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ON THE BREAKDOWN OF CHIRAL SYMMETRIES IN SUPERSYMMETRIC QCD

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A B S T R A C T

We investigate the breakdown of chiral symmetries in supersymmetric QCD with N colours and M flavours, using anomaly consistency conditions and an effective Lagrangian approach. For $M < N$, we obtain completely broken chiral symmetries. In the case of $M > N$, however, one needs at least unbroken $SU(N)_V \times SU(M-N)_R \times SU(M-N)_L \times U(1) \times U(1)$ to arrive at a consistent picture.

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Guessing the properties of supersymmetric QCD with N colours and M flavours has been a difficult task. The early conjecture of broken supersymmetry^{1),2)} through chiral fermion condensates was prematurely thought to be ruled out even in the massless case after the formulation of the index theorem^{3),4)}. Early investigations⁵⁾ using explicit effective Lagrangians confirmed the absence of supersymmetry breaking at non-zero mass, but indicated a non-smooth zero mass limit. This fact was confirmed by model independent arguments⁶⁾ showing that some supersymmetry preserving order parameters had to run to infinity in the chiral limit. Interpreting all this was not easy⁵⁾⁻⁷⁾.

With the introduction of a particular class of kinetic terms in the effective Lagrangians, a possible and interesting explanation of the massless limit emerged, at least in the large N limit: supersymmetry is broken and $SU(M)_L \times SU(M)_R \times U(1)_V \times U(1)_R$ breaks to $SU(M)_V \times U(1)_V$ ⁸⁾. This result could be extended to finite N for $M < N$ ^{9),10)}. The further extension of the effective Lagrangian approach to $M \geq N$ led to the conclusion of unbroken supersymmetry and seemed to indicate that also some of the chiral symmetries remained unbroken¹⁰⁾. The case $M = N$ has some special properties both with respect to index counting¹¹⁾ in the massless limit and the properties of the naïve effective Lagrangian¹⁰⁾ - it could be a critical point of the theory.

In this paper we want to investigate the breakdown of chiral symmetries of supersymmetric QCD combining the effective Lagrangian approach with a search for solutions to 't Hooft's anomaly consistency conditions¹²⁾. In the context of supersymmetric QCD, these consistency conditions have been analyzed in general for completely unbroken chiral symmetries and particular solutions were found^{13),14)}. For special values of N and M (e.g., again $N = M$) solutions are available as well¹⁵⁾⁻¹⁷⁾. The existence of such solutions, however, are only a necessary condition for unbroken chiral symmetries, i.e., solutions for special values of M and N might not be dynamically realized. For acceptable solutions to the anomaly conditions, we therefore require an additional smoothness argument¹²⁾: "M independence in the sense of 't Hooft", which implies the same structure of the massless fermions as one varies M . This smoothness argument excludes critical points for various values of M . The argument¹²⁾ has been proven to be very restrictive in ordinary QCD (except for the trivial one with completely broken chiral symmetries) where no solutions with this property have been found. In the case of supersymmetric QCD, we were able to find two non-trivial solutions with unbroken subgroups $SU(N) \times SU(M-N)_L \times SU(M-N)_R \times U(1) \times U(1)$ and $SU(M)_L \times SU(N)_R \times SU(M-N)_R \times U(1) \times U(1)$. The latter is not completely M independent, it only holds for $M > N$ and has a phase transition at $M = N$. The former connects smoothly to $SU(N)_V \times U(1)_V$ for

$M < N$. In the following, we will present and discuss these new solutions to the anomaly consistency conditions. In a second step, we will use the effective Lagrangian approach to compare these solutions with the trivial one where all the chiral symmetries are broken for all M . Assuming the validity of the effective Lagrangian approach, we can then show that one of the non-trivial solutions is dynamically realized.

We consider a supersymmetric $SU(N)$ Yang-Mills theory with vector superfield (A_μ, λ, D) in the adjoint representation and M left-handed chiral superfields $\Phi_i = (\phi, \psi, F)$ and $\bar{\Phi}_i = (\bar{\phi}, \bar{\psi}, \bar{F})$ in the $\bar{N} + N$ representation of $SU(N)$. At the classical level, this model possesses an $SU(M)_L \times SU(M)_R \times U(1)_V \times U(1)_R \times U(1)_X$ global symmetry under which the fields transform as follows :

$$\begin{aligned}
 A_\mu & (1, 1, 0, 0, 0) \\
 \lambda & (1, 1, 0, -M, 2N) \\
 \psi & (M, 1, 1, N, 1) \\
 \varphi & (M, 1, 1, N-M, 2N+1) \\
 \bar{\psi} & (1, \bar{M}, -1, N, 1) \\
 \bar{\varphi} & (1, \bar{M}, -1, N-M, 2N+1)
 \end{aligned} \tag{1}$$

$U(1)_R$ and $U(1)_X$ are R symmetries. In the quantum theory, $U(1)_X$ is destroyed by anomalies, and we will not consider it in our analysis.

