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TRANSVERSE HADRONIC ENERGY EMISSION IN HARD SCATTERING PROCESSES^{*)}

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A B S T R A C T

We discuss the distribution of transverse energy associated with soft multigluon emission from incoming quarks in hard processes such as high-mass lepton pair and hadron jet production, with particular emphasis on the dependence on detector angular configuration and the effects of non-leading QCD corrections.

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1. INTRODUCTION

A fundamental prediction of Quantum Chromodynamics (QCD) is that in any hard process involving incoming coloured partons (quarks or gluons), some of the energy of the partons should be radiated away as soft gluon bremsstrahlung¹⁾. Leaving aside the controversial possibility of some drastic effect due to spectator partons²⁾, one would expect an associated distribution of radiated transverse energy, E_T , for hard hadron collisions which depends on the hard process scale Q and the QCD scale Λ but otherwise is approximately universal in the range $\Lambda \ll E_T \ll Q$. In this range two types of large logarithms, $\ln(Q/\Lambda)$ and $\ln(Q/E_T)$, may appear in each order of perturbation theory.

Using techniques developed to sum these large terms to all orders, interesting predictions have been presented both for the hadronic transverse energy distribution in lepton pair production (the Drell-Yan process)³⁾ and as a possible more general explanation of the observed transverse energy distribution averaged over all types of collisions⁴⁾. It should be emphasized that in the latter case, the predictions apply only to initial-state bremsstrahlung: transverse energy associated with outgoing partons that have undergone hard collisions must be treated separately or subtracted experimentally and we do not consider it here. In the Drell-Yan process, on the other hand, there are no additional outgoing partons and results based on initial-state bremsstrahlung can be applied directly.

In this paper we investigate two potentially important sources of corrections to predictions of this type: the effects of limited angular coverage by the detector of the radiated hadronic energy, and non-leading QCD corrections.

The angular coverage of the detector is important because the double leading logarithms summed by the above-mentioned techniques correspond to soft, almost collinear, gluon emission. In practice, however, the detector does not cover the collinear regions, both because of engineering difficulties and also because of the contamination of those regions by debris from soft spectator interactions. If the region covered starts at an angle θ_0 to the collinear directions, then collinear logarithms are liable to get replaced by $\ln\theta_0$; predictions neglecting this fact will be strongly modified for $E_T < Q\theta_0$. We investigate this more quantitatively in Sect. 2, using a technique which allows us to study also the interesting effects of varying the azimuthal angular coverage.

In Sect. 3 we study the effects of QCD contributions that are non-leading in terms of the number of large logarithms they contain. For this purpose we can take the expected corrections for the Drell-Yan lepton pair transverse momentum distribution and evaluate from them the corresponding corrections to the hadronic transverse energy distribution. We find that although individual next-to-leading contributions are large their net effect in this case is small.

Sect. 4 contains a summary of our results and some concluding remarks.

2. DEPENDENCE OF E_T DISTRIBUTION ON ANGULAR REGION COVERED

To determine the dependence of observed cross sections on detector properties, it is convenient to use the generating functional technique developed by Brown⁵⁾. If $d\sigma_n^{\text{exc}}$ is the exclusive differential cross section for the emission of n objects (in our case, soft gluons) into a phase space element represented by $dp_1 \dots dp_n$, the generating functional G is defined as

$$G[f(p)] = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\sigma_n^{\text{exc}} f(p_1) \dots f(p_n) \quad (2.1)$$

where $f(p)$ is an arbitrary function of four-momentum. (The $1/n!$ arises because we integrate over the whole of phase space for each emitted object.) Then

$$d\sigma_n^{\text{exc}} = \prod_{i=1}^n dp_i \left. \frac{\delta}{\delta f(p_i)} G[f(p)] \right|_{f=0} \quad (2.2)$$

and the total cross section is

$$\sigma^{\text{tot}} = \sum_n \frac{1}{n!} \int d\sigma_n^{\text{exc}} = G[1] \quad (2.3)$$

Suppose now that $\varepsilon(p)$ is the detection probability for an object of momentum p . An ideal detector covering the polar angular region $\theta_0 < \theta < \pi - \theta_0$ and the azimuthal region $0 < \phi < \Phi$, for example, would correspond to

$$\varepsilon(p) = \Theta(\sin \theta - \sin \theta_0) \Theta(\Phi - \phi) \quad (2.4)$$

The differential cross-section for the detection of n objects in the phase space element $dp_1 \dots dp_n$, is then

$$d\sigma_n^{\text{det}} = \prod_{i=1}^n dp_i \varepsilon(p_i) \frac{\delta}{\delta f(p_i)} G[f(p)] \Big|_{f(p)=1-\varepsilon(p)} \quad (2.5)$$

[cf. Eq.(2.2). We need $f = 1 - \varepsilon$ to give a probability factor of $(1 - \varepsilon)$ for every object produced but not detected.] Consequently the differential cross-section for total detected transverse energy E_T is

$$\begin{aligned} \frac{d\sigma^{\text{det}}}{dE_T} &= \sum_n \frac{1}{n!} \int d\sigma_n^{\text{det}} \delta(E_T - \sum_{j=1}^n E_j \sin \theta_j) \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau e^{-i\tau E_T} \left\{ \sum_n \frac{1}{n!} \int d\sigma_n^{\text{det}} \prod_{j=1}^n \exp(i\tau E_j \sin \theta_j) \right\} \\ &= \frac{1}{2\pi} \int d\tau e^{-i\tau E_T} G[1 - \varepsilon(p) + \varepsilon(p) \exp(i\tau E \sin \theta)] \end{aligned} \quad (2.6)$$

We see from Eq.(2.6) that, although it remains true for any detector [any $\varepsilon(p)$] that

$$\int \frac{d\sigma^{\text{det}}}{dE_T} dE_T = G[1] = \sigma^{\text{tot}}, \quad (2.7)$$

the form of the observed E_T distribution depends on the detector^{*)}. One further mathematical step that will be useful is to express Eq.(2.6) in terms of the single particle inclusive distribution $N(p) \equiv C_1(p)$ and the multi-particle correlations $C_n(p_1, \dots, p_n)$. We have⁵⁾

$$G[f(p)] = \sigma^{\text{tot}} \exp \left\{ \sum_{n=1}^{\infty} \frac{1}{n!} \int C_n(p_1, \dots, p_n) \prod_{j=1}^n [f(p_j) - 1] dp_j \right\} \quad (2.8)$$

