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THE NEUTRON ELECTRIC DIPOLE MOMENT IN LEFT-RIGHT SYMMETRIC GAUGE MODELS

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A B S T R A C T

Vector boson and Higgs contributions to the neutron electric dipole moment d_n are calculated to lowest order in a general gauge theory. To evaluate the exchange diagrams we use a simple harmonic oscillator bound state model of the neutron. We present estimates of the various contributions for the minimal $SU(2)_L \times SU(2)_R \times U(1)$ model with a single scalar multiplet coupling to quarks. These estimates are illustrated by a detailed numerical study of a specific model of weak mixing angles. Allowing for maximally CP violating phases the present experimental limit $|d_n| < 4 \cdot 10^{-25} \text{ e} \cdot \text{cm}$ would imply $M_{W_2} \gtrsim 4 \text{ TeV}$ (for $m_t = 30 \text{ GeV}$) for the mass of the heavy charged vector boson W_2 . Whereas the neutral Higgs contributions turn out to be negligible the charged Higgs contribution may become sizeable for large m_t .

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1. INTRODUCTION

During the nearly 20 years since the original discovery [1] CP violating effects have only been detected in the K^0 system. Ever since that time the electric dipole moments (EDMs) of elementary particles, in particular of the neutron, have been investigated intensively as another possible manifestation of CP violation [2]. At present, the neutron EDM d_n is known to be smaller than $4 \cdot 10^{-25}$ e·cm (90% c.l.) in absolute magnitude and there are good prospects for further improvement [3]. Such a small value can be understood in the standard model [4] with the Kobayashi-Maskawa mechanism [5] for CP violation which leads to $|d_n| < 10^{-30}$ e·cm as a generous upper bound. This small bound is essentially due to the fact that the EDMs of individual quarks vanish up to second loop order [6] in the standard model.

The standard model has scored many successes at present energies. Nevertheless, it is meaningful to investigate possible alternatives and to search for deviations from the standard model. An especially attractive extension is provided by the gauge models based on $SU(2)_L \times SU(2)_R \times U(1)$ [7]. In particular, this gauge group allows for spontaneous P and CP violation. The differences with respect to the standard model can be parametrized by the scale of parity violation. A calculation of the gauge boson contributions to the $K_L K_S$ mass difference Δm_{LS} suggests [8] $M_{W_2} > 1.6$ TeV for the mass of the heavy charged vector boson W_2 . We have recently shown [9] that, barring some rather artificial cancellations, the neutral Higgs exchange contribution to Δm_{LS} also implies a scale of parity violation at least in the TeV range.

In order to pin down possible deviations from the standard model one is naturally led to consider those special cases where the predictions of the standard model depend on certain unique features of the theory. The natural suppression of induced flavour changing neutral currents is one such instance. With the same motivation we have performed a detailed calculation of d_n in the left-right symmetric gauge theory. As in any generic gauge theory, the EDMs of quarks are non-vanishing already at the one loop level. At the same time, one must include the vector boson and scalar exchange contributions involving a diquark system in the

neutron because they are of the same order of magnitude as the loop diagrams. The standard way to relate the EDMs of quarks to the experimentally accessible d_n is based on the non-relativistic quark model. It is therefore consistent to calculate the exchange contributions in the non-relativistic harmonic oscillator model of baryons [10] which gives reasonable results for the low-lying levels. Since our main concern is to obtain an approximate bound on the scale of parity violation in $SU(2)_L \times SU(2)_R \times U(1)$ we shall not be concerned with secondary effects such as QCD corrections.

The only explicit calculation of d_n in a left-right symmetric gauge theory was performed by Beall and Soni [11] who, however, computed only the gauge boson loop contributions in a four-quark version of the model. Their work needs to be supplemented for several reasons. First of all, even with four quarks the neglected exchange contributions are of equal importance as the loop diagrams. Moreover, the realistic six-quark case requires inclusion of Higgs diagrams (loop and exchange) since the time-honoured approximation $m_t \ll M_{W_1} \approx 80$ GeV can hardly be upheld any longer. Finally, the relation between d_n and CP violating parameters of the K^0 system given in ref. [11] as a lower bound for d_n is based on the assumption of only four quarks and does not hold in the general six-quark case [12].

In Sect. 2 we present the vector boson and scalar loop contributions to the EDM of an arbitrary fermion in a general gauge theory. The non-relativistic quark model relates the quark EDMs to d_n . In Sect. 3 we employ a Foldy-Wouthuysen transformation to derive an effective Schrödinger Hamiltonian for the general gauge theory of Sect. 2. With this Hamiltonian the exchange contributions to d_n can be calculated in the harmonic oscillator quark model. Whereas the charged Higgs exchange contribution vanishes, the vector boson and neutral Higgs parts turn out to be comparable in size to the corresponding loop contributions. In Sect. 4 we specialize to the minimal $SU(2)_L \times SU(2)_R \times U(1)$ model with only one scalar multiplet coupling to quarks. Only in this case the results of the previous sections can be expressed in terms of masses, mixing angles and some additional phases. In order to suppress Higgs induced CP violation in the K^0 system

we furthermore assume symmetric quark mass matrices [9]. This assumption can naturally be fulfilled in a theory with only spontaneous CP violation and it relates the right and left handed mixing matrices. It is shown that the neutral Higgs loop contribution to d_n is severely suppressed. In Sect. 5 we give order of magnitude estimates for the remaining contributions to d_n in terms of quark masses and KM mixing angles. The neutral Higgs exchange turns out to be completely negligible.

The qualitative estimates are corroborated by a detailed numerical study of the only existing six-quark model based on $SU(2)_L \times SU(2)_R \times U(1)$ where all parameters of the mixing matrix can be expressed naturally in terms of quark mass ratios and one CP violating phase of scalar vacuum expectation values [13]. In Sect. 6 we summarize our conclusions. The appendix contains some details of the minimal model which are needed for our calculations.

2. LOOP CONTRIBUTIONS TO THE EDM OF AN ELEMENTARY FERMION

The EDM of a spin $\frac{1}{2}$ fermion f is defined by means of the form factor decomposition of the electromagnetic current j_μ :

$$\langle p', s' | j_\mu(x) | p, s \rangle = \frac{m e^{i(p'-p) \cdot x}}{(2\pi)^3 \sqrt{\omega \omega'}} \bar{u}(p', s') \Gamma_\mu(p', p) u(p, s) \quad (2.1)$$

with*)

$$\Gamma_\mu = F_Q(q^2) \gamma_\mu + F_{EDM}(q^2) \sigma_{\mu\nu} \gamma_5 q^\nu + \dots \quad (2.2)$$

$$q = p' - p$$

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*) Our conventions are those of J.D. Björken and S.D. Drell, Relativistic Quantum Fields, McGraw-Hill, 1965.

Hermiticity of j_μ implies reality of the charge form factor $F_Q(q^2)$ (normalized to the charge of f at $q^2 = 0$) and of the EDM form factor $F_{\text{EDM}}(q^2)$, whereas CP invariance would require a purely imaginary $F_{\text{EDM}}(q^2)$. The EDM d_f is defined as

$$d_f = -F_{\text{EDM}}(0) \quad (2.3)$$

leading to a non-relativistic electric dipole Hamiltonian $H_{\text{EDM}} = -d_f \vec{\sigma} \cdot \vec{E}$ in an external electric field $\vec{E}$ where $\vec{\sigma}/2$ is the spin of fermion f .