One obvious solution to the anomaly condition is the trivial one where the chiral symmetries are broken to $SU(M)_V \times U(1)_V$, where $SU(M)_V$ is the diagonal subgroup of $SU(M)_L \times SU(M)_R$. Such a breakdown could occur due to scalar condensates $\langle \phi_{\alpha i}^{\dagger} \bar{\phi}_i^a \rangle = \delta_{\alpha}^a (i=1, \dots, N, \alpha, a=1, \dots, M)$. The corresponding Goldstone bosons are the pseudoscalar components of the bound states $\phi_{\alpha i}^{\dagger} \bar{\phi}_i^a = T_{\alpha}^a$ in the $(M, \bar{M})$ representation of $SU(M)_L \times SU(M)_R$ and they satisfy the anomaly conditions. If supersymmetry is unbroken, there will be massless fermions as partners of these Goldstone bosons, but they satisfy the anomaly constraints trivially. This trivial solution is of course perfectly M independent and we will hereafter call it solution I.

The additional non-trivial solutions which we present now are solutions with partially broken chiral symmetries, i.e., a subset of the anomalies is satisfied by Goldstone bosons, but the anomalies of the unbroken chiral symmetries require in addition massless fermions. The first non-trivial solution (called solution II) has unbroken $SU(N)_D \times SU(M-N)_L \times SU(M-N)_R \times U(1)_A \times U(1)_B$ where $SU(N)_D$ is again a diagonal subgroup. The charges A and B are combinations of V, R and the diagonal generators

$$Q_{L,R} = \begin{pmatrix} M-N & & & & \\ & \ddots & & & \\ & & M-N & & \\ \hline & & & N & \\ & & & & 0 \end{pmatrix} \begin{pmatrix} 0 & & & & \\ & M-N & & & \\ & & -N & & \\ & & & \ddots & \\ & & & & -N \end{pmatrix} \begin{matrix} \\ \\ \\ L, R \end{matrix} \quad (2)$$

of $SU(M)_{L(R)}$ respectively :

$$\begin{aligned} A &= Q_L - Q_R + R \\ B &= Q_L + Q_R + (N-M)V \end{aligned} \quad (3)$$

The thirteen anomaly conditions of the unbroken symmetries are satisfied by the following massless fermions (in $SU(N)_V \times SU(M-N)_L \times SU(M-N)_R \times U(1)_A \times U(1)_B$ representations)

$$\begin{aligned} \chi_r^a & (\bar{N}, M-N, 1, 0, -M) \\ \bar{\chi}_\alpha^s & (N, 1, \bar{M-N}, 0, M) \\ K & (1, 1, 1, M, 0) \\ \bar{K} & (1, 1, 1, M, 0) \\ O_a^\alpha & (N^2-1, 1, 1, M, 0) \end{aligned} \quad (4)$$

where $a, \alpha=1, \dots, N$ and $r, s=1, \dots, M-N$. These states can be written as composite fermions in terms of the fundamental particles (1), bound by the strong $SU(N)$ force :

$$\chi_r^a = \psi_r^i \bar{\varphi}_i^a ; \quad \bar{\chi}_\alpha^s = \bar{\psi}_i^s \varphi_\alpha^i \quad (5)$$

$$O_{\alpha}^a = 2\psi_{\alpha}^i \bar{\varphi}_i^a + \bar{\psi}_i^a \varphi_{\alpha}^i - \text{Trace} \quad (5) \text{ cont.}$$

where $i=1, \dots, N$ denotes the index of the strong gauge group. For the two massless states K and $\bar{K}$, we have three candidates :

$$\begin{aligned} \tilde{V}^{\alpha_{N+1} \dots \alpha_{N+M}} &= 2\psi_{\alpha_1}^{i_1} \varphi_{\alpha_2}^{i_2} \dots \varphi_{\alpha_N}^{i_N} \epsilon_{i_1 \dots i_N} \epsilon^{\alpha_1 \dots \alpha_M} \\ \tilde{\bar{V}}_{a_{N+1} \dots a_{N+M}} &= \bar{\psi}_{i_1}^{a_1} \bar{\varphi}_{i_2}^{a_2} \dots \bar{\varphi}_{i_N}^{a_N} \epsilon^{i_1 \dots i_N} \epsilon_{a_1 \dots a_M} \end{aligned} \quad (6)$$

which exist only for $M > N$ and

$$\sum_{\alpha} (2\psi_{\alpha}^i \bar{\varphi}_i^{\alpha} + \bar{\psi}_i^{\alpha} \varphi_{\alpha}^i) \quad (7)$$

