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Systematics of pion-nucleus optical potentials from analysis of

elastic scattering

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Most of the available data on the elastic scattering of pions by nuclei between 14 and 80 MeV is analyzed with a complex local optical potential which is parametrized by a bias-free Fourier-Bessel series, and is used as a means of extracting as much information from the data as possible without the explicit use of a model for the interaction. A typical shape of a negative real potential at large radii with a positive one at smaller radii is obtained and a systematic dependence on energy, charge of the pion, and isospin of the target is observed. The Kisslinger potential is shown to reproduce all these systematics whereas the Laplacian potential fails to do so. Some results for energies of up to 230 MeV are also analyzed. The dependence of reaction cross sections on the real potential is discussed.

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[NUCLEAR REACTIONS pion-nucleus "model-independent" optical potentials
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I. INTRODUCTION

With the availability of accurate and extensive experimental results on the elastic scattering of low to medium energy pions by nuclei¹⁻¹¹ it has become feasible to analyze such data with pion-nucleus optical potentials.¹² The pion-nucleon interaction in this energy region is dominated by the (3,3) resonance and, therefore, analyses have been performed using the Kisslinger potential¹³ or some variants of it, with generally reasonable success. This potential is usually written in terms of the densities of neutrons and protons in the target nucleus (and their derivatives), and it contains as many as 13 parameters which cannot all be determined from the scattering and reaction data. Therefore, several assumptions are often made and χ^2 fits to scattering data as well as to pion-nucleon data^{14,15} provide the values of the remaining parameters of the potential. Obviously, making a fit to the data with the help of a model does not necessarily reveal the true information content of the data; some information could be masked out in such a procedure or alternatively one may be led to conclusions which result from the model itself rather than from the data. This problem is particularly interesting in the case of the pion-nucleus interaction because of the peculiar properties of the Kisslinger potential which could have pathological behavior under quite realistic conditions.^{16,17}

The explicit use of a model in analyses of elastic scattering is, of course, not confined to pions. For many years phenomenological optical potentials have been used in analyses of elastic scattering of nucleons and light ions by nuclei, and they eventually provided the quantities which are well determined by the data such as volume integrals and radial moments of the potentials. The more fundamental theories of the optical

potential¹⁸ are seldom used directly in analyzing experimental data; a typical approach is to compare the predictions of such models with the systematics of empirical potentials. In this respect the latter serve as an intermediate stage between the data and theory, and this is particularly true for the so-called "model-independent" potentials^{19,20} where very little is assumed beforehand regarding the shape of the potentials.

Returning to pion scattering, it is known that the pion-nucleus potential is non-local and therefore a truly phenomenological approach does not, at first, look profitable. On the other hand, direct application of more fundamental theories to analyses of experimental data seem even more remote than in the case of nucleons mainly for practical reasons. Consequently, any attempt to study the information content of state-of-the-art experimental results on pion scattering by way of large scale fits to the data must rely on the use of local potentials which, hopefully, will provide useful information on the proper non-local interaction. It is worth mentioning at this point that in the widely used Kisslinger potential it is explicitly assumed that the p-wave component is of a zero range. Finite range is usually allowed in momentum-space descriptions which, unfortunately, are seldom used in large-scale fits to experimental data.

In the present work an attempt is made to extract information on the pion-nucleus potential directly from the data without explicitly assuming any model for the interaction, with the aim of creating an intermediate stage between experimental results and theory. This is done with the help of an empirical "model-independent" local potential, which could then be compared, for example, with the equivalent local potential²¹ resulting from the Kisslinger potential or with the truly local Laplacian

potential.²² Most of the available data below 80 MeV is analyzed together with some sample experimental results extending up to 230 MeV. A very consistent picture is obtained in this manner which shows several distinct and systematic trends in the shape of the potentials and their dependence on energy, charge and isospin, and these are then confronted with the Kisslinger and Laplacian models.

The method of the present work is outlined in Sec. II and the results are presented and discussed in Sec. III. Section IV offers some theoretical considerations which explain in broad terms all the observed trends and allow choosing the more suitable interaction model. The results are summarized in Sec. V where a discussion of further experiments is also included.

