U(1) FOUR-DIMENSIONAL GAUGE THEORY ON A SIMPLICIAL LATTICE

J.-M. Drouffe ^{*)}

CERN - Geneva

K.J.M. Moriarty and C.N. Mouhas

Department of Mathematics
Royal Holloway College
Englefield Green, Surrey, TW20 OEX, U.K.

A B S T R A C T

Monte Carlo simulations for pure U(1) gauge theory on a simplicial lattice are reported. The simplicial lattice formalism is briefly reviewed. Our results are compared with other U(1) calculations and are found to be similar. The Wilson loops and the related string tensions for squares and triangles are examined. It appears that at $\beta \approx 0.85$ a second-order phase transition occurs which deconfines electric charges in the weak coupling region.

^{*)} Permanent address : S.Ph.T., CEN Saclay, 91191 Gif-sur-Yvette Cedex, France.

1. INTRODUCTION

In a recent paper⁽¹⁾ gauge theories on a simplicial lattice were introduced and applied to the group $SU(2)$. In the present paper, we compare $U(1)$ simplicial lattice theory with other $U(1)$ calculations^(2,3). In table I we summarize some of the important geometrical properties of the simplicial lattice, which are described in detail in Ref. 1. Referring to Fig. 1 of Ref. 1, note that in the two-planes defined by $(\vec{\epsilon}_{ij}, \vec{\epsilon}_{jk})$ [where $\vec{\epsilon}_{ij} = \vec{\epsilon}_i - \vec{\epsilon}_j$ and $\vec{\epsilon}_1, \dots, \vec{\epsilon}_{d+1}$ are the d -dimensional simplicial lattice basis vectors obeying $\vec{\epsilon}_1 + \dots + \vec{\epsilon}_{d+1} = 0$] the lattice is triangular, whereas in the two-planes defined by $(\vec{\epsilon}_{ij}, \vec{\epsilon}_{kl})$ [$i, j, k, l = 1, \dots, d+1$ and all different] we have a square lattice. Thus, there exist different kinds of two-planes which may be used to estimate the string tension. This should provide an easier check of rotational invariance.

As can also be seen from table I, there are now more plaquettes per site with more orientations. Thus, the breaking of the Lorentz invariance is expected to be less important and the strong-coupling expansions to provide more accurate results with less terms. Similarly, a link can be seen to interact with more neighbours, so that mean-field results are more reliable and Monte Carlo simulations thermalize more quickly.

2. THE RESULTS

The program⁽⁴⁾ used was designed to measure the average action per triangle as well as the expectation values of Wilson loops for

triangles and squares. A lattice with 6 sites per dimension was used (with periodic boundary conditions) with the heat bath algorithm⁽⁵⁾ to produce thermal equilibrium. It was established that both ordered and disordered starts lead to the same result and consequently only ordered starts were used.

The calculations were performed partly on the VAX 11/780 machine at Royal Holloway College and partly on the CDC 7600 at the University of London Computer Centre (which is about 10 times faster than the VAX). The execution time of the program took about 2300 CPU seconds for 70 iterations on the CDC machine although in most cases this was much more than was needed to establish thermal equilibrium (see Fig.1).

We take the gauge action to be a sum over unoriented triangular plaquettes such that

$$\begin{aligned} S[U] &= \beta \sum_{\Delta} S_{\Delta} \\ &= \beta \sum_{\Delta} (1 - \text{Re tr } U_{\Delta}) \end{aligned}$$

where U_{Δ} are products of gauge group elements taken round the prescribed contours, each link (i,j) being associated with a $U(1)$ angle θ_{ij} , ranging from 0 to 2π with

$$U_{ij} = \exp(i\theta_{ij}),$$

and

$$U_{ij} = (U_{ji})^{-1}.$$

We define the partition function by

$$Z(\beta) = \int \left(\prod_{(i,j)} dU_{ij} \right) \exp(-S[U]) ,$$

where the $U(1)$ normalized invariant Haar measure is given by

$$\int dU_{ij} = \int_0^{2\pi} \frac{d\theta_{ij}}{2\pi} .$$

The coupling β is related to the bare coupling constant by

$$\beta = \frac{4}{\sqrt{5}} \frac{1}{g_0^2} .$$

Fig. 2 shows the average action per triangle plotted against β . In the region $\beta < 0.70$ and $\beta > 1.0$ it was found that twenty or so iterations were sufficient for thermalization as the lattice settled down after the first four or five iterations (see Fig. 1). In calculating the average action only results every second iteration were considered for our averages in order to eliminate correlations. However, near the critical region $0.70 \leq \beta \leq 1.0$ it was found that about 100 iterations were required. In this region, we carried out 150 iterations through our lattice and averaged over the last 50 iterations.

Using standard techniques^(1,6) the average action per triangle is calculated to be

$$\langle E_{\Delta} \rangle = 1 - \frac{1}{2}\beta + \frac{1}{16}\beta^3 + O(\beta^5) , \quad (1)$$

and

$$\langle E_{\Delta} \rangle = \frac{1}{4\beta} + O(\beta^{-2}) , \quad (2)$$

for strong- and weak-coupling, respectively.