*) With straightforward modifications, the same formalism may be applied to quantities other than transverse energy and to systems of several detectors (or a detector with several independent elements).

and hence

$$\frac{d\sigma}{dE_T}^{\text{det}} = \frac{\sigma^{\text{tot}}}{2\pi} \int d\tau \exp \left\{ -i\tau E_T - \sum_{n=1}^{\infty} \frac{1}{n!} \int C_n(p_1, \dots, p_n) \times \right. \\ \left. \prod_{j=1}^n \varepsilon(p_j) \left[1 - \exp(i\tau E_j \sin \theta_j) \right] dp_j \right\} \quad (2.9)$$

We now apply this formalism to the case of soft gluon bremsstrahlung from an incoming quark-antiquark pair undergoing a hard process with energy scale Q . In the double leading logarithmic approximation (DLA) the single-gluon distribution is

$$N(p) \simeq C_F \frac{\alpha_s}{\pi^2} \frac{1}{p^2} \left\{ \frac{1}{\theta^2} + \frac{1}{(\pi - \theta)^2} \right\} \Theta(Q - p) \quad (2.10)$$

($C_F = 4/3$) and there are no multigluon correlations, so Eq.(2.9) becomes

$$\frac{d\sigma}{dE_T}^{\text{det}} \underset{\text{DLA}}{\simeq} \frac{\sigma^{\text{tot}}}{2\pi} \int d\tau \exp \left\{ -i\tau E_T - \frac{C_F \alpha_s}{\pi^2} \int_0^Q \frac{dp_T}{p_T} \int_{p_T/Q}^1 \frac{d\theta}{\theta} \int_0^{2\pi} d\phi \right. \\ \left. \cdot \left[1 - \exp(i\tau p_T) \right] \cdot \left[\varepsilon(p, \theta, \phi) + \varepsilon(p, \pi - \theta, \phi) \right] \right\} \quad (2.11)$$

In Eq.(2.11) σ_{tot} is the hard scattering total cross-section ($\sigma_{\text{tot}} = 4\pi\alpha^2 e_q^2 / 9Q^2$ in lowest order for the Drell-Yan process), $p_T = E \sin \theta = p \sin \theta$ and α_s is taken for the moment to be a constant. For a detector characterized by Eq.(2.4) with $\theta_0 \ll 1$ we find

$$\frac{d\sigma}{dE_T}^{\text{det}} \simeq \frac{\sigma^{\text{tot}}}{2\pi} \int d\tau \exp \left\{ -i\tau E_T + S(\tau, Q, \Phi) - S(\tau, Q\theta_0, \Phi) \right\} \quad (2.12)$$

where

$$S(\tau, Q, \Phi) = \frac{2C_F \alpha_s}{\pi^2} \Phi \int_0^Q \frac{dp_T}{p_T} \ln \left(\frac{p_T}{Q} \right) \left[1 - \exp(i p_T \tau) \right] \quad (2.13)$$

$$S(\tau, Q, \Phi) \approx -\frac{C_F \alpha_s}{\pi^2} \Phi \left\{ \left[\ln(\tau Q) + \gamma - i\frac{\pi}{2} \right]^2 + \frac{\pi^2}{6} \right\} \quad (2.14)$$

when $\tau Q \gg 1$

where γ is Euler's constant.

Remembering that we are concerned with the region $E_T \ll Q$ and hence $\tau Q \gg 1$, we may use the asymptotic expression (2.14) to evaluate the right-hand side of Eq.(2.12) analytically in two limiting cases :

i) Neglecting the angular cut-off θ_0 , i.e. neglecting the lack of collinear coverage of the detector. This will be appropriate for $Q\theta_0 \ll E_T \ll Q$, if such a region exists. We may then set $S(\tau, Q\theta_0, \Phi) = 0$ in Eq.(2.12) to obtain

$$\frac{d\sigma^{\text{det}}}{dE_T} \approx \frac{\sigma^{\text{tot}}}{E_T} \frac{e^{-\gamma z}}{\Gamma(z)} \left(\frac{E_T}{Q} \right)^{\frac{1}{2}z} \quad (2.15)$$

where

$$z = \frac{2 C_F \alpha_s}{\pi^2} \Phi \ln \left(\frac{Q}{E_T} \right) \quad (2.16)$$

ii) In the region $E_T \ll Q\theta_0$, for which both $\tau Q \gg 1$ and $\tau Q\theta_0 \gg 1$ in Eq.(2.12) and we now find

$$\frac{d\sigma^{\text{det}}}{dE_T} \approx \frac{\sigma^{\text{tot}}}{E_T} \frac{e^{-\gamma z_0}}{\Gamma(z_0)} \left(\frac{E_T}{Q\sqrt{\theta_0}} \right)^{z_0} \quad (2.17)$$

where

$$z_0 = \frac{2 C_F \alpha_s}{\pi^2} \Phi \ln \left(\frac{1}{\theta_0} \right) \quad (2.18)$$

Fig. 1 shows the type of transverse energy distribution given by Eqs.(2.15) and (2.17). Notice that the two expressions, valid for $E_T \gg Q\theta_0$ and $E_T \ll Q\theta_0$ respectively, agree at $E_T = Q\theta_0$ ($z = z_0$), so we have boldly extrapolated them

both to that point. Since $\sigma^{\text{tot}} \propto Q^{-2}$ for a hard process involving no intrinsic energy scale, the scaled transverse energy distribution $Q^3 d\sigma^{\text{det}}/dE_T$ is, for fixed α_s , a universal function of E_T/Q ⁶⁾, although of course the form of this function depends on the detector parameters θ_0 and Φ .