II. METHOD

The interaction between an elastically scattered pion and a target nucleus is described by a local complex optical potential $U(r)$ which is inserted in the Klein-Gordon equation

$$\hbar^2 c^2 (\nabla^2 + k^2) \psi = [2E(U + V_C) - V_C^2] \psi, \quad (1)$$

where $\hbar k$ is the c.m. momentum and E the c.m. total energy of the pion and V_C is the Coulomb potential due to the finite charge distribution of the target nucleus. Quadratic terms involving the potential U are usually omitted. The elastic scattering calculated from the above equation is compared with experimental differential cross sections and the potential U is varied in a χ^2 parameter-search procedure in order to achieve the best fit of the calculated cross sections to the experimental values. The optical potential is described by a bias-free Fourier-Bessel (FB) series,^{19,20} for $r < R_C$,

$$U(r) = \sum_{n=1}^N a_n j_0(n\pi r/R_C) \quad (2)$$

and is set to zero beyond the cut-off radius R_C which is chosen to be several fm larger than the nuclear radius. a_n are complex coefficients. This form of potential is sufficiently flexible to yield unconventional shapes, if necessary, and the fit procedure starts with all coefficients a_n set to zero so that the potential indeed takes whatever shape is required to fit the data. In the fit process the number of terms in the potential is increased as long as the fit to the data improves and the parameters are well-determined. In most cases the real potential was found to have the peculiar shape of being negative at large radii and positive further inside the nucleus, a shape which differs markedly from conventional optical potentials. In contrast, the imaginary potential was found to be rather featureless, representing smooth absorption throughout the nucleus, which becomes poorly determined towards the nuclear center. The imaginary part of the potential was, therefore, parametrized also as a Woods-Saxon (WS) potential,

$$\text{Im}U(r) = -W/[1 + \exp(r-R_I)/a_I] \quad (3)$$

representing smooth absorption throughout the nucleus, or alternately as a derivative of the WS function also represents smooth absorption which, however, becomes very small at the center. Most of the results presented here were obtained with these smooth absorption parametrizations.

Another advantage of the FB method, in addition to its flexibility, is the ease of calculating²³ the uncertainties of any quantity which is derived from the FB coefficients a_n . Expressing the uncertainties of the coefficients a_n with the help of the covariance matrix which is obtained numerically in the course of the χ^2 best-fit procedure,

$$\frac{\partial a_m}{\partial a_n} = 2(M^{-1})_{mn} \chi_{\text{min}}^2/F, \quad (4)$$

where χ_{min}^2/F is the minimum χ^2 obtained divided by the number of degrees of freedom and

$$(M)_{mn} = \frac{\partial^2(\chi^2)}{\partial a_m \partial a_n}, \quad (5)$$

one finds, for example, that the uncertainty in the real potential as a function of position is given by

$$\delta \text{Re}U(r) = \left\{ \sum_{m,n} \frac{\delta a_m \delta a_n}{\delta a_m \delta a_n} j_0(n\pi r/R_C) j_0(n\pi r/R_C) \right\}^{1/2}. \quad (6)$$

The function χ^2 which is minimized is, as usual,

$$\chi^2 = \sum_i \left\{ [\sigma_{\text{exp}}(\theta_i) - \sigma_{\text{cal}}(\theta_i)] / \Delta\sigma(\theta_i) \right\}^2, \quad (7)$$

where $\sigma_{\text{exp}}(\theta_i)$ and $\sigma_{\text{cal}}(\theta_i)$ are the experimental and calculated differential cross sections at the angle θ_i , respectively. $\Delta\sigma(\theta_i)$ are the corresponding uncertainties in the experimental cross sections. The total reaction cross section σ_R may also be calculated and compared with experiment and then the term

$$\chi_R^2 = [\sigma_R(\text{exp}) - \sigma_R(\text{cal})]^2 / (\Delta\sigma_R)^2 \quad (8)$$

is added to (7). In such cases the reaction cross section acts, in fact, as a constraint on the fits to the angular distribution of the elastic scattering. Similar expressions provide the uncertainties in the imaginary potential when the FB option is used for it.

Fits to the data are usually started with $U = 0$, by having all the coefficients $a_n = 0$ and also $W = 0$ in the smooth absorption version.

The geometrical parameters of the imaginary potential (R_I, a_I) are usually chosen, only in the initial phases of the parameter search, close to the corresponding values for the charge distribution. Calculations are repeated for several values of the cut-off radius R_C and number of terms N to ensure not only the best χ^2/F but also convergence of the potential.