We do not know at this stage which two of the three we should select, but we will come back to this point later. The chiral group is partially broken which could be achieved through the condensates

$$\begin{aligned} T_{\alpha}^a &= \bar{\varphi}_i^a \varphi_{\alpha}^i \\ V_{\alpha_1 \dots \alpha_N} &= \varphi_{\alpha_1}^{i_1} \dots \varphi_{\alpha_N}^{i_N} \epsilon_{i_1 \dots i_N} \\ \bar{V}^{a_1 \dots a_N} &= \bar{\varphi}_{i_1}^{a_1} \dots \bar{\varphi}_{i_N}^{a_N} \epsilon^{i_1 \dots i_N} \end{aligned} \quad (8)$$

Observe the connection between the Goldstone bosons and the massless fermions which could be used to argue for unbroken supersymmetry.

Solution II has been found by complementarity arguments^{12),18),19)} applied to supersymmetric QCD; the above solutions exist both in the so-called broken and confining pictures; the massless fermions in the confining picture contain the condensates of the broken picture as constituents. The solution is perfectly "M independent". For $M < N$ it connects smoothly to $SU(N)_V \times U(1)$ as unbroken global symmetry group.

Solution III has also been found by complementarity arguments. It has (for $M > N$) the unbroken symmetry group $SU(M)_R \times SU(N)_L \times SU(M-N)_L \times U(1)_C \times U(1)_D$ with $C = Q_L + R$ and $D = Q_L + (N-M)_V$ where Q_L is defined in Eq.(2). The thirteen anomaly conditions are satisfied by the following composite fermions

$$\begin{aligned} \bar{\psi}_i^a \varphi_\alpha^i & \quad (\bar{M}, N, 1, N, M-N) \\ \psi_r^i \varphi_i^{*\alpha} & \quad (1, \bar{N}, M-N, 0, -M) \\ \bar{V} & \quad (1, 1, 1, M, 0) \end{aligned} \quad (9)$$

where again V can either represent the expression given in Eq.(6) or a suitable trace of $\psi\phi^*$. Observe that in Eq.(9), $a=1,\dots,M$, $\alpha=1,\dots,N$ and $r=1,\dots,M-N$ are the indices for the respective chiral groups. The breakdown of the original flavour group can be realized with the V condensate given already in Eq.(8). Solution III is actually a double solution, where the second solution is obtained by interchanging L and R . This solution is however not completely M independent, it only exists for $M \geq N$. A smooth extrapolation to $M < N$ would correspond to unbroken $SU(M) \times SU(M) \times U(1)$ but the anomaly conditions are not fulfilled there. The technical reason for this fact is the non-existence of a colour-singlet bound state V for $M < N$. For solution III to be realized, the theory is required to have a critical point at $M = N$. Since we had already seen that $M = N$ has very special properties for the breakdown of supersymmetry, we decided to include solution III in our discussions.

We thus have three acceptable classes of solutions to the anomaly consistency conditions. Solution I corresponds in a certain way to maximally broken chiral symmetry, whereas solutions II and III represent two different cases of maximally conserved chiral symmetries^{*)}, satisfying our constraints. Two general results of our analysis should be stressed at this point :

i) there are no M independent solutions with completely unbroken $SU(M) \times SU(M) \times U(1) \times U(1)$ (such solutions only exist for special cases like $M = N$).

ii) for $M < N$, the only M independent solution corresponds to the trivial one with $SU(M)_D \times U(1)_V$ symmetry if one assumes complementarity. This concludes our discussion of the anomaly consistency conditions.

^{*)} Subgroups of the chiral symmetries in solutions II and III are of course also solutions with a suitably chosen subset of massless fermions.

In a next step, we would like to use arguments based on an effective Lagrangian approach, to decide which of the three solutions might be realized dynamically. Let us first consider solution I. Here the chiral symmetry is completely broken. An effective Lagrangian for this case would thus describe the Goldstone and pseudo-Goldstone bosons of this model. They are given by

$$\begin{aligned} T_{\alpha}^a &= \varphi_{\alpha}^i \bar{\varphi}_i^a \\ S &= \lambda_j^i \lambda_i^j \end{aligned} \quad (10)$$