At the one loop level in a general gauge theory d_f receives contributions from the gauge boson loop diagrams of Fig. 1 and from the scalar loop diagrams of Fig. 2. The vector boson fields W_μ and the scalar fields H are assumed to be mass eigenfields at the tree level. We have adopted the unitary gauge as the most economical one. The relevant Lagrangian for the gauge boson-fermion and the Higgs-fermion couplings is

$$\begin{aligned} -\mathcal{L}_{\text{int}} = & \bar{\Psi} \gamma_\mu \left[G_L^W \frac{1-\gamma_5}{2} + G_R^W \frac{1+\gamma_5}{2} \right] \Psi W^\mu + \\ & + \bar{\Psi} \left[G_L^H \frac{1-\gamma_5}{2} + G_R^H \frac{1+\gamma_5}{2} \right] \Psi H + \text{H.c.} \end{aligned} \quad (2.4)$$

where $G_{L,R}^W, G_{L,R}^H$ are matrices in the (flavour) space of all fermions f_i ($i = 1, \dots, n$) and there is one term in (2.4) for each mass eigenfield W^μ or H . For Hermitian W^μ, H

$$\begin{aligned} G_L^{W\dagger} &= G_L^W, \quad G_R^{W\dagger} = G_R^W \\ G_L^{H\dagger} &= G_R^H \end{aligned} \quad (2.5)$$

and the Hermitian conjugation is to be omitted in (2.4). The couplings of the photon to vector boson and scalar fields are the standard ones of a gauge theory.

The EDM of an elementary fermion f_i to one loop order is given by

$$d_{f_i} = d_i(W) + d_i(H) \quad (2.6)$$

with the W loop contribution (Fig. 1)

$$d_i(W) = \frac{1}{16\pi^2 M_W^2} \sum_{j=1}^n m_j \operatorname{Im} (G_{L,ij}^W G_{R,ij}^{W*}) \cdot [(q_j - q_i) I_1(r_j, s_i) + q_j I_2(r_j, s_i)] \quad (2.7)$$

$$r_j = \frac{m_j^2}{M_W^2}, \quad s_i = \frac{m_i^2}{M_W^2}$$

and the H loop contribution (Fig. 2)

$$d_i(H) = \frac{-1}{16\pi^2 M_H^2} \sum_{j=1}^n m_j \operatorname{Im} (G_{L,ij}^H G_{R,ij}^{H*}) \cdot [(q_j - q_i) I_3(r_j, s_i) + q_j I_4(r_j, s_i)] \quad (2.8)$$

$$r_j = \frac{m_j^2}{M_H^2}, \quad s_i = \frac{m_i^2}{M_H^2}$$

where q_j is the electric charge of f_j . The kinematical functions $I_k(r, s)$ are defined as

$$I_1(r, s) = \frac{1}{2} + \int_0^1 dx \frac{(1-x)[3(1-x) - sx^2]}{1-x + rx - sx(1-x)} \simeq \frac{2}{(1-r)^2} \left(1 - \frac{11r}{4} + \frac{r^2}{4} - \frac{3r^2 \ln r}{2(1-r)} \right) \quad (2.9a)$$

$$\begin{aligned} I_2(r, s) &= \int_0^1 dx \frac{x [4(1-x) + x(r-s)]}{1-x + rx - sx(1-x)} \simeq \\ &\simeq \frac{2}{(1-r)^2} \left(1 + \frac{r}{4} + \frac{r^2}{4} + \frac{3r \ln r}{2(1-r)} \right) \end{aligned} \quad (2.9b)$$

$$\begin{aligned} I_3(r, s) &= \int_0^1 dx \frac{x(1-x)}{1-x + rx - sx(1-x)} \simeq \\ &\simeq \frac{1}{2(1-r)^2} \left(1+r + \frac{2r \ln r}{1-r} \right) \end{aligned} \quad (2.9c)$$

$$\begin{aligned} I_4(r, s) &= \int_0^1 dx \frac{x^2}{1-x + rx - sx(1-x)} \simeq \\ &\simeq \frac{-1}{2(1-r)^2} \left(3-r + \frac{2 \ln r}{1-r} \right) \end{aligned} \quad (2.9d)$$

where the approximate equations refer to the physically relevant case of light external fermions ($s \approx 0$). Equations (2.7), (2.9a) and (2.9b) for the vector boson loop diagrams agree with the results of Beall and Soni except for an overall sign and a misprint in eq. (6) of ref. [11].

The standard way to relate the quark EDMs d_u , d_d to d_n is to appeal to the non-relativistic quark model which gives

$$d_n^{\text{loop}} = (4d_d - d_u)/3 \quad (2.10)$$

for the one loop contribution d_n^{loop} .

3. EXCHANGE CONTRIBUTIONS TO d_n

To estimate the exchange contributions to d_n shown in Fig. 3 we use the non-relativistic harmonic oscillator quark model [10] with the Hamiltonian

$$H_{HO} = -\frac{1}{2m_q} (\Delta_R + \Delta_\lambda + \Delta_\rho) + \frac{1}{2} m_q \omega^2 (\vec{\rho}^2 + \vec{\lambda}^2) \quad (3.1)$$

where

$$\begin{aligned} \vec{R} &= \frac{1}{\sqrt{3}} (\vec{x}_1 + \vec{x}_2 + \vec{x}_3) \\ \vec{\lambda} &= \frac{1}{\sqrt{6}} (\vec{x}_1 + \vec{x}_2 - 2\vec{x}_3) \\ \vec{\rho} &= \frac{1}{\sqrt{2}} (\vec{x}_1 - \vec{x}_2) . \end{aligned} \quad (3.2)$$

The $\vec{x}_i$ ($i = 1, 2, 3$) are the quark coordinates in the baryon, ω is the level splitting and m_q is the constituent quark mass taken to be equal for u and d quarks. The baryonic states are classified according to SU(6) and must be totally symmetric under a permutation of both SU(6) and spatial indices because of the antisymmetry in colour.

The oscillator ground state is a 56-plet of SU(6) with positive parity. The non-relativistic Hamiltonian for particles in a constant electric field $\vec{E}$

$$H_{ED} = - \sum_{i=1}^3 q_i \vec{x}_i \cdot \vec{E} , \quad (3.3)$$

with q_i the charge of the i -th quark, connects the ground state to the first excited state only which is a 70-plet of SU(6) with negative parity. Because of the CP violating part H_{CP} of the weak Hamiltonian described below the neutron gets a small admixture of the opposite parity 70-plet. The resulting EDM d_n^{ex} is given to first order in H_{CP} by

$$\begin{aligned}
 \langle H_{ED} \rangle_n &= 2 \operatorname{Re} \sum_i \frac{\langle n | H_{ED} | i \rangle \langle i | H_{CP} | n \rangle}{m_n - m_i} = \\
 &= - \langle d_n^\alpha \vec{\sigma}_n \cdot \vec{E} \rangle_n
 \end{aligned}
 \tag{3.4}$$

where $|n\rangle$ denotes the pure SU(6) neutron state and $|i\rangle = |70\rangle$ the SU(6) intermediate states with corresponding masses m_n, m_i . Ignoring all corrections to the simple oscillator picture the masses of the intermediate states are $m_i = m_n + \omega$ because H_{ED} induces exactly one oscillator excitation.