These results are in good agreement with the Monte Carlo results and are also displayed in Fig. 2. The phase transition appears to be second-order as in the case of the hypercubical lattice.^(2,3)

The geometry of the Wilson loops we considered is shown in Fig. 3 of Ref. 1 together with their numbering scheme. The traingular loops (types 1,2,3,4,6,9,13) are expected to obey a leading-order strong-coupling behaviour⁽⁷⁾

$$\left(\frac{1}{2} \beta \right)^{\text{AREA}} .$$

This is confirmed in Fig.3 . In order to extract the area-law behaviour we form the logarithmic ratios

$$\begin{aligned} \chi_1^\Delta &= \frac{2}{\sqrt{3}} \ln \left| \frac{W_4^\Delta}{W_6^\Delta} \right| , \\ \chi_2^\Delta &= \frac{2}{\sqrt{3}} \ln \left| \frac{W_1^\Delta W_6^\Delta}{W_2^\Delta W_3^\Delta} \right| , \end{aligned} \tag{3}$$

and

$$\begin{aligned} \chi_3^\Delta &= \frac{1}{\sqrt{3}} \ln \left| \frac{W_9^\Delta}{W_{13}^\Delta} \right| , \\ \chi_4^\Delta &= \frac{2}{\sqrt{3}} \ln \left| \frac{W_4^\Delta W_4^\Delta}{W_1^\Delta W_9^\Delta} \right| . \end{aligned}$$

Then, assuming a relationship of the form

$$\ln W(I,J) \sim (\text{CONSTANT} - \chi[\text{AREA}] - \lambda[\text{PERIMETER}]),$$

the constant and the perimeter terms in the above combinations are seen to cancel. The Monte Carlo results for these ratios are shown in Fig.4 . Also shown in Fig.4 is the leading-order strong-coupling behaviour of the string tension

$$\chi_K^\Delta = - \frac{4}{\sqrt{3}} \{ \ln \beta + 2\beta^2 + \frac{31}{16} \beta^4 + O(\beta^6) \} , \tag{4}$$

In Fig. 5 we show the Wilson loops for squares following the numbering scheme of Fig. 3 in Ref.1. The leading-order strong-coupling expansion for $\langle E_{\square} \rangle$ is given by

$$\langle E_{\square} \rangle = 1 - 2 \left(\frac{\beta}{4} \right)^4 + O(\beta^6) . \quad (5)$$

This behaviour is shown in Fig. 5 .

The area-law behaviour of the square Wilson loops is shown in Fig. 6 . It is extracted from the ratios

$$\begin{aligned} \chi_1^{\square} &= \ln \left| \frac{W_3^{\square}}{W_4^{\square}} \right| , \\ \chi_2^{\square} &= \ln \left| \frac{W_2^{\square} W_6^{\square}}{W_3^{\square} W_4^{\square}} \right| , \end{aligned} \quad (6)$$

where again the constant and the perimeter terms cancel. The leading-order behaviour of the square Wilson loops is given by

$$K^{\square} = -4 \ln \beta , \quad (7)$$

in the strong-coupling region. These results are shown in Fig. 6. As Figs. 4 and 6 are very similar this shows that we get similar results for both triangles and squares. Hence rotational invariance is established.

3. CONCLUSIONS

In this paper we have attempted a brief comparison of $U(1)$ simplicial lattice theory against other $U(1)$ theories based on hypercubical lattices. The results prove to be quite similar. The simplicial lattice has the advantage in computer time needed for Monte Carlo calculations.

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	HYPERCUBICAL	SIMPLICIAL
Volume per site	a^d	$a^d \sqrt{d+1}$
Degrees of freedom per site	d	$\frac{1}{2}d(d+1)$
Triangular plaquettes per site	—	$\frac{1}{3}d(d^2-1)$
Square plaquettes per site	$\frac{1}{2}d(d-1)$	$\frac{1}{8}d(d^2-1)(d^2-2)$

Table I. Comparison of hypercubical and simplicial lattice geometries in d -dimensions.

FIGURE CAPTIONS

Fig. 1. The evolution of the average action per triangle $\langle E_{\Delta} \rangle$ for pure $U(1)$ gauge theory on a simplicial lattice with six sites in each direction as a function of the number of iterations through the lattice for ordered starting lattices for various values of the inverse coupling constant β .

Fig. 2. The average action per triangle $\langle E_{\Delta} \rangle$ for pure $U(1)$ gauge theory as a function of the inverse coupling constant squared β . The curves represent the leading-order strong- and weak-coupling expansions of Eqns. 1 and 2, respectively.

Fig. 3 . The Wilson loops W_i^A ($i = 1, 2, 3, 4, 6, 9$ and 13) for pure $U(1)$ gauge theory as a function of the inverse coupling constant squared β . The leading-order strong-coupling behaviour of Eqn. 1 is also shown.

Fig.4 . The string tensions χ_i^Δ ($i = 1,2,3$) of Eqn. 3 for pure U(1) gauge theory as a function of the inverse coupling constant β . Also shown is the leading-order strong-coupling expansion of Eqn. 4.

Fig.5 . The Wilson loops $W_i^\square$ ($i = 1,2,3,4,6$ and 9) for pure U(1) gauge theory as a function of the inverse coupling constant squared β . The leading-order strong-coupling expansion of Eqn. 5 is also shown.

Fig.6 . The string tensions $\chi_i^\square$ ($i = 1,2$) of Eqn. 6 for pure U(1) gauge theory as a function of the inverse coupling constant β . Also shown is the leading-order strong-coupling expansion of Eqn. 7.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6