As an example, R_c was between 4.4 and 5.0 fm for ^{12}C and between 9.3 and 10.3 fm for ^{208}Pb . The number of terms N was usually between 2 (at the lowest energies) and 5 with the total number of parameters never exceeding 8. As the number of data points in most cases is about 20 or more, the parameter values obtained in the fits have almost always been well determined. Some fits were repeated with very different choices of initial values to ascertain that the final potentials did not depend on the procedure used.

III. RESULTS

The data analyzed consists mostly of published results¹⁻⁹ but some unpublished data,^{10,11} particularly for π^- , was also included in the present work. The target nuclei studied were ^{12}C , ^{13}C , ^{16}O , ^{40}Ca , ^{48}Ca , ^{90}Zr , and ^{208}Pb covering different energy ranges, e.g. 14-80 MeV for ^{12}C , 20-230 MeV for ^{40}Ca , and 20-80 MeV for ^{208}Pb . The quality of the fits to the data achieved with the FB potential was very good, typical values of χ^2/F being 1.5-2 with some cases with values as low as 0.5. In a few cases it was impossible to achieve χ^2/F smaller than ≈ 3 . For 50 MeV pions on ^{12}C and on Fe experimental total reaction cross sections are also available,²⁴ and for these cases the χ^2 fits have also been repeated with the q_R constraint [Eq. (8) added to (7)].

Figures 1 and 2 provide examples of the fairly extensive data used and of the very good fits between calculation and experiment which have been achieved. Figure 3 shows examples for the energy dependence of the optical potentials for π^+ , where the dots with the error bars (6) represent the real potential and the smooth curves represent the imaginary potential, drawn with its sign reversed for clarity of display. Errors are not plotted for the imaginary potential as no simple error analysis

is available in the smooth absorption version of the potential, but the scatter of the results obtained from several repeats of the fits with different real potentials suggests that the imaginary potentials are determined to about 10% over the range plotted. When the derivative of the WS function is used for the imaginary potential, the resulting potentials are almost identical to the ones plotted, with large differences observed only at radii smaller than 1.5 fm, which is a region where the potential is not determined by the data. This is also confirmed by using a FB series for the imaginary potential, where an error analysis is readily available. The real potential was always found to be independent of the particular option chosen for the imaginary part.

The shape of a negative real potential at large radii which becomes positive further inside the nucleus is the typical shape found for all targets studied, at energies well below 100 MeV. In the few cases which have been analyzed above 100 MeV (see below) the real potential becomes attractive and almost featureless throughout the nucleus, together with a greatly increased imaginary potential. It is interesting to note the increase in the errors towards the interior of the nucleus, reflecting the reduced transparency both due to a potential barrier and absorption. Very similar effects were observed in the applications of the FB method to analyses of elastic scattering of α particles,^{19,20} although in the latter case the reduced transparency is mostly due to absorption.

The dependence of the real potential on the bombarding energy is clearly observed in Fig. 3. The depth of the external attractive part remains fairly constant (except possibly at the lowest energies) whereas the internal repulsive part becomes considerably smaller as the energy increases, a trend which is enhanced in ^{90}Zr compared to ^{12}C . The point

where the real potential crosses the zero is seen to shift to smaller radii as the energy increases. The imaginary potential increases with energy, particularly towards 80 MeV.

Figures 4 and 5 show examples for the dependence of the optical potentials on the charge of the projectile and isospin of the target. For ^{12}C and ^{16}O the potentials are virtually the same for π^+ and π^- but for ^{40}Ca , which is also a self-conjugate nucleus with zero isospin, there are distinct differences between the potentials for π^+ and π^- reminiscent of the energy dependence observed in Fig. 3, which could arise from the Coulomb potential. This effect is also observed at other energies, as is evident in Fig. 6. Effects due to the isospin of the target nucleus are observed when comparing the potentials for ^{48}Ca with those for ^{40}Ca and the ^{14}C with ^{12}C ones. For π^+ the negative parts are similar for ^{40}Ca and ^{48}Ca but for π^- a shift to larger radii is observed in the negative part of the ^{48}Ca real potential relative to ^{40}Ca . This is consistent with the expectations that the p-wave interaction of π^+ is mostly with protons and that of π^- is mostly with neutrons, and that the 28 neutrons in ^{48}Ca extend to larger radii than do the 20 neutrons in ^{40}Ca . One should note that the proton distributions in ^{40}Ca and ^{48}Ca are known²⁵ to be remarkably similar. By comparing the difference between the potentials for π^+ and π^- on ^{48}Ca with the corresponding difference for ^{40}Ca , isospin effects are observed in the former in addition to the charge-dependent effects which are seen in the latter (assuming a zero isospin for ^{40}Ca). Very large charge and isospin effects are observed on ^{208}Pb , where for π^- the potential remains negative throughout most of the nucleus even at the low energy of 50 MeV. Regarding the imaginary potential, it is observed that it is always larger for π^- than for π^+ except

