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THE SELF-ENERGY OF A CONFINED QUARK*

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Abstract

We present the results of a numerical calculation of the $O(\alpha_s)$ self-energy for a massless quark confined to a spherical cavity.

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In a previous paper^[1] (henceforth referred to as I) we developed methods for calculating QCD loop diagrams in a fixed spherical cavity with confining (i.e. "bag") boundary conditions. As an example, we outlined the calculation of the $O(\alpha_s)$ quark self-energy Σ (Fig. 1) in some detail. Here we present a numerical computation of Σ which explicitly demonstrates the power of our methods. Specifically, we find that

- 1) After having removed the logarithmically divergent boundary independent contribution Σ^0 , the final expression for the remaining part $\tilde{\Sigma} = \Sigma - \Sigma^0$, is well-suited for a precise numerical evaluation.
- 2) Only the first few terms in the multiple reflection expansion (M.R.E.) of the propagators give significant contribution to Σ , showing that it is fairly insensitive to the detailed shape of the surface.
- 3) Analyticity in ω provides several strong constraints that can be used to check the numerical calculations.

After reviewing a few results from I*, we outline the calculation and discuss the above points. We consider only the lowest $1 S_{1/2}$ state for mass $m=0$. A fuller account of our work including results for $m \neq 0$ and excited states will be given elsewhere^[2]. Furthermore, we will here only briefly discuss the phenomenological implications of our results. There are three previous calculations of Σ ^[3,4,5]. At the end we will comment on them.

As described in I, we work in a covariant (Feynman) gauge, and

*Our notation will always follow I.

separate the quark and gluon propagators, $S(\vec{x}, \vec{x}', \omega)$ and $D_{\mu\nu}(\vec{x}, \vec{x}', \omega)$ into free field and boundary dependent parts, i.e. $S = S^0 + \tilde{S}$ and $D = D^0 + \tilde{D}$. The most divergent part of any diagram is associated with the free propagators, and for Σ , we showed that the only divergence comes from the $S^0 D^0$ contribution, Σ^0 . This piece must be handled separately and carefully to satisfy the Ward identity which ensures gauge invariance. For $m=0$, Σ^0 was shown to be finite, and we derived^[1],

$$\Sigma^0 = \frac{\alpha_s}{3\pi} \int d^4p \bar{\phi}(\vec{p}) \not{p} \not{p}^2 \phi(\vec{p}) \delta(p^0 - \omega_q) \quad (1)$$

In the following, all "external" wavefunctions, $\psi(\vec{x})$ (and Fourier transforms $\phi(\vec{p})$), will be $1s_{1/2}$, and $\omega_q = \omega_{1s_{1/2}} \approx 2.04/R$. It is more convenient to use the $(\vec{x}, \omega)$ space expression

$$\Sigma^0 = \frac{16i\pi\alpha_s}{3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int d^3x d^3x' \bar{\psi}(\vec{x}) [\omega_q \gamma^0 + i \vec{\gamma} \cdot \vec{\nabla}_x] \psi(\vec{x}') \Delta^0(\vec{x} - \vec{x}', \omega) \Delta^0(\vec{x} - \vec{x}', \omega_q - \omega) \quad (2)$$

which can be derived from (1) by Fourier-transformation, or directly from the $(\vec{x}, \omega)$ Feynman rules. Eq.(2) is conveniently evaluated by partial integration, and is finite due to the bag boundary condition^[6]. In a condensed notation, the contribution to Σ from the boundary dependent parts of the propagators reads,

$$\tilde{\Sigma} \equiv \int_{-\infty}^{\infty} d\omega \tilde{\Sigma}(\omega) = -i\alpha_s \sum_J \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_0^R r_1^2 dr_1 \int_0^R r_2^2 dr_2 \tilde{\Sigma}_J(r_1, r_2, \omega) \quad (3)$$

where ω is the energy carried by the quark line, and J is the gluon total angular momentum. The explicit expression for $\tilde{\Sigma}_J(r_1, r_2, \omega)$ involves^[1]: A sum over space and time-like gluon contributions and over the three combinations $S^0 \tilde{D}$, $\tilde{S} D^0$ and $\tilde{S} \tilde{D}$; sums over angular

momenta and Dirac indices; partial wave expressions for external wavefunctions, propagators, and vertices.