Decomposing the 56- and 70-plets according to SU(3) and spin yields

$$\begin{aligned}
 56 &= {}_4 10 \oplus {}_2 8 \\
 70 &= {}_2 10 \oplus {}_4 8 \oplus {}_2 8 \oplus {}_2 1
 \end{aligned}
 \tag{3.5}$$

where 1, 8, 10 denote the dimensions of the SU(3) representations and the subscript indicates the dimension of the spin representation. Since H_{ED} does not change the spin and the third component of isospin only ${}_2 10$ and ${}_2 8$ contribute as intermediate states. The decomposition of the relevant states into spatial wave functions (ground state and first excited state oscillator wave functions), SU(3) and spin wave functions can be found in ref. [10].

We still have to determine the non-relativistic approximation to the CP violating part H_{CP} of the gauge boson and Higgs Lagrangian (2.4). We consider the effective four-fermion interactions and employ a Foldy-Wouthuysen transformation [14] to compute H_{CP} to first order in quark momentum/ m_q . The various exchange contributions will now be discussed separately.

i) W exchange

We only consider gauge theories with a natural GIM-mechanism [15]. In this case, neutral vector boson exchange does not contribute to d_n . From the charged current Lagrangian

$$-\mathcal{L}_{cc} = \bar{u} \gamma^\mu \left[G_L^W \frac{1-\gamma_5}{2} + G_R^W \frac{1+\gamma_5}{2} \right] d W_\mu^+ + \text{H.c.} \quad (3.6)$$

we obtain the CP violating non-relativistic Hamiltonian

$$H_{CP}^W = \frac{-i \text{Im}(G_L^W G_R^{W*})}{4 m_q M_W^2} \sum_{\substack{i,j=1 \\ i \neq j}}^3 \tau_i^+ \tau_j^- \{ (\vec{\sigma}_i + \vec{\sigma}_j) \cdot (\vec{p}_i - \vec{p}_j) \delta(\vec{x}_i - \vec{x}_j) \} \quad (3.7)$$

where $\vec{p}_i$ and $\vec{\sigma}_i/2$ are momentum and spin operators of the i-th quark and

$$\tau_i^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \tau_i^- = (\tau_i^+)^{\dagger}$$

act in isospin space of u and d quarks.

The relevant matrix elements are

$$\langle {}_2 10, s_z = \frac{1}{2}, m | H_{CP}^W | n, s_z = \frac{1}{2} \rangle = 0 \quad (3.8a)$$

$$\langle {}_2 8, s_z = \frac{1}{2}, m | H_{CP}^W | n, s_z = \frac{1}{2} \rangle = \frac{\text{Im}(G_L^W G_R^{W*}) m_q \omega^2}{4\sqrt{3} \pi^{3/2} M_W^2} \delta_{m0} \quad (3.8b)$$

where $m = 1, 0, -1$ is the third component of orbital angular momentum $\ell = 1$ and

$$\begin{aligned} & \langle n, s_z = \frac{1}{2} | H_{ED} | {}_2 8, s_z = \frac{1}{2}, m=0 \rangle = \\ & = \langle n, s_z = \frac{1}{2} | H_{ED} | {}_2 10, s_z = \frac{1}{2}, m=0 \rangle = - \frac{e E_z}{\sqrt{6} m_q \omega} . \end{aligned} \quad (3.9)$$

From eq. (3.4) one obtains the W exchange contribution

$$d_n^{\text{ex}}(W) = - \frac{\sqrt{2} m_q \omega}{12 \pi^{3/2} M_W^2} \text{Im}(G_L^W G_R^{W*}) . \quad (3.10)$$

ii) H⁰ exchange

Writing the neutral Higgs part of (2.4) in the form

$$- \mathcal{L}_Y^{H^0} = H^0 (\bar{u}_L G_u u_R + \bar{d}_L G_d d_R + \text{H.c.}) \quad (3.11)$$

we find the non-relativistic CP violating Hamiltonian

$$H_{\text{CP}}^{H^0} = \frac{i}{2 m_q M_{H^0}^2} \sum_{\substack{i,j=1 \\ i \neq j}}^3 A_i B_j [\vec{\sigma}_j \cdot \vec{p}_j, \delta(\vec{x}_i - \vec{x}_j)] \quad (3.12)$$

where

$$A_i = \begin{pmatrix} \text{Re } G_u & 0 \\ 0 & \text{Re } G_d \end{pmatrix}, \quad B_i = \begin{pmatrix} \text{Im } G_u & 0 \\ 0 & \text{Im } G_d \end{pmatrix} \quad (3.13)$$

are matrices in isospin space.

In this case both 210 and 28 contribute as intermediate states and we get

$$\begin{aligned} \langle 210, s_z = \frac{1}{2}, m | H_{\text{CP}}^{H^0} | n, s_z = \frac{1}{2} \rangle &= \frac{m_q \omega^2 \delta_{m0}}{8\sqrt{3} \pi^{3/2} M_{H^0}^2} \cdot \\ &\cdot (\text{Re } G_u \text{Im } G_d + \text{Re } G_d \text{Im } G_u - 2 \text{Re } G_d \text{Im } G_d) \end{aligned} \quad (3.14a)$$

$$\begin{aligned} \langle 2, 8, s_z = \frac{1}{2}, m | H_{CP}^{H^0} | n, s_z = \frac{1}{2} \rangle &= \frac{m_q \omega^2 \delta_{m0}}{8\sqrt{3} \pi^{3/2} M_{H^0}^2} \cdot \\ &\cdot (3 \operatorname{Re} G_u \operatorname{Im} G_d + \operatorname{Re} G_d \operatorname{Im} G_u + 2 \operatorname{Re} G_d \operatorname{Im} G_d) . \end{aligned} \quad (3.14b)$$

Using (3.9), we get from (3.4)

$$d_n^{\text{ex}}(H^0) = - \frac{\sqrt{2} m_q \omega}{12 \pi^{3/2} M_{H^0}^2} e (2 \operatorname{Re} G_u \operatorname{Im} G_d + \operatorname{Re} G_d \operatorname{Im} G_u) \quad (3.15)$$

as the exchange contribution due to neutral Higgs exchange.

iii) H^+ exchange

The Yukawa interaction

$$- \mathcal{L}_Y^{H^+} = H^+ (\bar{u}_R G_L d_L + \bar{u}_L G_R d_R) + \text{H. c.} \quad (3.16)$$

yields the non-relativistic CP violating Hamiltonian

$$H_{CP}^{H^+} = \frac{-i}{4 m_q M_{H^+}^2} \operatorname{Im} (G_L G_R^*) \sum_{\substack{i,j=1 \\ i \neq j}}^3 \tau_i^+ \tau_j^- [\vec{\sigma}_i \cdot \vec{p}_i + \vec{\sigma}_j \cdot \vec{p}_j, \delta(\vec{x}_i - \vec{x}_j)] . \quad (3.17)$$

In this case,

$$\langle 2, 10, s_z = \frac{1}{2}, m | H_{CP}^{H^+} | n, s_z = \frac{1}{2} \rangle = - \langle 2, 8, s_z = \frac{1}{2}, m | H_{CP}^{H^+} | n, s_z = \frac{1}{2} \rangle \quad (3.18)$$

and, consequently, the charged Higgs exchange contribution to d_n vanishes in our non-relativistic approximation because of (3.4) and (3.9).