in the case of ^{16}O and ^{12}C (Fig. 5). It also increases with the addition of neutrons, as can be seen by comparing ^{14}C to ^{12}C and ^{48}Ca to ^{40}Ca .

In the case of the $^{12},^{14}\text{C}$ isotopes (Fig. 5) one notes a shift of the attractive part towards larger radii also for the π^+ potential, when two neutrons are added. Unlike the situation with $^{48},^{48}\text{Ca}$, the proton density is known²⁵ to extend to larger radii in ^{14}C than in ^{12}C so the shift in the π^+ potential is indeed expected.

Figure 6 shows some additional examples for the strong dependence of the shape of the real potential on energy, extending well beyond 100 MeV. It is seen that the characteristic shape which is found at the lower energies disappears at the higher energies. The charge dependence observed for this isoscalar target nucleus, which at first seems surprising, persists at the higher energies, with clear indications for a sign reversal of the effect on the real potential at the (3,3) resonance. The potentials at 230 MeV look very similar to those at 180 MeV.

The main features of the "model-independent" local potentials obtained in the present analysis are summarized below. These features can be regarded as genuinely resulting from the data because no explicit model for the pion-nucleus interaction was employed in the analysis.

- (a) At energies of several tens of MeV the real potential is negative at large radii and positive at smaller radii.
- (b) As the energy increases the positive portion of the real potential becomes smaller fairly quickly, and it disappears around 80 MeV for light targets and around 50 MeV for heavy ones.
- (c) The depth of the external negative part of the real potential is, to approximately $\pm 10\%$, independent of energy and of the target nucleus, below 80 MeV.

- (d) The point where the real potential crosses the zero (if at all) moves to smaller radii as the energy increases.
- (e) The imaginary potential increases with increasing energy.
- (f) When changing from π^+ to π^- on a self-conjugate target nucleus, then except for the lightest nuclei the potential changes in a manner similar to the energy dependence mentioned above, as though the π^- corresponds to higher energies than the π^+ .

IV. THEORETICAL CONSIDERATIONS

It is interesting to look at more fundamental approaches to the pion-nucleus potential in light of the systematics of the empirical potentials discussed above.

Two of the commonly used more fundamental potentials which describe the pion-nucleus interaction are the Kisslinger¹³ potential and the so-called Laplacian²² potential. Both contain explicitly terms which originate in the p-wave pion-nucleon interaction. The Kisslinger potential is written as

$$U_K(r) = (\hbar^2 c^2 / 2E) [q(r) + \vec{\nabla} \cdot \alpha(r) \vec{\nabla}] , \quad (9)$$

where q results from the s-wave part and α results from the p-wave part of the pion-nucleon interaction, q and α expressed in terms of the nuclear densities of the target nucleus.^{12,15} In the Laplacian model the potential is local

$$U_L(r) = (\hbar^2 c^2 / 2E) [q(r) - k^2 \alpha - \frac{1}{2} \nabla^2 \alpha] \quad (10)$$

with q and α the same as for the Kisslinger potential. When both potentials are employed in fits to experimental results it is usually found that any single case can be described fairly successfully by either one of these potentials, and only when many cases are handled together is it sometimes found⁵ that the parameters of the potentials vary rather

erratically over the periodic table. Figure 7 shows the best fit Laplacian potential and the equivalent local Kisslinger potential (see below) obtained for 14 MeV π^+ scattered by ^{12}C . The fits obtained are as good as with the FB method and indeed the real potentials are qualitatively similar to the FB potential shown in Fig. 3. (Note, however, the regions of negative absorption.) It is, therefore, difficult to distinguish between the two potentials on the basis of a single fit. In order to further assess these two more fundamental potentials we turn now to the six points (a)-(f) which summarize the systematics of the empirical FB potentials.