The analytic structure of $\tilde{\Sigma}(\omega)$ is shown in Fig. 2. The cuts with branch points at $\omega=0$ and $\omega=\omega_q$ come from $\tilde{S}(\omega)$, $S^0(\omega)$ and $\tilde{D}(\omega_q-\omega)$, $D^0(\omega_q-\omega)$ respectively, and the poles occur at the eigenfrequencies of the quark and gluon cavity modes (including those of longitudinal and timelike gluons). To avoid the poles it is convenient to perform a Wick rotation^[1] to a contour parallel to the imaginary axis. As long as $\omega_q - \omega_0 < \text{Re}\omega < \omega_q$, where $\omega_0 R \approx 2.08$ is the lowest (longitudinal and time-like) gluon pole, we pick up no extra pole terms.* Any contributions from encircled cuts will cancel between eqs. (2) and (3) since the full propagators are meromorphic. Some care is needed when using eq.(2), since, in contrast to eq.(3), ω does not refer to the quark energy but to the average of two terms $\sim S^0(\omega) D^0(\omega_q - \omega)$ and $S^0(\omega_q - \omega) D^0(\omega)$. (The expression for the individual terms are considerably more complicated). This implies that the result of rotating eq.(2) to a contour with $\text{Re}\omega = \alpha\omega_q$, $0 < \alpha < 1$, should be added to the average of eq.(3) integrated along two contours, one with $\text{Re}\omega = \alpha\omega_q$ and the other with $\text{Re}\omega = (1-\alpha)\omega_q$. The contour with $\alpha=0.5$ requires no averaging and is also favored by being maximally far away from the poles as well as the points $\omega=0$ and $\omega=\omega_q$ where expressions like $\omega h_{\ell}^{(1)}(\omega R) j_{\ell}(\omega r)$ might cause numerical difficulties.

The result for Σ as calculated along the $\alpha=0.5$ contour is**

*The pole in the timelike gluon propagator $\tilde{D}_{00}(\omega_q - \omega)$ at $\omega_q - \omega = 0$ cancels against a term in the one gluon exchange energy for color singlets. See Appendix A of I or Ref. 4 for discussion.

**One can easily show that $\tilde{\Sigma}_J(\omega)$ is a real analytic function so

$$-i \int_{-\infty}^{\infty} d\omega \rightarrow 2 \text{Re} \int_0^{\infty} d\eta \quad \text{where } \omega = \alpha\omega_q + i\eta.$$

$$\Sigma = \Sigma^0 + \tilde{\Sigma} = (-0.126 + 1.036 \pm 0.001) \frac{\alpha_S}{R} = (0.910 \pm 0.001) \frac{\alpha_S}{R} \quad (4)$$

In Fig.(3) we show the integrand in eq.(3) for $J = 0-3$ and in Fig.(4) the corresponding integrals. The above result is obtained by two extrapolations. For fixed J the tails of the ω -integrals are calculated by the first three terms in a Taylor expansion in $1/\omega$, $a\omega^{-4} + b\omega^{-5} + c\omega^{-6}$ where a , b and c are fitted in the range $\omega = 10-20$. Finally the J summation is extrapolated to $J \geq 9$ by fitting the Taylor series (in $1/J$) $aJ^{-2} + bJ^{-3} + cJ^{-4}$ for $2 \leq J \leq 8$. For details, see ref. 2.

We have taken several precautions in order to avoid numerical mistakes. The entire program was written in complex notation for clarity and simplicity and to avoid phase errors; the propagators and wavefunctions were separately checked by computing various known matrix elements;^[2] the coefficients of the radial integrations were checked by hand for several sets of angular momenta. We also performed the following global checks on the calculation:

a) The discontinuity in $\tilde{\Sigma}(\omega)$ across the upper cut was calculated in two different ways. First by directly performing the $\vec{x}$ and $\vec{x}'$ integration in eq.(2) and using $\text{disc}\tilde{\Sigma}(\omega) = \text{disc}\Sigma^0(\omega)$. Second, from eq.(3)*, summing the partial wave series for $J \leq 4$. The two calculations agree to 0.1%. This checks the numerical calculation, and helps in checking the overall sign (see below).