The total exchange contribution $d_n^{\text{ex}} = d_n^{\text{ex}(W)} + d_n^{\text{ex}(H^0)}$ depends on the parameter $m_q \omega$ of the harmonic oscillator model. This parameter is not very well determined because it depends on what input data are used for its determination. In the literature [10] one finds values for $\sqrt{m_q \omega}$ ranging approximately from 200 to 450 MeV showing the intrinsic limitations of the harmonic oscillator picture. For the numerical discussion we shall use $\sqrt{m_q \omega} = 300$ MeV.

We close this section with two remarks. In addition to the inherent uncertainties of the harmonic oscillator model and the non-relativistic approximation in general there are gluonic corrections to the weak Hamiltonian. These corrections were neglected for both loop and exchange diagrams. For the W exchange contribution the corresponding uncertainty has been estimated to be less than a factor two [16]. Finally, a comparison of (2.7), (2.8) and (3.10), (3.15) immediately shows that loop and exchange contributions to d_n are indeed of similar magnitude.

4. RESULTS FOR THE MINIMAL $SU(2)_L \times SU(2)_R \times U(1)$ MODEL

The general results of the previous sections will now be discussed within the framework of the minimal $SU(2)_L \times SU(2)_R \times U(1)$ model [17]. All essential details of this model can be found in the appendix.

From (2.7) and the charged current Lagrangian (A.14) we obtain the EDMs of u and d quarks due to W loop diagrams

$$d_u(W) = \frac{eg^2 \sin 2\xi}{64\pi^2 M_1^2} \sum_{j=d,s,b} m_j \text{Im}(e^{i\lambda} K_{L,u j} K_{R,u j}^*) \cdot$$

$$\cdot \left\{ I_1(m_j^2/M_1^2, m_u^2/M_1^2) + \frac{1}{3} I_2(m_j^2/M_1^2, m_u^2/M_1^2) - \right.$$

$$\left. - M_1^2/M_2^2 \left[I_1(m_j^2/M_2^2, m_u^2/M_2^2) + \frac{1}{3} I_2(m_j^2/M_2^2, m_u^2/M_2^2) \right] \right\} \quad (4.1)$$

$$\begin{aligned}
 d_d(W) &= \frac{eg^2 \sin 2\xi}{64 \pi^2 M_1^2} \sum_{j=u,c,t} m_j \operatorname{Im} (e^{i\lambda} K_{L,jd} K_{R,jd}^*) \cdot \\
 &\cdot \left\{ I_1(m_j^2/M_1^2, m_d^2/M_1^2) + \frac{2}{3} I_2(m_j^2/M_1^2, m_d^2/M_1^2) - \right. \\
 &\left. - M_1^2/M_2^2 \left[I_1(m_j^2/M_2^2, m_d^2/M_2^2) + \frac{2}{3} I_2(m_j^2/M_2^2, m_d^2/M_2^2) \right] \right\}.
 \end{aligned}
 \tag{4.2}$$

The W exchange contribution (3.10) to d_n reads

$$d_n^{\text{ex}}(W) = \frac{eg^2 \sin 2\xi \sqrt{2m_q \omega}}{48 \pi^{3/2} M_1^2} (1 - M_1^2/M_2^2) \operatorname{Im}(e^{i\lambda} K_{L,ud} K_{R,ud}^*). \tag{4.3}$$

Unlike the mixing phase λ , the quantity $\operatorname{Im}(e^{i\lambda} K_L K_R^*)$ is, of course, gauge independent. Moreover, the appearance of this quantity in the above equations shows that the neutral vector bosons Z_1, Z_2 do not contribute to d_n to this order: the corresponding quark mixing matrices are unity (GIM mechanism [15]) and the neutral gauge boson mixing matrix is real.

Turning now to the Higgs sector, we briefly summarize the scalar particle content of the minimal model. There is a single charged Higgs field ϕ^+ [(A.16)] coupling to quarks. Among the neutral scalar fields that have Yukawa couplings to quarks there is a field ϕ_1 , the analogue of the standard model Higgs field, and two additional flavour mixing fields ϕ_2, ϕ_3 [see (A.18)]. Inspection of the Higgs potential shows that ϕ^+, ϕ_1, ϕ_2 and ϕ_3 are mass eigenstates, up to mixings of the order M_1/M_2 . Moreover, the masses of ϕ^+, ϕ_2 and ϕ_3 are approximately equal and of the order of the scale of parity violation.

From (2.8) and (A.17) we get the ϕ^+ loop contributions

$$d_u(\phi^+) = \frac{e N^2 |vw| V^2}{4 \pi^2 (|v|^2 - |w|^2)^2 M_{\phi^+}^2} \sum_{j=d,s,b} m_j (m_j^2 - m_u^2) \text{Im} (e^{i\lambda} K_{L,uj} K_{R,uj}^*) \cdot$$

$$\cdot \left[I_3 (m_j^2 / M_{\phi^+}^2, m_u^2 / M_{\phi^+}^2) + \frac{1}{3} I_4 (m_j^2 / M_{\phi^+}^2, m_u^2 / M_{\phi^+}^2) \right] \quad (4.4)$$

$$d_d(\phi^+) = \frac{e N^2 |vw| V^2}{4 \pi^2 (|v|^2 - |w|^2)^2 M_{\phi^+}^2} \sum_{j=u,c,t} m_j (m_j^2 - m_d^2) \text{Im} (e^{i\lambda} K_{L,jd} K_{R,jd}^*) \cdot$$

$$\cdot \left[I_3 (m_j^2 / M_{\phi^+}^2, m_d^2 / M_{\phi^+}^2) + \frac{2}{3} I_4 (m_j^2 / M_{\phi^+}^2, m_d^2 / M_{\phi^+}^2) \right] \quad (4.5)$$

where v, w, V, N are defined in the appendix. We recall from Sect. 3 that there is no ϕ^+ exchange contribution in the harmonic oscillator model.

For the computation of the neutral Higgs contributions we assume $M_{\phi_2} = M_{\phi_3} \equiv M_{H^0}$ as suggested by the Higgs potential. From (2.8) and (A.19) we find in this case

$$d_u(\phi_1, \phi_2, \phi_3) \sim \text{Im} (G_{2,uj}^u G_{2,ju}^u + G_{3,uj}^u G_{3,ju}^u) \quad (4.6)$$

$$d_d(\phi_1, \phi_2, \phi_3) \sim \text{Im} (G_{2,dj}^d G_{2,jd}^d + G_{3,dj}^d G_{3,jd}^d)$$

and because of [see (A.20)]

$$G_2^u = -i G_3^u, \quad G_2^d = i G_3^d \quad (4.7)$$

these contributions vanish. In general, for $M_{\phi_2} \neq M_{\phi_3}$, the neutral Higgs loop contributions are therefore suppressed by a factor $(M_{\phi_2}^2 - M_{\phi_3}^2) / (M_{\phi_2}^2 + M_{\phi_3}^2)$.

However, (3.15) and (4.7) imply a non-vanishing neutral Higgs exchange contribution

$$d_n^{\text{ex}}(\phi_2, \phi_3) = - \frac{e\sqrt{2m_q\omega}}{4\pi^{3/2}M_{H^0}^2} \text{Im}(G_{2,uu}^u G_{2,dd}^d). \quad (4.8)$$

Summing up all contributions we arrive at the final result

$$d_n = \frac{4}{3} (d_d(W) + d_d(\phi^+)) - \frac{1}{3} (d_u(W) + d_u(\phi^+)) + \\ + d_n^{\text{ex}}(W) + d_n^{\text{ex}}(\phi_2, \phi_3) \quad (4.9)$$

in the minimal model.