The Laplacian potential being a local one may be directly compared with the FB potential, and it can be easily seen that it indeed possesses the characteristics (a)-(e) listed above. The change of sign of the real potential is brought about by the $-\nabla^2 \alpha$ term whereas the strong energy dependence of the positive part results from the $-k^2 \alpha$ term, which is also the source of the energy dependence of the zero-crossing point. The almost constant depth of the negative part of the FB potential suggests that there is no strong dependence of α on energy below 80 MeV. Assuming a simple WS shape for $\alpha(r)$ and using parameters from fits to pionic atoms,¹⁵ Eq. (10) does reproduce almost quantitatively the energy dependence displayed in Fig. 3. However, Eq. (10) does not have any Coulomb effects other than those explicit in Eq. (1), and so it is incapable of reproducing the charge-dependent effects observed for ^{40}Ca (Figs. 4 and 6).

For the Kisslinger potential one uses the Krell and Ericson transformation²¹ which leads from Eq. (4) to an effective local potential for the transformed wave function $\tilde{\psi} = (1-\alpha)^{1/2} \psi$,

$$U_{\text{eff}} = (\hbar^2 c^2 / 2E) \left[q - k^2 \alpha - \frac{1}{2} \nabla^2 \alpha - \frac{(\nabla \alpha)^2}{4(1-\alpha)} \right] / (1-\alpha), \quad (11)$$

where the $-k^2 \alpha$ and the $-\frac{1}{2} \nabla^2 \alpha$ terms are similar to those in (10). Again assuming a simple WS function for $\alpha(r)$, Eq. (11) reproduces very well the characteristics (a)-(e), but the important difference between the Laplacian and the Kisslinger potentials is in the charge effect (f). In the transformation from (9) to (11) the $1-\alpha$ denominator applies also to the full local term, including the Coulomb terms. As in Eq. (1), where the FB potential is inserted, the Coulomb term appears explicitly without such a denominator, the FB potential is to be identified with Eq. (11) less $[V_C - V_C^2 / 2E] (1 - 1/(1-\alpha))$. Hence, in the Kisslinger model, the FB potential corresponds to

$$U = (\hbar^2 c^2 / 2E) \left[q - \left\{ k^2 - \frac{2E}{\hbar^2 c^2} \left(V_C - \frac{V_C^2}{2E} \right) \alpha - \frac{1}{2} \nabla^2 \alpha - \frac{(\nabla \alpha)^2}{4(1-\alpha)} \right\} / (1-\alpha) \right]. \quad (12)$$

In this expression the energy-dependent $-k^2 \alpha$ term is modified by a Coulomb term which qualitatively explains the last empirical point (f). The difference between the Kisslinger-based FB potentials for π^+ and π^- on an isoscalar target can be written as

$$U^+ - U^- = \frac{4E\alpha}{\hbar^2 c^2} \left(V_C - \frac{V_C^2}{2E} \right) / (1-\alpha), \quad (13)$$

where V_C is the Coulomb potential for π^+ . The reversal of the sign of $\text{Re} U^+ - \text{Re} U^-$ at 180 MeV (Fig. 6), which is also observed at 230 MeV, could be related to the reversal of the sign of $\text{Re} \alpha$ at the (3,3) resonance. On the basis of this Coulomb effect it is suggested that the Kisslinger potential and not the Laplacian one is the more suitable for describing the pion-nucleus interaction. It is interesting to note that it has been recently shown¹⁵ that from fits to pionic atom data the Kisslinger-based

potential is clearly preferred in comparison to the Laplacian-based one when fits are attempted over wide regions of the periodic table.

Comparing the sharp structure of the potentials in Fig. 7 with the more moderate slopes of the experimentally determined FB potentials in Figs. 3-6, it is suggested that probably a finite range version of the Kisslinger potential is to be preferred for a more fundamental approach. The absence in the empirical potentials of regions with negative absorption is a reflection of the fact that the present data is insensitive to this feature of the potential.