*To compare with eq.(2) we must again average over points symmetric around $\omega = 0.5\omega_q$

b) For $\omega \approx 0$, $\tilde{\Sigma}(\omega)$ is dominated by the pole in $\tilde{D}_{00}(\omega_q - \omega)$ and $D_{LONG}(\omega_q - \omega)$ $\omega_q - \omega \approx 2.08$. The residues of these poles can be calculated analytically (up to a one dimensional integral) using general properties of the propagators^[2]. The analytic result agrees with the computer calculation in both modulus and phase. This checks, in particular, the relative sign of the timelike and longitudinal gluon propagators, and is part of the overall sign check (see below).

c) Redoing the entire calculation by averaging contours with $\text{Re}\omega = 0.25\omega_q$ and $\text{Re}\omega = 0.75\omega_q$, we obtained, within the quoted error, the same final answer as for the contour $\text{Re}\omega = 0.5\omega_q$. This requires the change in $\tilde{\Sigma}$ to cancel the change in Σ^0 . Since these are calculated independently and with entirely different methods (note that no extrapolations are needed to calculate Σ^0), we consider this test as a very strong one. We also explicitly checked that the change in $\tilde{\Sigma}$ when integrated along various contours, partial wave by partial wave, was equal to the total discontinuity integrated along the real axis between the contours.

d) The sign of our result is consistent with the following chain of arguments. The sign of $\text{Im } \Sigma^0$ is fixed to be negative by unitarity. The signs of the real and imaginary parts of $\tilde{\Sigma}$ for ω real are fixed relative to Σ^0 by the check a) described above. Finally factors of i introduced by Wick rotation are checked when the residue of the pole at $\omega = -0.04$ (check b)) was found to agree with the analytic calculation.

In the Table we give the contributions to $\Sigma \tilde{\Sigma}(J)$ from successive $J > 1$ terms in the multiple reflection expansion.* As discussed in I, the rapid convergence of the M.R.E. suggests that $\tilde{\Sigma}$ is insensitive to small variations in the shape of the surface.

*The M.R.E. does not converge for $J = 0, 1$ and small n , but being a low momentum effect, this does not invalidate our argument^[2]. For calculational convenience, we have grouped the quark reflections in pairs.

The (gauge invariant) electromagnetic energy shift, ΔE^{cm} $\approx 0.83 \alpha/R$, is easily obtained by retaining only the SD^0 contributions to Σ .

To compare our results with those of Refs (3-5), we consider the quantity,

$$\Delta E = \Sigma + \frac{1}{2} \Delta E^{\text{INT}} \approx (0.903 \pm 0.001) \frac{\alpha_s}{R} \quad (5)$$

where ΔE^{INT} is the $O(\alpha_s)$ (spin-independent) electric contribution to the $\bar{q}q$ interaction energy^[1]. For overall color singlets, ΔE is gauge invariant, while Σ is not. In our formalism ΔE^{INT} is to be calculated using the propagator,^[4]

$$D(\vec{x}, \vec{x}') = \lim_{\omega \rightarrow 0} D_{00}(\vec{x}, \vec{x}', \omega) = \frac{1}{4\pi |\vec{x} - \vec{x}'|} + \frac{1}{4\pi R} \sum_{l=1}^{\infty} \frac{l+1}{l} \left(\frac{rr'}{R^2} \right)^l P_l(z) \quad (6)$$

$$+ \frac{1}{4\pi} \left(\frac{r^2 + r'^2}{2R^3} - \frac{9}{5R} \right)$$

giving $\Delta E^{\text{INT}} \approx -0.013 \alpha_s/R$. Our result $\Delta E \approx 0.90 \alpha_s/R$ is to be compared with the previously obtained values $.16 \alpha_s/R$ ^[3], $.40 \alpha_s/R$ ^[4], and $0.84 \alpha_s/R$ ^[5,7]. At first sight the situation is not very satisfactory. However, we believe that the methods used in refs. 3 and 4 are less reliable than ours and those of refs. 5 and 7. We are encouraged that our result agrees with the latter to within 10%. Note, however, that this discrepancy is much greater than our estimate of the total error in the numerical calculation.

The reason we doubt the reliability of Refs. (3) and (4) is the fact that the potentially linearly divergent term, Σ^0 , is calculated numerically without ensuring that the Ward identity is satisfied. It is well known from ordinary perturbation theory that an arbitrary

cutoff procedure can in this case cause finite ambiguities in the answer.* Our eqs. (1) and (2) do respect the Ward identity as described in I.

The calculation in Ref.(5) uses the same general strategy as ours and we have no objections in principle. There are, however, important technical differences. For one, Ref.(5) uses Coulomb gauge. The calculation of Σ^0 in this gauge is quite complicated. It is based on a recently developed and not so well explored formalism.^[8] [To our knowledge there is no calculation of any known quantity such as the hydrogen Lamb-shift, using the methods of Ref.(8).] Also, in Ref.(5) the ω -integration and the partial wave summation is performed prior to the radial integration. This complicates the computation since very high J are needed ($J \sim 300$) to get accurate results for $r \simeq r' \simeq R$, and no constraints similar to those points a) - c) above can be derived.

