2}$, $T_c \sim 10^{-1}$. We have not analyzed the potential in between the region around zero and $\phi = 1$. We take it for granted that a smooth behaviour between these two areas could be always obtained with a sufficient number of terms in the superpotential $f(\phi)$. We have therefore presented a consistent picture for producing inflation within an $N = 1$ supergravity framework. At high temperatures, the minimum is not at the origin but approaches it very fast as T decreases. At T_H , the relevant temperature at which the transition driving inflation occurs, finite temperature corrections are negligible.

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NOTE ADDED

We have recently received a paper by Linde (Academy of Sciences of the USSR preprint "Primordial inflation without primordial monopoles") also considering finite temperature corrections. For reasons contained in the present work, we tend to disagree with him on this point.

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FIGURE CAPTIONS

Fig. 1 The potential $V(\phi, \phi^*)$ at $T = 0$ near to the origin for our particular choice of parameters, with an upper cut. This cut is made at the maximum of the barriers which separate the local minimum at the origin from other minima different from the one on the real axis in which we are interested. The height of the mentioned barriers is $V - V(0) = 1.6 \times 10^{-19}$ ($M = M_p/8\pi = 1$ in our units). The barrier on the real axis is eight times lower. It can be seen here that the real axis is the preferred path out from the origin.

Fig. 2 We present the difference of the effective T^2 dependent potential on the real axis around the origin, with respect to $V(0) = 0.16 \times 10^{-12}$, the value of the $T = 0$ potential at the origin, for various $T \leq 10^{-1}$, when our choice of parameters is used. The units are always such that $M = M_p/8\pi = 1$. The difference is of the order of $10^{-19} - 10^{-18}$. We can see that the minimum at high temperatures lies on the negative real axis near to the origin (~ 0.013 for $T = 1 \times 10^{-1}$) and approaches it very fast as temperature decreases (it sits practically there at $T = 1 \times 10^{-2}$). The potential at the Hawking temperature $T_H \sim 10^{-7}$ coincides with the one at $T = 0$.

FIGURE 1

FIGURE 2