V. DISCUSSION

A "model-independent" method where a local optical potential is described by a Fourier-Bessel series has been successfully applied to the analysis of elastic scattering of pions by nuclei at energies of up to 230 MeV. Very good fit to the data is always obtained and the potential is found to vary in a systematic manner both with energy and along the periodic table, and it displays several characteristic features which are easily traced to the p-wave component of the pion-nucleon interaction. This empirical local potential does not pretend to represent the true pion-nucleus optical potential which is known to be strongly non-local. However, it serves as a means of extracting as much information from the data as possible with-out the risk of introducing some assumptions into the analysis which may later be rediscovered in the results. An example for this point is the characteristic shape of the real potential below 80 MeV which is typical of a $-k^2 \alpha - \frac{1}{2} \nabla^2 \alpha$ term and which has been shown to be indeed required by the data. A more subtle point is the observed Coulomb effect on isoscalar target nuclei, which could naively be used to reject the whole procedure. Upon a closer examination of this effect, which again is clearly required by the data, one finds that it is a

signature of the Kisslinger potential as opposed to the Laplacian one, thus strongly favoring the former for the more fundamental approach to the optical potential.

The usefulness of the empirical potentials is a separate question. Clearly if the true potential is non-local then the empirical local potential cannot be used to calculate wave functions. The main purpose of the empirical potentials must remain in being an intermediate stage between experiment and the more fundamental theories which are rarely employed in large-scale confrontation with experimental results.

An unusual feature observed in the present calculations is a large sensitivity of the calculated total reaction (or non-elastic) cross section σ_R to the details of the real potential. In the more conventional case of nucleons and light ions a good optical model fit to the elastic scattering data usually predicts well-determined values for the reaction cross section, and it is not expected that much can be gained from measurements of σ_R in addition to good measurements of the angular distribution for elastic scattering. This is not the case for the elastic scattering of pions. Figure 8 shows two examples for the improved accuracy of the real potential due to the inclusion of the χ_R^2 term [Eq. (8)] in the χ^2 . For ^{12}C we use the recent results of Navon et al.²⁴

and a noticeable improvement in the accuracy of $\text{Re}U$ is observed. In the case of ^{40}Ca there is no experimental determination of σ_R so an extrapolated value for σ_R was used with an assumed accuracy of $\pm 10\%$.

The improvement in the accuracy of the otherwise poorly determined $\text{Re}U$ is quite dramatic. For local potentials the coupling between $\text{Re}U$ and σ_R in cases of potential barriers can be understood from the following expression for σ_R ,

$$\sigma_R = \frac{-2}{\hbar v} \int |\psi|^2 \text{Im}U dr, \quad (14)$$

where v is the relative velocity. When there is a real potential barrier, $|\psi|^2$ becomes quite sensitive to its details and hence σ_R depends on $\text{Re}U$ much more than in the case of negative potentials where $|\psi|^2$ is determined mostly by $\text{Im}U$. It may, therefore, be interesting to extend measurement of σ_R to low energies and to use them as constraints in optical model analysis of elastic scattering of pions by nuclei. It might also be fruitful to study small differences between isotopes once well-determined empirical optical potentials are obtained from fits to the data and a more fundamental model is applied to describe those isotopic effects.

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Figure Captions

1. Experimental and best-fit calculated angular distributions for $\pi^{\pm}$ elastically scattered by ^{12}C at 80 MeV. The data are from Ref. 11.
2. Experimental and best-fit calculated angular distributions for $\pi^{\pm}$ elastically scattered by ^{40}Ca at 65 MeV. The data are from Ref. 7.
3. Energy dependence of the best-fit optical potentials for $\pi^{\pm}$ elastically scattered by ^{12}C and ^{90}Zr . The dots with error bars represent the real potential and the continuous curves represent the negative of the imaginary potential.
4. Charge and isospin dependence of the best-fit optical potentials for 50 MeV $\pi^{\pm}$ elastically scattered by ^{16}O , ^{40}Ca , ^{48}Ca and ^{208}Pb . See also caption to Fig. 3.
5. Charge and isospin dependence of the best-fit optical potentials for 65 MeV $\pi^{\pm}$ elastically scattered by ^{12}C , ^{14}C . See also caption to Fig. 3.
6. Energy and charge dependence of the best-fit optical potentials for $\pi^{\pm}$ elastically scattered by ^{40}Ca . See also caption to Fig. 3.
7. Best-fit Laplacian potential and local-equivalent Kisslinger potential for 14 MeV $\pi^{\pm}$ elastically scattered by ^{12}C . Real part - continuous curves, negative of the imaginary part - dashed curves.
8. The effects of using reaction cross sections as constraints in optical model fits. Q_R for ^{12}C is from Ref. 24. For ^{40}Ca an extrapolated value was used with an assumed accuracy of $\pm 10\%$. See also caption to Fig. 3.

Fig. 1

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8