Perhaps the most striking qualitative feature of the figure is that the cross-section diverges (integrably) as $E_T/Q \rightarrow 0$ instead of having a zero as predicted by Eq.(2.15), which is essentially the standard leading logarithm expression for the E_T distribution. As explained in the Introduction, this is because of the replacement of logarithms associated with collinear divergences by $\ln(1/\theta_0)$ in Eq.(2.18).

Turning to the dependence on the azimuthal coverage Φ of the detector, we see from Eqs.(2.16) and (2.18) that this enters in the combination $\alpha_s \Phi$ and therefore, to leading order, reducing the azimuthal coverage is equivalent to reducing the strong coupling constant. This in turn will suppress high-order multigluon contributions, which give more isotropic or multijet energy flow patterns relative to the lowest order one gluon term, quantifying the tendency for a narrower detector to select more jetlike configurations.

These predictions can be made somewhat more realistic by using a running value for the strong coupling constant. There are good arguments for taking the scale at the gluon bremsstrahlung vertex to be the transverse momentum p_T ⁷⁾, so we use

$$\alpha_s = \frac{1}{\beta_0 \ln(p_T^2/\Lambda^2)} \quad ; \quad \beta_0 = \frac{33 - 2N_f}{12\pi} \quad (2.19)$$

Eq.(2.13) becomes

$$S(\tau, Q, \Phi) = \frac{C_F \Phi}{\pi^2 \beta_0} \int_{\lambda}^Q \frac{dp_T}{p_T} [1 - \exp(-i\tau p_T)] \frac{\ln(p_T/Q)}{\ln(p_T/\Lambda)} \quad (2.20)$$

where λ is the cut-off to regularize the singularity of α_s at $p_T = \Lambda$ (since the important region of integration is $p_T \gg \Lambda$, λ never appears in leading logarithmic results). Corresponding to Eq.(2.14) we find

$$S(\tau, Q, \Phi) \underset{Q\tau \gg 1}{\sim} \frac{C_F \Phi}{\pi^2 \beta_0} \left\{ \ln \frac{Q}{\Lambda} \ln \left[\frac{-\ln(\tau \Lambda) - \gamma + i\pi/2}{\ln(Q/\Lambda)} \right] + \ln(\tau Q) + \gamma - \frac{i\pi}{2} \right\} \quad (2.21)$$

and in place of Eqs. (2.15) to (2.18) :

i) For $\Lambda \ll Q\theta_0 \ll E_T \ll Q$

$$\frac{d\sigma^{\text{det}}}{dE_T} = \frac{C_F}{E_T} \frac{e^{-\gamma z}}{\Gamma(z)} \exp \left[\frac{C_F \Phi}{\pi^2 \beta_0} \left\{ \ln \frac{Q}{\Lambda} \ln \ln \frac{E_T}{\Lambda} - \ln \frac{Q}{\Lambda} \ln \ln \frac{Q}{\Lambda} + \ln \frac{Q}{E_T} \right\} \right] \quad (2.22)$$

where z is given by Eq.(2.16) with $\alpha_s = \alpha_s(E_T^2)$.

ii) For $\Lambda \ll E_T \ll Q\theta_0 \ll Q$

$$\frac{d\sigma^{\text{det}}}{dE_T} = \frac{C_F}{E_T} \frac{e^{-\gamma z_0}}{\Gamma(z_0)} \exp \left[\frac{C_F \Phi}{\pi^2 \beta_0} \left\{ \ln \frac{Q\theta_0}{\Lambda} \ln \ln \frac{Q\theta_0}{\Lambda} - \ln \frac{Q}{\Lambda} \ln \ln \frac{Q}{\Lambda} - \ln \theta_0 \ln \ln \frac{E_T}{\Lambda} - \ln \theta_0 \right\} \right] \quad (2.23)$$

with z_0 given by Eq.(2.18) with $\alpha_s = \alpha_s(E_T^2)$.

These functions are shown in Fig. 2. We see that the qualitative features are similar to the fixed α_s case (Fig. 1). As before, the two expressions agree at $E_T = Q\theta_0$ and we therefore extrapolate both to that point. The scaling behaviour of $Q^3 d\sigma^{\text{det}}/dE_T$ versus E_T/Q is violated logarithmically when α_s runs, owing to the introduction of the intrinsic QCD scale Λ .

The predicted dependence on Φ , the azimuthal coverage of the detector, is illustrated in Fig. 3, this time for the case of negligible θ_0 ^{*)}. To interpret these results, it is useful to compare them with what would be seen if all events were perfect single jets or were perfectly isotropic in azimuth, with the same E_T distribution when $\Phi = 2\pi$. In the single-jet case, a detector subtending Φ would see all the transverse energy $\Phi/2\pi$ of the time, so

^{*)} In practice, the unavoidable contribution of soft spectator interactions would modify the distribution at low E_T for small θ_0 and might obscure the predicted turnover in this region at present energies.

$$\left. \frac{d\sigma^{1\text{-jet}}}{dE_T}(\Phi) \right|_{E_T=E} = \frac{\Phi}{2\pi} \left. \frac{d\sigma^{\text{det}}}{dE_T}(2\pi) \right|_{E_T=E} \quad (2.24)$$

while for isotropic events the detector would see $\Phi/2\pi$ of the transverse energy all the time, and so

$$\left. \frac{d\sigma^{\text{isotropic}}}{dE_T}(\Phi) \right|_{E_T=E} = \frac{2\pi}{\Phi} \left. \frac{d\sigma^{\text{det}}}{dE_T}(2\pi) \right|_{E_T=2\pi E/\Phi} \quad (2.25)$$

Eqs.(2.24) and (2.25) are shown in Fig. 3 by the dashed and dotted curves respectively. In general, the predicted distribution looks more like that for isotropic events at low E_T and more like that for jets at high E_T . As noted before, smaller values of Φ select more jetty events : the similarity to the one-jet distribution sets in at lower values of E_T . A detector with $\Phi = 2\pi$, on the other hand, would not be expected to see a jetty distribution at $Q = 90$ GeV until $E_T \gtrsim 30$ GeV. This contrasts with the case of the dilepton transverse momentum (Q_T) distribution in the Drell-Yan process⁸⁾, which is expected to be dominated by the single-jet configuration at lower values of Q_T . The fact that the gluon transverse momenta in isotropic multigluon emission add to E_T but cancel vectorially in Q_T means that such configurations will naturally predominate out to larger values of E_T than Q_T .