An analysis of the $K_L K_S$ mass difference in the minimal model shows that natural suppression of induced scalar contributions requires [9] $M_{H^0} \geq 4$ TeV. Unless M_{H^0} is much larger than this lower bound, the quark mass matrices must moreover be symmetric in a certain basis of weak eigenstates to guarantee naturally small CP violation in the K^0 system. This implies $K_R = K_L^*$ although the left and right handed mixing matrices are *a priori* independent. With this restriction all results of this section can be expressed in terms of masses, Kobayashi-Maskawa mixing angles and some additional phases.

5. NUMERICAL ESTIMATES

In this section we present numerical results for the various contributions to d_n in the minimal $SU(2)_L \times SU(2)_R \times U(1)$ model with $K_R = K_L^*$. To get an idea of the orders of magnitude involved we shall derive as a first step approximate formulas for the maximally possible values of these contributions. This amounts to assuming maximally CP violating phases. In the second part of this section we shall investigate a specific model to exhibit in detail the phase dependence of d_n .

Using $I_{1,2}(r,s) \approx I_{1,2}(0,0) = 2$ for small quark masses and neglecting terms proportional to M_1^2/M_2^2 , we obtain

$$d_u(W) \approx \frac{eg^2 \sin 2\xi}{24\pi^2 M_1^2} \sum_{j=d,s,b} m_j \operatorname{Im}(e^{i\lambda} K_{L,uj}^2) \quad (5.1)$$

$$d_d(W) \approx \frac{eg^2 \sin 2\xi}{64\pi^2 M_1^2} \left[\frac{10}{3} \sum_{j=u,c} m_j \operatorname{Im}(e^{i\lambda} K_{L,jd}^2) + \right. \\ \left. + m_t \operatorname{Im}(e^{i\lambda} K_{L,td}^2) (I_1(m_t^2/M_1^2, 0) + \frac{2}{3} I_2(m_t^2/M_1^2, 0)) \right] \quad (5.2)$$

as approximate W loop contributions.

At this point the question arises whether to use current or constituent masses for the intermediate quarks in the loop diagrams of Figs. 1, 2. There are arguments in favour of current masses (Shabalin in [2] and ref. [18]) but we have convinced ourselves that insertion of constituent masses does not change the results dramatically. For the rest of this paper we shall use the current quark masses [19] $m_u = 5.1$ MeV, $m_d = 8.9$ MeV, $m_s = 175$ MeV, $m_c = 1.35$ GeV and $m_b = 5.6$ GeV.

With the assumption of maximally CP violating phases we get $\operatorname{Im}(e^{i\lambda} K_{L,uj}^2)_{j=d,s,b} = (c_1^2, s_1^2 c_3^2, s_1^2 s_3^2)$ and $\operatorname{Im}(e^{i\lambda} K_{L,jd}^2)_{j=u,c,t} = (c_1^2, s_1^2 c_2^2, s_1^2 s_2^2)$ with the usual notation for the Kobayashi-Maskawa mixing angles [5]. Using the approximate relation (A.9) for ξ and taking $s_1 = 0.23$, $c_1 \approx c_2 \approx c_3 \approx 1$ one gets

$$|d_u(W)| \approx 1.5 \cdot 10^{-26} \text{ e.cm } \left(\frac{50 M_1}{M_2} \right)^2 \left(\frac{30 \text{ GeV}}{m_t} \right) \left(\frac{|V_{Wt}|}{V^2} \frac{m_t}{m_b} \right). \quad (5.3)$$

$$\cdot \begin{bmatrix} 1, & 1, & 1.8 (s_3/s_1)^2 \end{bmatrix}$$

d   s   b

$$|d_q(w)| \simeq 1.5 \cdot 10^{-25} \text{ e.cm} \left(\frac{50 M_1}{M_2} \right)^2 \left(\frac{30 \text{ GeV}}{m_t} \right) \left(\frac{|vw|}{V^2} \frac{m_t}{m_b} \right) \cdot$$

$$\cdot \left[\underset{u}{7 \cdot 10^{-2}}, \underset{c}{1}, \underset{t}{(s_2/s_1)^2} \cdot \left\{ \begin{array}{l} 1 \\ 1.7 \\ 2.3 \end{array} \text{ for } m_t = \begin{array}{l} 30 \text{ GeV} \\ 60 \text{ GeV} \\ 100 \text{ GeV} \end{array} \right\} \right] \quad (5.4)$$

where the various quark contributions are indicated separately. The combination $|vw|m_t/V^2 m_b$ has been chosen because in the minimal model m_b/m_t is generically of the order $|w/v| \simeq |vw|/V^2$ [see (A.24)].

With the same approximations we find for the W exchange contribution (4.3)

$$|d_n^{\text{ex}}(w)| \simeq 6.2 \cdot 10^{-25} \text{ e.cm} \left(\frac{50 M_1}{M_2} \right)^2 \left(\frac{30 \text{ GeV}}{m_t} \right) \left(\frac{|vw|}{V^2} \frac{m_t}{m_b} \right) \left(\frac{\sqrt{m_q w}}{0.3 \text{ GeV}} \right). \quad (5.5)$$

Taking $|w|/|v| \simeq m_b/m_t$ into account, the ϕ^+ loop contributions are approximately

$$d_u(\phi^+) \simeq \frac{e\sqrt{2} G_F |w|}{4\pi^2 M_{\phi^+}^2 |v|} \sum_{j=d,s,b} m_j (m_j^2 - m_u^2) \text{Im} (e^{i\lambda} K_{L,uj}^2) \cdot$$

$$\cdot \left[I_3 (m_j^2 / M_{\phi^+}^2, 0) + \frac{1}{3} I_4 (m_j^2 / M_{\phi^+}^2, 0) \right] \quad (5.6)$$

$$d_d(\phi^+) \simeq \frac{e\sqrt{2} G_F |w|}{4\pi^2 M_{\phi^+}^2 |v|} \sum_{j=u,c,t} m_j (m_j^2 - m_d^2) \text{Im} (e^{i\lambda} K_{L,jd}^2) \cdot$$

$$\cdot \left[I_3 (m_j^2 / M_{\phi^+}^2, 0) + \frac{2}{3} I_4 (m_j^2 / M_{\phi^+}^2, 0) \right].$$

(5.7)

With $I_3(r,s) \simeq I_3(0,0) = \frac{1}{2}$, $I_4(r,s) \simeq I_4(r,0) \simeq -\frac{3}{2} - \ln r$ ($r \ll 1$) and the same approximations as before we obtain

$$|d_u(\phi^+)| \simeq 2.1 \cdot 10^{-28} \text{ e.cm} \left(\frac{4 \text{ TeV}}{M_{\phi^+}} \right)^2 \left(\frac{30 \text{ GeV}}{m_t} \right) \left(\frac{|w| m_t}{|v| m_b} \right). \quad (5.8)$$

$$\cdot \left[\underset{d}{2 \cdot 10^{-6}}, \underset{s}{10^{-3}}, \underset{b}{(s_3/s_1)^2} \right]$$

$$|d_d(\phi^+)| \simeq 4.4 \cdot 10^{-26} \text{ e.cm} \left(\frac{4 \text{ TeV}}{M_{\phi^+}} \right)^2 \left(\frac{30 \text{ GeV}}{m_t} \right) \left(\frac{|w| m_t}{|v| m_b} \right) \cdot \left[\underset{u}{10^{-8}}, \underset{c}{3 \cdot 10^{-3}}, \underset{t}{\left(\frac{s_2}{s_1} \right)^2 \left(\frac{m_t}{30 \text{ GeV}} \right)^3} \left| 1 - 0.22 \ln \left(\frac{m_t}{30 \text{ GeV}} \cdot \frac{4 \text{ TeV}}{M_{\phi^+}} \right) \right| \right] \quad (5.9)$$

where the dependence on $\ln M_{\phi^+}$ is only shown for the t contribution. As was to be expected, the t contribution is by far the dominant one unless s_2/s_1 is extremely small.