We cannot fail to note that the quark self-energy we have computed is disturbingly large relative to the $O(1)$ energy, $0.90/R$ as compared to $2.04/R$ for $\alpha_s = 1$. It is, however, not clear that this number should be taken at face value. One strong indication to the contrary, is that if we make a cutoff at $\simeq 1$ GeV in the euclidian loop momentum q_E when calculating $\tilde{\Sigma}$, the value $.90\alpha_s/R$ drops substantially down to $\approx .4\alpha_s/R$. [To get this estimate we put $q_E^2 \approx \eta^2 + J(J+1)/\langle r^2 \rangle \leq 1 \text{ GeV}^2$ using $\langle r^2 \rangle \approx .7R^2$ and $R \simeq 3 \text{ GeV}^{-1}$ ^[9].] A cutoff of this kind is expected since confinement is a low momentum phenomenon. Another reason not to directly use our results in phenomenological applications, is that our calculation includes contributions from

* For example, if the loop integral is regulated by cutting off the Euclidean four momentum at Λ , then the correct result is obtained only if the external momentum is routed entirely through the quark propagator.

"spurious" intermediate quark modes corresponding to translations of the external state^[3]. For further discussion of these points, see ref.2.

In conclusion, we believe we have demonstrated the power and efficiency of our methods for calculating loop diagrams in QCD subject to confining boundary conditions. The phenomenological consequences of our work have as yet to be explored.

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REFERENCES

1. T. H. Hansson and R. L. Jaffe, MIT Report No. CTP #1026 (September 1982).
2. S. Goldhaber, T. H. Hansson and R. L. Jaffe, work in progress.
3. J. D. Breit, Columbia University Report No. CU-TP-229 (1982).
4. S. A. Chin, A. K. Kerman and X. H. Yang, Nucl. Phys. A382, 355 (1982).
5. J. Baacke, Y. Igarashi and G. Kasperidus, CERN Report No. TH.3509 (January 1983).
6. The $(\vec{x}, t)$ version of eq.(2) was shown to us by K. Johnson.
7. The value $1.2\alpha_s/R$ given in ref.(5) has been revised, J. Baacke, private communication, and erratum to CERN Report No. TH. 3509.
8. G.S. Adkins, Phys. Rev. D27, 1814 (1983).
9. C.E. Carlson, et.al., Phys. Rev. D27, 1556 (1983).

QUARK REFLECTIONS

GLUON REFLECTIONS

	0	1+2	3+4	5+6	$\sum_{n=1}^6 s^n$	$\tilde{s}$
0		.45464	-.00112	-.00003	.45348	.45348
1	.10360	.06519	.00010	-.00001	.06528	.06528
2	.07557	-.03862	.00005	.00001	-.03856	-.03856
3	.01689	-.01508	-.00004	.00001	-.01511	-.01511
4	.00493	-.00397	-.00003	.00000	-.00400	-.00400
5	.00116	-.00104	-.00002	.00000	-.00105	-.00105
6	.00030	-.00025	-.00001	.00000	-.00025	-.00025
7	.00007	-.00006	.00000	.00000	-.00006	-.00006
8	.00002	-.00001	.00000	.00000	.00000	.00000
$\sum_{n=1}^3 D^n$	.20252	.00618	.00005	.00002	.00625	.00625
$\tilde{D}$	.20254	.00616	.00005	.00002	.00623	.00623

Table: Contributions to $\tilde{\Sigma}$ for $J > 1$ from different terms in the multiple reflection expansion. The last row and column are calculated by using the exact expressions for $\tilde{D}$ and $\tilde{S}$ respectively. All higher reflection terms are zero to the given accuracy.

FIGURE CAPTIONS

- Fig.1 $O(\alpha_s)$ quark self-energy diagram.
- Fig.2 Analytic structure of the integrand in eq.(3). Gluon cuts and poles are denoted by wavy lines and crosses; quark cuts and poles by zig-zag lines and dots.
- Fig.3 Total and partial wave ($J = 0-3$) contributions to $d\tilde{\Sigma}/d\omega$.
- Fig.4 Contributions to $\tilde{\Sigma}$ for different values of the total gluon angular momentum J . The curve is used to extrapolate to $J \geq 9$ as described in the text.

FIGURE 1

FIGURE 2

FIGURE 3

FIGURE 4