3. NON-LEADING QCD CORRECTIONS

To discuss non-leading corrections we have to look at the gluon bremsstrahlung mechanism in more detail; to be specific we examine in this section the case of the Drell-Yan process.

We consider a quark-antiquark pair with centre of mass energy $\sqrt{s}$, radiating gluons with total transverse energy E_T and then annihilating to a virtual photon with mass Q^2 . Two large energy scales are present, $\sqrt{s}$ and E_T , which nevertheless have a large ratio. The restriction of the phase space for real gluon emission implied by selecting events with a given hadronic transverse energy causes logarithms of this ratio to appear in calculations and necessitates a sum over

diagrams to all orders. (Large logarithms containing the angular cut-off θ_0 may also appear if this is not negligible, as in Eqs.(2.13) and (2.17). For clarity, we take in this section the case of negligible θ_0).

The cross-section $d\sigma/dE_T$ (with a calorimeter covering the whole solid angle) appears as a convolution involving restricted quark and antiquark structure functions $D(\hat{s}, E_T, x)$. $D(\hat{s}, E_T, x)$ is the probability distribution for the evolution of the initial quark into another quark with momentum fraction x , virtuality limited by $\hat{s}$ and accompanied by gluons with total transverse energy E_T .

Then

$$\int dE_T D(\hat{s}, E_T, x) = D(\hat{s}, x) \quad (3.1)$$

$D(\hat{s}, x)$ is the normal integrated structure function. So

$$\frac{1}{\sigma_0} \frac{d\sigma}{dx_q dx_{\bar{q}} dE_T} = \int dE_{Tq} dE_{T\bar{q}} dE_{TV} \delta(E_T - E_{Tq} - E_{T\bar{q}} - E_{TV}) \cdot D[\hat{s}, E_{Tq}, x_q] \cdot D[\hat{s}, E_{T\bar{q}}, x_{\bar{q}}] \cdot V[\hat{s}, E_{TV}] \quad (3.2)$$

where V represents the central process $q\bar{q} \rightarrow \gamma^* \rightarrow \mu^+ \mu^-$. σ_0 is the E_T integrated cross-section, i.e., the normal Drell-Yan cross-section $4\pi\alpha^2 e_q^2 / 9Q^2$.

The evolution of $D(k^2, E_T, x)$ is given by equations similar to those of Bassetto, Ciafaloni and Marchesini⁹⁾. The large logarithms that we seek are found, in an axial or planar gauge, in ladder diagrams¹⁾ where the parton masses are ordered along the ladder. A Bethe-Salpeter equation for $D(k^2, E_T, x)$ can be derived by moving one rung down the ladder from the vertex. Only non-singlet evolution is important for both the leading and next-to-leading terms¹⁰⁾ as we are looking for gluons which are both collinear and soft. The Altarelli-Parisi kernel of the evolution $P(z)$ ¹¹⁾ can be set to $P_{qq}(z)$ and so the space-like quarks evolve to quarks further from the mass-shell along the ladder, radiating time-like gluons.

$$k^2 \frac{\partial}{\partial k^2} D(k^2, E_T, x) = \int_x^1 \frac{dz}{z} \left[\frac{\alpha_s \left(\frac{k^2(1-z)}{z} \right) P_{qq}(z)}{2\pi} \right] \frac{d^2 p_T}{\pi} \delta((1-z)k^2 - p_T^2) \cdot D(k^2, E_T - p_T, \frac{x}{z}) \quad (3.3)$$

Setting the argument of the D function as indicated on the right-hand side of Eq.(3.3) neglects changes of direction of the quark as it evolves. This is justifiable because as $z \rightarrow 1$, the radiated gluon carries very little of its parent's transverse momentum.

Taking a joint Fourier-Mellin transform

$$D(\hat{s}, \tau, n) = \int dE_T \int dx x^{n-1} e^{-i\tau E_T} D(\hat{s}, E_T, x) \quad (3.4)$$

gives

$$k^2 \frac{\partial}{\partial k^2} D(k^2, \tau, n) = \int_0^1 dz z^{n-1} \left[\frac{\alpha_s \left(\frac{k^2(1-z)}{z} \right) P_{qq}(z)}{2\pi} \right]_+ \frac{d^2 p_T}{\pi} \delta((1-z)k^2 - p_T^2) \times e^{-i\tau p_T} \cdot D(k^2, \tau, n) \quad (3.5)$$

The z integration can be split into a part describing soft and collinear gluon emission where we can set $z = 1$ in all slowly varying functions and a part describing the evolution of the integrated structure functions by collinear emission^{9),12)}.

$$k^2 \frac{\partial}{\partial k^2} D(k^2, \tau, n) = \int \frac{d^2 p_T}{\pi} \delta((1-z)k^2 - p_T^2) \left[\int_0^1 dz z^{n-1} \left(\frac{\alpha_s \left(\frac{k^2(1-z)}{z} \right) P_{qq}(z)}{2\pi} \right) D(k^2, \tau, n) + \int_0^1 dz z^{n-1} \left(\frac{\alpha_s \left(\frac{k^2(1-z)}{z} \right) P_{qq}(z)}{2\pi} \right) \left\{ e^{-i\tau p_T} - 1 \right\} D(k^2, \tau, n) \right] \quad (3.6)$$

$$k^2 \frac{\partial}{\partial k^2} D(k^2, \tau, n) = \int \frac{d^2 p_T}{\pi} \delta((1-z)k^2 - p_T^2) \left[\frac{\alpha_s(k^2)}{2\pi} A_n^{qq} + \int_0^1 dz \frac{\alpha_s \left(\frac{k^2(1-z)}{z} \right) P_{qq}(z)}{2\pi} \left\{ e^{-i\tau p_T} - 1 \right\} D(k^2, \tau, n) \right] \quad (3.7)$$

where A_n^{qq} is the usual non-singlet anomalous dimension. The solution is

$$D(\hat{s}, \tau, n) = D(\hat{s}_0, \tau, n) \times \exp - \int_{\hat{s}_0}^{\hat{s}} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} A_n^{qq} \\ \times \exp \int_{\hat{s}_0}^{\hat{s}} \frac{dk^2}{k^2} \int d p_T^2 \delta((1-z)k^2 - p_T^2) \frac{\alpha_s(p_T^2)}{2\pi} \int dz P_{qq}(z) \left[e^{-i\tau p_T} - 1 \right] \quad (3.8)$$