Finally, the neutral Higgs exchange contribution is at most

$$|d_n^{\text{ex}}(\phi_2, \phi_3)| \simeq 6.5 \cdot 10^{-31} \text{ e.cm} \left(\frac{4 \text{ TeV}}{M_{H^0}} \right)^2 \left(\frac{\sqrt{m_q \omega}}{0.3 \text{ GeV}} \right) \left(0.95 + \frac{m_t}{30 \text{ GeV}} \left(\frac{s_2}{s_1} \right)^2 \right) \quad (5.10)$$

and thus completely negligible.

The approximate formulas in this section can be used to derive "natural" bounds for M_2 and M_{ϕ^+} if we assume maximally CP violating phases and if we insist that each contribution separately obeys the experimental limit $|d_n| < 4 \cdot 10^{-25} \text{ e.cm}$.

From (5.5) we find in this way

$$M_2/M_1 \gtrsim 62 \left(\frac{30 \text{ GeV}}{m_t} \right)^{1/2} \left(\frac{\sqrt{m_q \omega}}{0.3 \text{ GeV}} \right)^{1/2}. \quad (5.11)$$

With $m_t = 30 \text{ GeV}$ and $0.2 \text{ GeV} \lesssim \sqrt{m_q \omega} \lesssim 0.45 \text{ GeV}$, eq. (5.11) yields a lower bound for M_2 of approximately 4-6 TeV.

Similarly, one obtains from (5.9) $M_{\phi^+} \gtrsim 1.3 (s_2/s_1) \text{ TeV}$ for $m_t = 30 \text{ GeV}$ and $M_{\phi^+} \gtrsim 2.7 (s_2/s_1) \text{ TeV}$ for $m_t = 60 \text{ GeV}$, respectively.

All the numerical results of this section were obtained by maximizing the various contributions with respect to the six in general independent CP violating phases (see Appendix). In order to illustrate these estimates we now turn to a specific model [13] of weak mixing angles which has a single CP violating phase $\alpha = \arg(vw)$. This model can be derived by imposing a horizontal symmetry and it is unique in the following sense: there is no other phenomenologically acceptable minimal $SU(2)_L \times SU(2)_R \times U(1)$ model with a non-trivial horizontal symmetry for three generations. The model is described in some detail in the appendix where it is shown that it suffices to consider $0 \leq \alpha \leq \pi/2$. Depending on whether the parameter g_{22} is positive or negative two different cases may be distinguished. For $m_t < m_b m_c / m_s$ there are moreover two different solutions for K_L for given α and $\text{sgn } g_{22}$.

The results of our numerical calculations in the framework of this model are displayed in Figs. 4-7 for $M_2 = 50 M_1$, $M_{\phi^+} = 4 \text{ TeV}$, $\sqrt{m_q \omega} = 0.3 \text{ GeV}$ and three different values of m_t . The W loop, W exchange and ϕ^+ loop contributions are shown separately.

Comparing the curves in Figs. 4-7 with our approximate estimates we conclude that due to specific phase relations the maximally possible values are in general not attained in our model. However, the order of magnitude estimates are obviously confirmed. Of course, it is always possible to have much smaller values for d_n .

In particular, for $\alpha = 0$ or $\pi/2$ one obtains $d_n = 0$ [see (A.22)]. One may even envisage the possibility that for special values of α the three relevant contributions cancel to produce a vanishing total d_n . Of course, such a possibility could hardly be called natural.

6. CONCLUSIONS

In the standard model of electroweak interactions the EDMs of quarks vanish up to second loop order [6]. This fact makes d_n an ideal observable to distinguish between the standard model and alternative theories because d_n is in general non-vanishing already at the one loop level. Together with the loop diagrams, one must also consider the single particle exchange diagrams where the particle is exchanged within a diquark system in the neutron. In a simple harmonic oscillator model of the neutron the charged Higgs exchange does not contribute, but the W and neutral Higgs exchange contributions are of similar magnitude as the corresponding loop contributions.

Turning to the minimal $SU(2)_L \times SU(2)_R \times U(1)$ model, we have found that the neutral Higgs loop contributions to d_n effectively cancel. To discuss the remaining non-vanishing contributions one may adopt two different attitudes. In order not to get into conflict with the experimental bound $|d_n| < 4 \cdot 10^{-25} \text{ e} \cdot \text{cm}$, one can resort to small enough CP violating phases or even postulate some definite phase relations to produce cancellations among the various contributions. In our opinion, the more natural approach is to require that the theoretical value for d_n obeys the experimental bound for all possible CP violating phases. In this way, one obtains "natural" lower bounds for the masses of the heavy charged gauge boson W_2 and of the charged Higgs boson ϕ^+ : $M_{W_2} \gtrsim 4\text{--}6 \text{ TeV}$, $M_{\phi^+} \gtrsim 1.3 (s_2/s_1) \text{ TeV}$ (both values for $m_t = 30 \text{ GeV}$). These numbers confirm the corresponding bounds on the scale of parity violation obtained from an investigation of the $K_L K_S$ mass difference [8,9]. To support our general estimates we performed a detailed calculation of d_n in a specific model [13] of weak mixing angles with a single independent CP violating phase.

The left-right symmetric gauge models can easily accommodate values for d_n in the vicinity of the current experimental bound. If experimental evidence for $|d_n| > 10^{-30}$ e.cm should be found it would be necessary to consider strong CP violation or to go beyond the standard model. It need hardly be emphasized that such a discovery would not prove the validity of the left-right symmetric extension [2,18,20]. However, gauge models based on $SU(2)_L \times SU(2)_R \times U(1)$ would certainly become even more attractive candidates for an enlargement of the standard model.

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APPENDIX

APPENDIX

THE MINIMAL $SU(2)_L \times SU(2)_R \times U(1)$ MODEL [17]

The gauge sector of the model is defined by the covariant derivative

$$D_\mu = \partial_\mu + ig_L \vec{T}_L \cdot \vec{W}_{L\mu} + ig_R \vec{T}_R \cdot \vec{W}_{R\mu} + ig' \frac{B-L}{2} B_\mu \quad (A.1)$$

with gauge fields $\vec{W}_{L\mu}$, $\vec{W}_{R\mu}$, B_μ , generators $\vec{T}_L$, $\vec{T}_R$, $(B-L)/2$ and coupling constants g_L , g_R , g' . An additional left-right symmetry is imposed on the theory to enforce $g_L = g_R \equiv g$. The left and right handed quarks $q_L = \begin{pmatrix} p \\ n \end{pmatrix}_L$, $q_R = \begin{pmatrix} p \\ n \end{pmatrix}_R$ (weak eigenstates) transform under the gauge group as $(1/2, 0, 1/6)$ and $(0, 1/2, 1/6)$, respectively.