We will denote by S, T these scalar components as well as the corresponding chiral superfields. With the restrictions of the exact and anomalous Ward identities, the superpotential for a possible effective Lagrangian is determined practically in a unique way⁵⁾

$$W = S \log (S^{N-M} \det T) - (N-M)S \quad (11)$$

which leads to a potential

$$\begin{aligned} P &= f(S, S^*) / \log (S^{N-M} \det T) / ^2 + \\ &+ \sum_{a, \alpha} g(T_{\alpha}^a, T_a^{* \alpha}) / S (T^{-1})_{\alpha}^a / ^2 \end{aligned} \quad (12)$$

where f and g depend on the choice of the kinetic terms in the effective Lagrangian⁸⁾. Supersymmetric minima correspond to $P = 0$, thus $\log(S^{N-M} \det T) = 0$ and $S(T^{-1})_{\alpha}^a = 0$ (with the qualification that the contributions of f and g might rule out supersymmetric solutions in certain cases). For $N < M$, supersymmetric solutions require $\det T = \infty$, a bizarre situation. With suitably chosen kinetic terms, one can, however, arrive at a consistent picture with broken supersymmetry and $S, \det T \neq 0$ and finite¹⁰⁾. Independent of these subtleties, one can, however, conclude that the chiral symmetries are broken to $SU(M)_D \times U(1)_V$ for $M < N$. This confirms our result obtained by the anomaly conditions, where all acceptable solutions had completely broken chiral symmetries for $M < N$.

For $M = N$, a supersymmetric solution is obtained for $S = 0$ and $\det T = 1$ corresponding to $SU(M) \times SU(M) \times U(1)_V \times U(1)_R \rightarrow SU(M) \times U(1)_V \times U(1)_R$.

$M > N$ requires $S = 0$ and $\det T/S^{M-N} = 1$ which can only be fulfilled if $\det T = 0$, which in turn implies unbroken chiral symmetries. The effective Lagrangian approach is thus inconsistent with solution I, although it contains all the necessary ingredients [Goldstone boson candidates given in Eq.(10)]. Assuming the validity of the effective Lagrangian approach, we rule out solution I^{*)}.

With unbroken chiral symmetries for $M > N$, the effective Lagrangian is no longer complete, new light degrees of freedom have to be taken into account, for example the superfields V and $\bar{V}$ from Eq.(8) if we want to discuss solution II. According to the construction [compare with Ref. 10)], the superpotential would then read

$$W = \alpha S \log (S^{N-M} \det_M T) + (M-N)S + \\ + \beta S \log (S^{N-M} V \bar{V} \det_{M-N} T) \quad (13)$$

with $\alpha + \beta = 1^+)$. Eq.(13) is only defined for $M > N$, since V and $\bar{V}$ cannot be defined for $M < N$ in a gauge-invariant way. The scalar potential that can be derived from Eq.(13) has many degenerate supersymmetric minima. They have all the common properties that the chiral symmetries are not completely broken. Some of them have actually completely unbroken chiral symmetries, which can, however, be ruled out by the anomaly constraints.

The situation remains therefore ambiguous. Let us, however, try to determine the case of maximally broken chiral symmetries that is consistent with Eq.(13). It corresponds to $\det_N T = 1$, $V = 1$, $S, \bar{V}, \det_{M-N} T = 0$ with $S^{N-M} \det_{M-N} T \bar{V}^\beta = 1$. The corresponding unbroken symmetry is $SU(N) \times SU(M-N)_L \times SU(M-N)_R \times U(1) \times U(1)$ which coincides with solution II. The fact that $\bar{V} = 0$ actually solves the problem of choosing two states out of three candidate states which we encountered in the

*) Our assumption depends only on the form of the superpotential which is constrained uniquely by the Ward identities. It is independent of the choice for the kinetic term in $\mathcal{L}_{\text{eff}}$, except for certain pathological choices¹⁰⁾.

+) The argument of the logarithm $V \bar{V} \det_{M-N} T$ is an abbreviation of $\epsilon^{\alpha_1, \dots, \alpha_M} \epsilon_{a_1, \dots, a_M} V^{\alpha_1, \dots, \alpha_N} \bar{V}^{a_1, \dots, a_N} T^{a_{N+1}, \dots, a_M} T^{\alpha_{N+1}, \dots, \alpha_M}$ which defines $\det_{M-N} T$.

discussion of the anomaly conditions (compare Eqs.(6) and (7) and the discussion there). Solution II is thus compatible with the effective Lagrangian approach. The same is true for solution III ($V=1$, S , $\bar{V}$, $\det_M T = 0$ and $S^{N-M} \det_{M-N} T (\det_N T)^{\alpha-\beta} \bar{V}^\beta = 1$). At the moment, we do not know how to decide between solutions II and III. One way to decide would be the consideration of the transition point $N = M$, where the simple Lagrangian predicts $SU(M) \times U(1)_V$ which would be inconsistent with solution III. The more complicated Lagrangian based on Eq.(13), however, does allow solutions $SU(M)_R \times SU(M)_L \times U(1)$ for $N = M$ consistent with solution III. We do not know how to decide this question, nevertheless solution II seems to be the more conservative choice.

To conclude : we have presented two new classes of "M independent" solutions to the anomaly conditions of supersymmetric QCD. They might be realized dynamically since the trivial M independent solution is inconsistent with the standard effective Lagrangian approach.

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