We must choose a starting point for the evolution $\hat{s}_0$ to be such that $\alpha_s(\hat{s}_0)/2\pi \ll 1$ but also such that no large logarithms $\ln(\sqrt{\hat{s}_0}\tau)$ arise. Thus we choose $\sqrt{\hat{s}_0} = 1/\tau$ with the condition $\tau \ll 1/\Lambda$. Then

$$D(\hat{s}_0, \tau, n) = D(\hat{s}_0, n) + O(\alpha_s(\hat{s}_0)) \quad (3.9)$$

and so

$$D(\hat{s}_0, \tau, n) \times \exp - \int_{\hat{s}_0}^{\hat{s}} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} A_n^{qq} = D(\hat{s}, n) \quad (3.10)$$

giving the correct normalization to Eq.(3.8) according to Eq.(3.1) (by setting $\tau = 0$). In fact we shall study the case $n = 1$ so that the integrated structure functions do not enter the equations since $A_1^{qq} = 0$. Then

$$D_q(\hat{s}, \tau, 1) = \exp S_q(\tau) \quad (3.11)$$

Because the antiquark structure function and the vertex term also take a similar form, we have

$$\frac{1}{G_0} \frac{d\sigma}{dE_T} = \frac{1}{2\pi} \int d\tau e^{iE_T\tau} \exp S(\tau) \quad (3.12)$$

with $S(\tau) = S_q + S_{\bar{q}} + S_v$. Eq.(3.12) is the first $Q^2/\hat{s}$ moment of Eq.(3.2) and σ_0 is now $4\pi\alpha^2 e_q^2/9\hat{s}$.

$\text{Exp}[S(\tau)]$ represents the squared quark form factor in τ space. $S(\tau)$ contains large logarithms $\ln(\sqrt{s}\tau) \equiv \ell$ and $\ln(\sqrt{s}/\Lambda) \equiv L$ so should be a good approximation in the region $1/\sqrt{s} \ll \tau \ll 1/\Lambda$.

Ignoring power corrections, we may write it in the general form

$$S(\tau) \sim \sum_{N=1}^{\infty} \sum_{M=0}^{N+1} d_N^M \frac{L^M}{L^N} \quad (3.13)$$

where $d_N^M \sim 0(1)$. The connection between these large logarithms in τ space and those in E_T space where $d\sigma/dE_T$ is observed are not straightforward¹³⁾. The leading "trajectory" of Eq.(3.13) with $M = N + 1$, $S_1(\tau)$, gives rise to terms $\alpha_s^p \ln^q(\sqrt{s}/E_T)$ with $q \leq 2p - 1$, i.e., the leading and non-leading double logarithms. Terms with $q \leq 2p - 2$ can arise from cross-terms between the $M = N + 1$ and the $M = N$ trajectory, $S_2(\tau)$ (which by itself gives rise to single logarithms $q \leq p - 1$). It is hoped that these latter terms are included correctly by calculating $S_2(\tau)$ using this exponentiating method.

Previous calculations have concentrated on $S_1(\tau)$ including pieces of $S_2(\tau)$. We now look numerically at the terms in $S_2(\tau)$ to see what difference they make to the E_T distribution.

Kodaira and Trentadue¹⁰⁾ recently proposed a form factor for the quark, useful in the measurement of the back-to-back energy-energy correlations in e^+e^- collisions and expected to apply also to the dilepton transverse momentum distribution in the Drell-Yan process¹⁴⁾. This form factor $T(b)$ is calculated in b space¹⁵⁾ conjugate to Q_T space where Q_T is the total transverse momentum of emitted gluons. A similar expression is obtained as follows for $S(\tau)$.

In the light-like gauge used by Kodaira and Trentadue, the leading term $N = 1, M = 2$ arises from evolution of the quark alone with

$$P(z) = \frac{2}{1-z} \cdot C_F \quad (3.14)$$

$$e^{-i\tau p_T} \rightarrow \Theta(p_T - 1/\tau)$$

$\alpha_s = \text{constant}$

$$S(\tau) \approx - \int_{1/\tau}^{\sqrt{s}} \frac{2C_F}{\pi} \frac{dp_T}{p_T} \alpha_s \ln \hat{s}/p_T^2$$

(3.14)
cont.

$$= - \frac{4C_F}{\pi} \alpha_s \ln^2(\sqrt{s} \tau)$$

This will generate the DLLA expression where one gluon, although still soft, carries essentially all of the transverse energy and the other gluons are both soft and collinear¹⁶⁾.

The leading trajectory $M = N + 1$ is generated from Eq.(3.14) by the running of α_s at the scale p_T^2 which effectively introduces higher powers of α_s

$$\alpha_s(p_T^2) = \alpha_s(\hat{s}) + \beta_0 \alpha_s^2(\hat{s}) \ln \hat{s}/p_T^2 + \dots \quad (3.15)$$

The non-leading trajectory $M = N$ arises from evolution of the antiquark and the vertex V as well as non-leading terms in the quark evolution. (In a non-light-like planar gauge as used by Dokshitzer, D'yakonov and Troyan¹⁾, the antiquark would evolve in the same way as the quark and there would be no large logarithms at the vertex).