The Higgs sector consists of a single field

$$\phi = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \quad (A.2)$$

transforming as $(1/2, 1/2, 0)$ and in addition of (at least two) scalar multiplets

$$\Delta_{L,R} = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}_{L,R} \quad (A.3)$$

transforming as $(1, 0, 1)$ and $(0, 1, 1)$, respectively. We shall explicitly use only one pair $\Delta_{L,R}$, but all conclusions will be independent of the number of these fields. On the other hand, the restriction to a single field ϕ is essential.

The gauge group is broken down to $U(1)_{EM}$ by the scalar vacuum expectation values

$$\langle \phi \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} v & 0 \\ 0 & w \end{pmatrix}, \quad \langle \Delta_{L,R} \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ u_{L,R} & 0 \end{pmatrix}. \quad (A.4)$$

After spontaneous symmetry breaking the massive charged vector bosons $W_{1,2}^+$ are related to the gauge fields $W_{L,R}^+$ as

$$\begin{pmatrix} W_1^+ \\ W_2^+ \end{pmatrix}_\mu = \begin{pmatrix} \cos \xi & -\sin \xi e^{i\lambda} \\ \sin \xi e^{-i\lambda} & \cos \xi \end{pmatrix} \begin{pmatrix} W_L^+ \\ W_R^+ \end{pmatrix}_\mu \quad (\text{A.5})$$

where the left-right mixing angle ξ and the phase λ are given by

$$\tan 2\xi = \frac{2|vw|}{|u_R|^2 - |u_L|^2}, \quad e^{i\lambda} = -vw^*/|vw|. \quad (\text{A.6})$$

Although $\lambda \neq 0$ in general, λ can always be transformed away through a suitable gauge transformation which renders the $W_L W_R$ mass matrix real. It is therefore misleading [11] to consider $\lambda \neq 0$ as an independent source of CP violation.

The charged vector boson masses are

$$M_{1,2}^2 = \frac{g^2}{4} \left[V^2 + |u_R|^2 + |u_L|^2 \pm \sqrt{(|u_R|^2 - |u_L|^2)^2 + 4|vw|^2} \right] \quad (\text{A.7})$$

with $V^2 = |v|^2 + |w|^2$. In the physically relevant limit $|u_L|^2 \ll V^2 \ll |u_R|^2$ one gets approximately

$$M_1^2 \simeq \frac{g^2 V^2}{4}, \quad M_2^2 \simeq \frac{g^2}{4} (2|u_R|^2 + V^2) \quad (\text{A.8})$$

so that

$$V^2 \simeq (\sqrt{2} G_F)^{-1}, \quad \xi \simeq \frac{2|vw|}{V^2} \frac{M_1^2}{M_2^2}. \quad (\text{A.9})$$

The Yukawa couplings of quarks are defined by

$$- \mathcal{L}_Y = \bar{q}_L \Gamma_1 \phi q_R + \bar{q}_L \Gamma_2 \tilde{\phi} q_R + \text{H.c.} \quad (\text{A.10})$$

where $\tilde{\phi} = \tau_2 \phi^* \tau_2$ and Γ_1, Γ_2 are n_G -dimensional matrices (n_G denotes the number of generations) which are Hermitian due to left-right symmetry. From (A.4) and (A.10) one obtains the quark mass matrices

$$\begin{aligned} M_p &= \frac{1}{\sqrt{2}} (v \Gamma_1 + w^* \Gamma_2) \\ M_n &= \frac{1}{\sqrt{2}} (w \Gamma_1 + v^* \Gamma_2). \end{aligned} \quad (A.11)$$

Diagonalization of M_p, M_n yields the physical quark fields $u_{L,R}$ and $d_{L,R}$:

$$\begin{aligned} p_{L,R} &= U_{L,R}^p u_{L,R}, \quad n_{L,R} = U_{L,R}^n d_{L,R} \\ \hat{M}_u &= U_L^{p\dagger} M_p U_R^p, \quad \hat{M}_d = U_L^{n\dagger} M_n U_R^n \end{aligned} \quad (A.12)$$

with diagonal, positive definite $\hat{M}_u, \hat{M}_d$.

The charged current Lagrangian

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} (\bar{p}_L \gamma^\mu n_L W_{L\mu}^+ + \bar{p}_R \gamma^\mu n_R W_{R\mu}^+) + \text{H.c.} \quad (A.13)$$

can now be expressed in terms of mass eigenfields as

$$\begin{aligned} -\mathcal{L}_{cc} &= \frac{g}{\sqrt{2}} (\cos \xi \bar{u}_L \gamma^\mu K_L d_L - e^{-i\lambda} \sin \xi \bar{u}_R \gamma^\mu K_R d_R) W_{1\mu}^+ + \\ &+ \frac{g}{\sqrt{2}} (e^{i\lambda} \sin \xi \bar{u}_L \gamma^\mu K_L d_L + \cos \xi \bar{u}_R \gamma^\mu K_R d_R) W_{2\mu}^+ + \text{H.c.} \end{aligned} \quad (A.14)$$

where $K_{L(R)} = U_{L(R)}^{p\dagger} U_{L(R)}^n$ are the left (right) handed mixing matrices. In order to count the number of independent phases [12,21] we first fix the gauge to set $\lambda = 0$. K_L, K_R each contain $n_G(n_G+1)/2$ phases but $2n_G - 1$ of them can be transformed away by proper phase transformations on the massive quark fields u_i, d_i ($i = 1, \dots, n_G$). In the general case there are thus $n_G^2 - n_G + 1$ measurable phases. Even knowing the number of independent phases one has to be careful in

choosing a proper parametrization for K_L, K_R . As already emphasized by Chang [21] a widely used form [11] for K_L, K_R for $n_G = 2$ is incorrect.

If M_p and M_n are symmetric in a certain basis of weak eigenstates one can choose $K_R = K_L^*$ and the number of phases is reduced to $n_G(n_G + 1)/2$.

Minimizing the most general Higgs potential [17,22] consistent with gauge and left-right symmetry one finds $|u_L| \sim v^2/|u_R|$. In order to simplify our formulas we shall put $u_L = 0$ in the following.

