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FOUR-DIMENSIONAL SUPERSYMMETRIES FROM ELEVEN-DIMENSIONAL SUPERGRAVITY *)

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A B S T R A C T

For the spontaneous compactification of eleven-dimensional supergravity, it is found that the local obstruction due to the Weyl tensor of the small seven-dimensional manifold can lead only to $N = 0, 1, 2, 4, 8$ supersymmetries in four dimensions.

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As eleven-dimensional supergravity ¹⁾ spontaneously compactifies to four dimensions ²⁾, the number N of surviving supersymmetries is determined ³⁾ by the restricted "Weyl-holonomy" group H of the small seven-dimensional manifold M_7 . For a solution of the type we proposed earlier ²⁾ one finds $N = 8$, $H = 1$ (the trivial one-element group) when M_7 is the round seven-sphere or the seven-torus ³⁾, $N = 4$, $H = SU(2)$ when $M_7 = K3 \times T_1$ ⁴⁾ and $N = 1$, $H = G_2$ when M_7 is the squashed seven-sphere ³⁾. For Englert's solution ⁵⁾ with $M_7 = S^7$ one finds $N = 0$ ⁶⁾. It is natural to ask what other values of N can be obtained as M_7 ranges over all seven-dimensional compact manifolds. We find the possible values of N to be $N = 0$ and the powers of two : $N = 1, 2, 4, 8$, whereas $N = 3, 5, 6, 7$ never occur. We emphasize that we consider here only the "local obstructions implied by the Weyl tensor" and disregard any "global obstructions" such as those mentioned in Ref. 4).

The proof of our statement is very simple. N equals the number of "covariantly" constant spinors on M_7 , such that the corresponding integrability condition is ³⁾

$$C_{\mu\nu ab} [\gamma^a, \gamma^b]_{\alpha\beta} \psi_\beta = 0$$

with $C_{\mu\nu ab}$ the Weyl tensor ($\mu, \nu = 1, \dots, 7$ are world indices ; $a, b = 1, \dots, 7$ are tangent space indices), γ^a, γ^b the 8×8 ($\alpha, \beta = 1, \dots, 8$) Dirac matrices in seven dimensions. The 8×8 matrices $C_{\mu\nu ab} [\gamma^a, \gamma^b]$ generate under commutation the Lie algebra $\mathfrak{h}$ of the restricted Weyl-holonomy group H . $\mathfrak{h}$ is necessarily a subalgebra of $\mathfrak{so}(7)$ [the Lie algebra of the Riemannian structure group $\text{Spin}(7)$]. ψ_β provides an eight-dimensional irreducible spinor representation space $\underline{8}$ of $\text{Spin}(7)$ and hence of $\mathfrak{so}(7)$. The elements of $\mathfrak{h}$ will automatically yield zero iff they act on a singlet of $\mathfrak{h}$. Thus N equals the number of independent singlets obtained from branching the $\underline{8}$ representation of $\mathfrak{so}(7)$ into representations of $\mathfrak{h}$. We therefore go one by one through all possible subalgebras of $\mathfrak{so}(7)$ and indicate in each case the branching of the $\underline{8}$ representation of $\mathfrak{so}(7)$ [for the smaller subalgebras more than one branching is possible depending on the details of their embedding in $\mathfrak{so}(7)$]. The results of this procedure are presented in the Table.

We notice from the last column that only the values announced at the beginning (viz. $N = 0, 1, 2, 4, 8$) appear. This is not to say that there exist seven-manifolds corresponding to all these possibilities ⁷⁾ of $\mathfrak{h}$, rather that there are none with $N = 3, 5, 6, 7$. We have disregarded above Abelian $u(1)$ pieces. They can arise from two sources : i) fitting a subalgebra [e.g., $\mathfrak{su}(4) \rightarrow \mathfrak{su}(3) \oplus u(1)$, $\mathfrak{su}(3) \rightarrow \mathfrak{su}(2) \oplus u(1)$], or ii) the $u(1)$ subalgebras of $\mathfrak{su}(2)$ (the I_3 part).

It is readily checked that neither source gives new values for N (whether or not such manifolds exist). This can also be seen directly by considering the $u(1)$ subalgebras of $so(7)$ in the Dirac 8 representation.

We draw attention to the entries in the Table at $(g_2, N = 1)$, $[su(2), N = 4]$ and (trivial Lie algebra, $N = 8$). They are realized for the squashed seven-sphere, $K3 \times T^3$, and for the round seven-sphere or for the seven-torus as mentioned at the beginning [see Refs. 3) and 4)].

In conclusion then, the spontaneous compactification of eleven-dimensional supergravity is selective not only as to the number of surviving space-time dimensions ²⁾, but also as to the number of surviving supersymmetries.

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h	h multiplets into which $\underline{8}$ of $SO(7)$ branches	N = number of h singlets
$so(7)$	8	0
g_2	$7 + 1$	1
$su(4)$	$4 + \bar{4}$	0
$su(2) \oplus su(2) \oplus su(2)$	$(1,2,2) + (2,2,1)$	0
$su(3)$	$3 + \bar{3} + 1 + 1$	2
$sp(4)$	$4 + 4$	0
$su(2) + su(2)$	$(2,2) + (1,3) + (1,1)$	1
	$(2,2) + (1,2) + (1,2)$	0
	$(1,2) + (1,2) + (2,1) + (2,1)$	0
	$(2,2) + (2,2)$	0
$su(2)$	$2 + 2 + 1 + 1 + 1 + 1$	4
	$3 + 3 + 1 + 1$	2
	$3 + 2 + 2 + 1$	1
	$7 + 1$	1
	$4 + 4$	0
	$2 + 2 + 2 + 2$	0
0 (trivial Lie algebra)	$1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$	8

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