Approximating the Fourier transform as in Eq.(3.14), but including next-to-leading corrections (we discuss the effects of this approximation later), gives

$$S(\tau) \approx - \frac{2C_F}{\pi} \int_{1/\tau}^{\sqrt{s}} \frac{dp_T}{p_T} \left\{ \ln \frac{\hat{s}}{p_T^2} \left[\alpha_s(p_T^2) + \frac{K}{2\pi} \alpha_s^2(p_T^2) \right] - \frac{3}{2} \alpha_s(p_T^2) + 2 \left(\gamma - i\frac{\pi}{2} \right) \alpha_s(1/\tau^2) \right\} \quad (3.16)$$

$$S(\tau) = \frac{2C_F}{\pi\beta_0} \left\{ L \ln \left(1 - \frac{L}{L} \right) + L \right\} - \frac{2C_F}{\pi\beta_0} \left(\gamma - i\frac{\pi}{2} \right) \frac{L}{L-L}$$

(3.17)

$$- \frac{3}{2} \frac{C_F}{\pi\beta_0} \ln \left(1 - \frac{L}{L} \right) - \frac{C_F K}{2\pi^2\beta_0^2} \left[\ln \left(1 - \frac{L}{L} \right) + \frac{L}{L-L} \right]$$

$$+ \frac{C_F \beta_1}{\pi \beta_0^3} \left\{ \frac{L}{L-l} + \frac{L}{L-l} \ln \left(1 - \frac{l}{L} \right) + \frac{L}{L-l} \ln 2L + \frac{1}{2} \ln^2 \left(1 - \frac{l}{L} \right) + \ln \left(1 - \frac{l}{L} \right) \ln 2L \right\} \quad (3.17)$$

cont.

$$S(\tau) \equiv S_1(\tau) + S_{21}(\tau) + S_{22}(\tau) + S_{23}(\tau) + S_{24}(\tau)$$

Several terms are necessary in $S_2(\tau)$ to include consistently all terms $O(l/L)^n$.

We see that $S_1(\tau) + S_{21}(\tau)$ are the leading terms of the expansion of the expression used in Eq.(2.21) :

$$S(\tau) \approx L \ln \left(1 - \frac{\hat{l}}{L} \right) + \hat{l} \quad (3.18)$$

where $\hat{l} = l + \gamma - i\pi/2$. The use of $\hat{l}$ as expansion parameter in Eq.(3.17) is arbitrary since $\ln(1 - (a+l)/L)$ can be expanded as

$$\ln \left(1 - \frac{l}{L} \right) + \frac{al}{L-l} \quad (3.19)$$

An expansion in terms of $\hat{l}$ is perhaps more logical since the presence of $\gamma - i\pi/2$ is purely an artefact of Fourier transforming to $\hat{\tau}$ space in order to take account of the δ function $\delta(E_T - E_{Tq} - E_{T\bar{q}} - E_{TV})$. When $\hat{l}$ is the expansion parameter, we find a cross-section $d\sigma/dE_T$ which vanishes as $E_T \rightarrow 0$ (when all θ is covered) as the phase space for gluon production shrinks. Using l instead does not reproduce this.

The other terms in $S_2(\tau)$ arise from various sources :

i) $S_{22}(\tau)$ contains the famous $3/2$ whose inclusion gives a better approximation to the matrix element by including the production of energetic but still collinear gluons. This term is known to exponentiate¹⁶⁾. It is obtained by a better treatment of the first order Altarelli-Parisi kernel $P(z) \propto (1+z^2)/(1-z)$ rather than approximating it by $2/(1-z)$.

In the planar gauge of Ellis and Stirling

$$\int_0^1 \frac{1+z^2}{1-z + p_T^2/\hat{s}(1-z)} dz \approx \ln \hat{s}/p_T^2 - \frac{3}{2} \quad (3.20)$$

The same result is achieved in a light-like axial gauge by considering evolution of the antiquark line and the vertex as well as corrections to the quark evolution¹⁰⁾.

ii) $S_{23}(\tau)$ contains two-loop corrections to the Altarelli-Parisi kernel $P(z)$. This term is rather controversial¹⁷⁾. The form of K used by Kodaira and Trentadue is the coefficient of that part of $P_1(z)$ which behaves like $(1+z^2)/(1-z)$, calculated by Curci, Furmanski and Petronzio¹⁸⁾.

$$K = C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - N_F \frac{10}{18} \quad (3.21)$$

iii) $S_{24}(\tau)$ is effectively $S_1(\tau)$ multiplied by two-loop corrections to α_s (assuming its argument still to be p_T^2).

If $S(\tau) = A(\tau) + iB(\tau)$, then, because $A(\tau) = A(-\tau)$, $B(\tau) = -B(-\tau)$

$$\frac{1}{G_0} \frac{dG}{dE_T} = \frac{1}{\pi} \int_0^\infty d\tau \cos(E_T \tau - B(\tau)) \exp A(\tau) \quad (3.22)$$

This is the form used in the numerical calculation. Fig. 4 shows the effect of different terms in $S(\tau)$ on $\exp A(\tau)$. We find that $S_{23}(\tau)$ and $S_{24}(\tau)$ are small and that $S_{21}(\tau)$ and $S_{22}(\tau)$, although sizeable [but not as large as $S_1(\tau)$], tend to cancel out.

In order to calculate $d\sigma/dE_T$, we must integrate in principle over all τ which requires an extension of $S(\tau)$ outside the region where it is a good approximation. Since $\exp A(\tau)$ cuts off the large τ region, it is only at the lower limit, where $\tau < 1/Q$, that we need worry.

$$\frac{1}{G_0} \int_0^\infty \frac{dG}{dE_T} dE_T = e^{S(0)} \quad (3.23)$$

A normalization of 1 implies $A(0) = B(0) = 0$. The calculation in section two [Eq.(2.14)] continued $S(\tau)$ as it stood in the region of negative $\ln(Q\tau)$ and so $S(0) \rightarrow -\infty$ and

$$\frac{1}{\epsilon_0} \int_0^\infty \frac{d\epsilon}{dE_T} dE_T = 0 \quad (3.24)$$

$d\sigma/dE_T$ then in fact becomes negative for $E_T > Q$, but this will not matter in the region of applicability $E_T \ll Q$ and we still expect $1/\sigma \int_0^Q (d\sigma/dE_T) dE_T \approx 1$ ¹⁹⁾.