Spontaneous symmetry breakdown leads to six would-be-Goldstone bosons

$$\begin{aligned}
 \phi_{G,1}^+ &\sim w^* \phi_{12} - v \phi_{21}^+ , \phi_{G,1}^- \\
 \phi_{G,2}^+ &\sim v^* \phi_{12} - w \phi_{21}^+ - \sqrt{2} u_R^* \delta_R^+ , \phi_{G,2}^- \\
 \phi_{G,1}^0 &\sim u_R^* \delta_R^0 - u_R \delta_R^{0+} \\
 \phi_{G,2}^0 &\sim v^* \phi_{11} - v \phi_{11}^+ + w \phi_{22}^+ - w^* \phi_{22} .
 \end{aligned} \tag{A.15}$$

Considering the physical scalar fields we shall restrict ourselves to those fields involving components of the multiplet ϕ in (A.2) because only they can couple to quarks. This leaves us with a single charged field

$$\phi^+ = N \left(v^* \phi_{12} + w \phi_{21}^+ + \frac{|v|^2 - |w|^2}{\sqrt{2} u_R} \delta_R^+ \right) \tag{A.16}$$

where $N = [v^2 + ((|v|^2 - |w|^2)/\sqrt{2}|u_R|)^2]^{-\frac{1}{2}}$. The corresponding Yukawa Lagrangian is

$$-\mathcal{L}_Y^+ = (\bar{u}_L G_R d_R + \bar{u}_R G_L d_L) \phi^+ + \text{H.c.}$$

with

$$\begin{aligned}
 G_R &= \frac{\sqrt{2} N}{|v|^2 - |w|^2} (V^2 \hat{M}_u K_R - 2vw^* K_L \hat{M}_d) \\
 G_L &= \frac{\sqrt{2} N}{|v|^2 - |w|^2} (2vw^* \hat{M}_u K_L - V^2 K_R \hat{M}_d).
 \end{aligned} \tag{A.17}$$

We choose the three neutral Higgs fields involving components of ϕ as [9]

$$\begin{aligned}
 \phi_1 &= \frac{1}{\sqrt{2} V} (v^* \phi_{11} + v \phi_{11}^\dagger + w^* \phi_{22} + w \phi_{22}^\dagger) \\
 \phi_2 &= \frac{1}{\sqrt{2} V} \left[\left| \frac{v}{w} \right| (w^* \phi_{22} + w \phi_{22}^\dagger) - \left| \frac{w}{v} \right| (v^* \phi_{11} + v \phi_{11}^\dagger) \right] \\
 \phi_3 &= \frac{i}{\sqrt{2} V} \left[\left| \frac{v}{w} \right| (w^* \phi_{22} - w \phi_{22}^\dagger) + \left| \frac{w}{v} \right| (v^* \phi_{11} - v \phi_{11}^\dagger) \right].
 \end{aligned} \tag{A.18}$$

The corresponding Yukawa couplings are given by

$$\begin{aligned}
 -\mathcal{L}_Y^0 &= \frac{\phi_1}{V} (\bar{u}_L \hat{M}_u u_R + \bar{d}_L \hat{M}_d d_R) + \\
 &+ \sum_{i=2}^3 \phi_i (\bar{u}_L G_i^u u_R + \bar{d}_L G_i^d d_R) + \text{H.c.}
 \end{aligned} \tag{A.19}$$

with

$$\begin{aligned}
 G_2^u &= -i G_3^u = -(|v|^2 - |w|^2)^{-1} (2|vw| \hat{M}_u / V + e^{i\lambda} V K_L \hat{M}_d K_R^\dagger) \\
 G_2^d &= i G_3^d = -(|v|^2 - |w|^2)^{-1} (2|vw| \hat{M}_d / V + e^{-i\lambda} V K_L^\dagger \hat{M}_u K_R)
 \end{aligned} \tag{A.20}$$

where (A.6) has been used.

ϕ_1 is obviously the analogue of the single Higgs field of the standard model, whereas ϕ_2 and ϕ_3 have flavour changing couplings to the quarks. From a study of the Higgs potential one finds a single field with small mass $\sim V$. Up to mixing angles of the order M_1/M_2 , this field is just ϕ_1 . To the same accuracy, ϕ_2 and ϕ_3 are nearly degenerate heavy Higgs fields. Up to terms of order V , the masses of ϕ_2 , ϕ_3 and also ϕ^+ are all equal and $\sim |u_R|$.

Finally, we discuss some properties of a specific model [13] of weak mixing angles which we employ for the numerical discussion of Sect. 5. Imposing an arbitrary horizontal symmetry on the minimal $SU(2)_L \times SU(2)_R \times U(1)$ theory one obtains a single model that is compatible with empirical constraints on quark masses and mixing angles. The model is defined by the following mass matrices:

$$M_p = \begin{pmatrix} v g_{11} & 0 & w^* h_{13} \\ 0 & v g_{22} & w^* h_{23} \\ w^* h_{13}^* & w^* h_{23}^* & v g_{33} \end{pmatrix}, \quad M_n = M_p (v \leftrightarrow w) \quad (A.21)$$

and we restrict [13] ourselves to $|g_{11}| \geq |g_{33}| \geq |g_{22}|$. If we choose the phases of quark fields such that $g_{11} \geq 0$, $v \geq 0$, $h_{13} \geq 0$, $h_{23} \geq 0$ the mass matrices are symmetric and consequently $K_R = K_L^*$. For given quark masses the model still depends on the parameters $\alpha = \arg w$, $\text{sgn } g_{22}$ and $\text{sgn } g_{33}$.

From Sects. 4 and 5, we know that all relevant contributions to d_n are proportional to $\text{Im}(e^{i\lambda} K_L^2) = -\text{Im}(e^{-i\alpha} K_L^2)$ keeping in mind (A.6) and our choice of phase for v . One can then derive the following relations:

$$\begin{aligned} \text{i)} \quad & \alpha \rightarrow -\alpha \Rightarrow K_L \rightarrow K_L^*, d_n \rightarrow -d_n \\ \text{ii)} \quad & \alpha \rightarrow \pi - \alpha \Rightarrow K_L \rightarrow i K_L^*, d_n \rightarrow -d_n \\ \text{iii)} \quad & g_{33} \rightarrow -g_{33}, \alpha \rightarrow \frac{\pi}{2} - \alpha \Rightarrow K_L \rightarrow e^{i\pi/4} K_L^*, d_n \rightarrow -d_n \end{aligned} \quad (A.22)$$

For the computation of d_n (and of the mixing angles as well) it is therefore sufficient to consider $0 \leq \alpha \leq \pi/2$ and $g_{33} > 0$. Note that d_n ($g_{22} < 0$) cannot be related to d_n ($g_{22} > 0$) in a similar way and thus must be considered separately.

For $m_t \leq m_b m_c / m_s$ the angle α is restricted [13] by

$$|\sin 2\alpha| \lesssim \frac{m_s m_t}{m_c m_b} \quad (\text{A.23})$$

and there are two different solutions for K_L for given α and $\text{sgn } g_{22}$. For $m_t > m_b m_c / m_s$, α is not restricted and there is a unique solution for K_L in terms of α and $\text{sgn } g_{22}$.

As a final remark we note that

$$\left| \frac{W}{V} \right| \simeq \frac{m_b}{m_t} \quad (\text{A.24})$$

to a very good approximation. The approximate equality (A.24) can be argued to be a general feature of the minimal model (single ϕ multiplet) if the smallness of mixing angles is to have a natural explanation.

Figure Captions

- Fig. 1 : Gauge boson loop diagrams (unitary gauge) for the EDM of fermion i .
- Fig. 2 : Scalar loop diagrams.
- Fig. 3 : Exchange diagrams contributing to d_n : W exchange (a), charged Higgs exchange (b), neutral Higgs exchange (c,d). The photon must be appended to quark lines in all possible ways.
- Fig. 4 : W loop, ϕ^+ loop and W exchange contributions to d_n for $m_t = 30$ GeV and $g_{22} > 0$ [see (A.23)].
- Fig. 5 : Same as Fig. 4 with $m_t = 60$ GeV and $g_{22} > 0$.
- Fig. 6 : Same as Fig. 4 with $m_t = 60$ GeV and $g_{22} < 0$.
- Fig. 7 : Same as Fig. 4 with $m_t = 100$ GeV and $g_{22} > 0$.

Fig. 4

Fig. 5

Fig. 6

Fig. 7