In all of the following figures, we have put $A = B = 0$ for $0 < \tau < 1/Q$. It was found that using other methods of continuation made no difference, e.g., $A = B = 0$ for $0 < \tau < 2/Q$ or $\ln(Q\tau) \rightarrow \ln(Q\tau + 1)$.

Because we are considering a one-dimensional Fourier transform, however, the region around $\tau = 0$ gives a large contribution to the cross-section. In fact, it contributes approximately $1/\pi E_T \sin(E_T/Q) \sim 3 \times 10^{-3}$ and this dominates $d\sigma/dE_T$ for $E_T \gtrsim 20 - 30$ GeV. This means that our approximation $E_T \ll Q$ is breaking down here and we cannot apply the analysis beyond this point. The sensitivity of the E_T distribution to small τ is unfortunate because we might hope that large p_T gluons are more correctly treated in a calculation of $d\sigma/dE_T$ rather than $d\sigma/dQ_T^2$, the dilepton transverse momentum distribution. The calculation of $d\sigma/dQ_T^2$ allows all of the gluon p_T s to become very large and their sum to violate total energy conservation ³⁾, whereas here the p_T s are always small as $p_{T1} < E_T < Q$. However, the Q_T distribution has a kinematic bdb which suppresses the small b region and the E_T distribution has no such factor.

Fig. 5 shows that $d\sigma/dE_T$ calculated from $S(\tau)$ differs by a negligible amount from that calculated from $S_1(\tau)$.

$S_{21}(\tau)$ takes into account multi-gluon configurations whose p_T s add up to E_T and thus tends to increase the cross-section at high E_T , making it less "jetty". $S_{22}(\tau)$ represents energetic collinear gluons which shift more of the cross-section to low E_T . The two effects tend to cancel out, leaving $e^{S_1(\tau) + S_2(\tau)}$ slightly more peaked in τ space than $e^{S_1(\tau)}$, i.e., $d\sigma/dE_T$ is slightly broadened by non-leading terms.

Kodaira and Trentadue's work shows the different effect on non-leading terms in a calculation of $d\sigma/dQ_T^2$ and the quark form factor in b space, $\exp T(b)$. $T_{21}(b)$ arising from the momentum conserving δ function contributes mostly to the cross-section at $Q_T = 0$ but has little effect at large Q_T . $T_{22}(b)$ broadens the distribution in b space considerably, so over-all $d\sigma/dQ_T^2$ is more peaked in Q_T space when the next-to-leading terms are included²⁰⁾.

A perturbative calculation of $d\sigma/dE_T$ or $d\sigma/dQ_T^2$ relies on the fact that the form factor (in Fourier transform space) is damped at large values of τ or b ; corresponding to the region $\tau, b \sim 1/\Lambda$ [$p_T \sim O(\Lambda)$] where uncalculable non-perturbative effects are present. The broadening in b of $T(b)$ on the inclusion of non-leading terms is a problem because it makes this region much more important.

We have shown that this does not happen for the E_T distribution but the opposite problem appears. The sensitivity of the cross-section to the region $\tau \gtrsim 1/Q$ (i.e. $\ell \sim 1$) means that even less leading terms than those included here may become important, e.g., the $M = N - 1$ trajectory which gives terms like $1/L(\ell/L)^N$ in $S(\tau)$.

We have shown that $M = N$ terms $(\ell/L)^N$ tend to cancel but there is no reason why this should be the case for other less leading terms. While the assumption that the entire quark form factor exponentiates is unjustified beyond the leading approximation, we might hope to gain some feeling for the effects of less leading terms by considering those which would result from a better approximation to the Fourier transform :

$$S(\tau) = \int_{\lambda^2}^{\hat{s}} \frac{d p_T^2}{p_T^2} \frac{\alpha_s(p_T^2)}{2\pi} \int_0^{1-p_T^2/\hat{s}} dz P_{qq}(z) \left[e^{-i\tau p_T} - 1 \right] \quad (3.25)$$

In fact, we consider terms generated by exact integration of the lowest order expression for $P_{qq}(z)$, i.e., non-leading terms generated from $S_1(\tau) + S_{21}(\tau)$. These terms will certainly appear because the quark form factor has been exponentiated in τ space [the first such term is $\pi^2/3L (1-\ell/L)^{-2}$].

We consider a calorimeter of azimuthal angular acceptance Φ (covering all θ) and take

$$A(\tau) = \frac{\Phi}{2\pi} \int_{\lambda^2}^{\hat{s}} \frac{d p_T^2}{p_T^2} \frac{C_F}{\pi} \ln \hat{s}/p_T^2 \left[\cos(p_T \tau) - 1 \right] \alpha_s(p_T^2) \quad (3.26)$$

$$B(\tau) = \frac{\Phi}{2\pi} \int_{\lambda^2}^{\hat{s}} \frac{d\rho_T^2}{\rho_T^2} \frac{C_F}{\pi} \ln \hat{s}/\rho_T^2 \sin(p_T \tau) \mathcal{L}_3(\rho_T^2) \quad (3.26)$$

cont.

The results are not sensitive to the value of λ which is taken to be 2Λ in the graphs shown (Fig. 6). The $d\sigma/dE_T$ resulting from Eq.(3.26) is considerably broader than that resulting from Eq.(3.22) using $\Phi/2\pi [S_1(\tau) + S_{21}(\tau)]$. Also Eq.(3.26) does not reproduce the single gluon cross-section at high E_T . Eq.(3.22) is much closer to it because at high E_T it approximates the Fourier transform of the one-gluon distribution as all of the $L(\hat{\lambda}/L)^n$ terms drop out except the first $\sim -\hat{\lambda}^2/L$. Eq.(3.26) contains in addition terms of $O(1/L)$ which spoil the agreement. When the $3/2$ term is included in both Eq.(3.22) and Eq.(3.26), then the agreement between them is much improved because this term tends to suppress large values of E_T .

Consider finally the effect of the controversial $S_{23}(\tau)$ term in Eq.(3.17). With the K value used by Kodaira and Trentadue, $S_{23}(\tau)$ tends to peak the cross-section slightly more in τ space, i.e., cause a slight broadening of $d\sigma/dE_T$.

Ellis, Richards and Stirling¹⁷⁾ have analyzed the exact $O(\alpha_s^2)$ back-to-back energy-energy correlation function and they suggest the following form for K :

$$K = C_A \left(\frac{103 \pm 11}{18} - \frac{\pi^2}{6} \right) - \frac{10}{18} N_F - (10.0 \pm 1.2) C_F \quad (3.27)$$

Using this K value, $S_{23}(\tau)$ would tend to narrow the cross-section very slightly in E_T space. Fig. 7 shows that neither of these forms for K has any noticeable effect on the cross-section.

4. CONCLUSIONS

We have studied the distribution of transverse hadronic energy radiated by an incoming quark-antiquark pair which annihilates at a large mass scale. The method used is a resummation of large logarithms.

i) In the DLLA, we show the dependence of this distribution on the angular acceptance of the calorimeter. A calorimeter which misses the forward and backward directions in θ will see a modified distribution below $E_T \sim Q\theta_0$, masking the zero of $d\sigma/dE_T$ as the phase space for soft gluon emission vanishes. The smaller the acceptance in ϕ of the calorimeter, the more jetty the distribution will appear.

ii) We have calculated the transverse energy distribution including leading and next-to-leading logarithms in Fourier transform space. The two-loop part of the next-to-leading correction is negligible, even if its numerical value is not certain. The two $O(\alpha_s)$ next-to-leading terms contrive to almost cancel, so $d\sigma/dE_T$ is only slightly broader than a leading order calculation would suggest. This analysis gives a correct treatment in E_T space of the leading and next-to-leading double logarithms, i.e., $\alpha_s^n \ln^{2n-1} \hat{s}/E_T^2$ and $\alpha_s^n \ln^{2n-2} \hat{s}/E_T^2$.

Parts of less leading terms are also included, e.g., two-loop corrections to $P(z)$ appear at $\alpha_s^n \ln^{2n-3} \hat{s}/E_T^2$ since this term begins at $(\lambda/L)^2$ in τ space. To correctly treat $\alpha_s^n \ln^{2n-3} \hat{s}/E_T^2$ in full, however, we would need to include terms proportional to $1/L$ in τ space, e.g., the π^2 terms arising from analytic continuation which are believed²¹⁾ to take the form $\exp(C_F \pi^2 \alpha_s / 2\pi)$ [and would appear differently in a discussion of the energy-energy correlation in e^+e^- ²²⁾].

We have considered non-leading terms in τ space which arise as a consequence of taking the Fourier transform. They tend to cause a broadening of $d\sigma/dE_T$ but their effect is small. Other non-leading terms might arise from diagrams which introduce a correlation between gluons and these would spoil the factorizing and exponentiating behaviour of the form factor [see Eq.(2.9)].

In general, we see that the effects of non-leading terms on the E_T distribution are mainly confined to the larger E_T region, where a leading-logarithmic approach would be expected to be least reliable. This contrasts with the case of the Drell-Yan transverse momentum (Q_T) distribution where there are also substantial effects at lower Q_T .

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FIGURE CAPTIONS

- Fig. 1 : The transverse energy distribution at fixed $\alpha_s = 0.2$, $Q = 90$ GeV, $\theta_{\min} = 0.1$. The solid line joins Eqs.(2.15) and (2.17) at $E_T = Q\theta_{\min}$. The dashed line shows the extrapolation of these equations outside their region of validity.
- Fig. 2 : The transverse energy distribution for running α_s . $Q = 90$ GeV, $\Lambda = 0.1$ GeV, $N_f = 4$, $\theta_{\min} = 0.1$. The solid line joins Eqs.(2.22) and (2.23) at $E_T = Q\theta_{\min}$. The dashed line shows the extrapolation of these equations outside their region of validity.
- Fig. 3 : The dependence of the transverse energy distribution on Φ . For each value of Φ , the solid line shows the QCD prediction (DLA), the dashed line, $d\sigma^{\text{1-jet}}/dE_T$ and the dotted line, $d\sigma^{\text{isotropic}}/dE_T$. $Q = 90$ GeV, $\Lambda = 0.5$ GeV, $N_f = 4$.
- Fig. 4 : The behaviour of $\exp A(\tau)$. The dashed line is obtained using only $S_1(\tau)$, the dotted line $S_1(\tau) + S_{22}(\tau)$ and the solid line uses $S(\tau) \equiv S_1(\tau) + S_2(\tau)$.
- Fig. 5 : The transverse energy distribution obtained using Eq.(3.12) with $S_1(\tau) + S_2(\tau)$ (solid line) and only $S_1(\tau)$ (dashed line) $\sqrt{s} = 90$ GeV, $\Lambda = 0.2$ GeV, $N_f = 4$.
- Fig. 6 : A comparison of the transverse energy distributions obtained from Eq.(3.22) with $S(\tau) = \Phi/2\pi [S_1(\tau) + S_{21}(\tau)]$ (solid line), and Eq.(3.26) (dashed line), for $\Phi = \pi$, $\Lambda = 0.2$ GeV, $\sqrt{s} = 90$ GeV, $N_f = 4$. The single gluon distribution is shown as a dotted line.
- Fig. 7 : A comparison of the transverse energy distribution obtained using different values for the constant K in Eq.(3.17). The solid line uses $S(\tau)$ without $S_{23}(\tau)$. The dashed line uses $S_1(\tau) + S_{23}(\tau)$ with $K = C_A(67/18 - \pi^2/6) - N_f \cdot 10/18$. The dotted line uses $S_1(\tau) + S_{23}(\tau)$ with $K = C_A(103/18 - \pi^2/6) - 10/18 N_f - 10 C_F$.

FIG. 1

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