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QUARKS AND THE DEUTERON ASYMPTOTIC D STATE

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ABSTRACT

Form factor effects associated with the nucleon size reduce significantly calculated values of the deuteron asymptotic D to S state ratio. We show that tensor forces due to pions and gluons exchanged between quarks compensate for such reductions.

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1. - INTRODUCTION

The problem of how quark degrees of freedom influence nuclear properties is presently under very active investigation. Much attention has been given to the problem of constructing nuclear forces from quark interaction and Pauli principle effects¹⁾. However, mesonic exchange effects^{2),3)} are still believed to provide the intermediate range attraction although there are other suggestions⁴⁾.

One long-standing difficulty is the reconciliation of the form factors used in boson exchange calculations with ideas corresponding to the size of the nucleon. For example, the form factor used in the recent computations of the Bonn group^{5),6)} corresponds to a nucleon radius of about 0.4 fm, and the Paris group²⁾ treats the nucleon as point-like for separations greater than 0.8 fm. On the other hand, there are several successful phenomenologies of baryon structure⁶⁾⁻¹¹⁾ in which the nucleon is treated as a bag with radius (R) about 1 fm. If form factors derived from such sizes are employed in nucleon-nucleon potentials the previously obtained^{2),3)} description of phase shifts would be lost⁵⁾.

A specific case is the computation of the asymptotic D to S state ratio (η) of the deuteron. Ericson and Rosa-Clot¹²⁾ showed that the effects of the one pion exchange (OPEP) could very accurately account for the experimental value [e.g., Ref. 13)] of η ($= 0.0271 \pm 0.0004$) [see also Appendix 4, Ref. 12)]. However, the calculations were made using point-like nucleons. When de Kam¹⁴⁾ and Ericson and Rosa-Clot modified the OPEP by including form factors to represent the finite nucleon size, significant reductions in the calculated value of η were obtained. Indeed, the use of nucleon bags larger than about $R = 0.6$ fm would destroy the agreement with observation even if experimental errors much larger than the quoted ones are used. Similar problems result in computations of the quadrupole moment¹²⁾.

However, the conventional manner of including finite nucleon size effects through modifications of the vertex function (form factor) alone is not complete. If the quark structure is mostly responsible for the extent of a nucleon large enough to cause reductions in the computed value of η , then explicit interactions between quarks and the antisymmetrization of the quark wave function must be included in addition to the conventional terms. For example, consider OPEP. In chiral bag models^{7),8),10),11),15)} the pion-nucleon interaction is assumed to arise from pion-quark interactions. (Many aspects of pion-nucleon scattering and interactions including chiral symmetry and current algebra can be reproduced in this manner.) One can therefore compute OPEP by allowing a quark in one of the nucleons to exchange a pion with any quark in other nucleon. When two nucleons

do not overlap (separation distance between centres is greater than $2R$) the result is the familiar point-like Yukawa interaction^{7),15)}. This is analogous to electrodynamics. When two finite sources of charge do not overlap, the Coulomb interaction between them is the same as for two point sources (in the appropriate locations) and form factor effects play no role. When two bags overlap, there are two kinds of terms (Fig. 1). The direct term, Fig. 1a, which includes wave functions for confined quarks leads to OPEP with a form factor modification. (The form factor is the Fourier transform of the relevant quark density.) This term is included in Refs. 12) and 14). However, in an overlapping system one must use six-quark wave functions that are antisymmetrized. Thus, there is an additional term, the exchange term, Fig. 1b. This term is analogous to the Fock term of atomic and nuclear physics. It is of the same general size as the term of Fig. 1a, but it is not included in the calculations of Refs. 12) and 14). Another term that is not included in conventional calculations, but allowed for overlapping nucleons, is the exchange of a single gluon.

Thus, the relevant question is: do quark interaction and antisymmetrization effects compensate for the loss in η induced by using form factors ?

2. - THE MODEL

In order to assess the importance of tensor interactions between quarks in situations in which nucleons overlap, it is necessary to use a model to describe the quark wave functions and their interactions. The hybrid model of Kisslinger¹⁶⁾ and co-workers¹⁷⁾ is employed here. The basic idea is to divide the co-ordinate space of the two-nucleon wave function into two regions defined by a matching radius r_0 . For separations $r > r_0$, one uses a typical nucleon-nucleon NN wave function derived from, for example, the Paris²⁾ or Reid soft-core potential¹⁹⁾. The nucleons are treated as point-like for $r \geq r_0$. However, the NN wave function is set to zero for $r < r_0$ and is not used for small separation distances. Instead one discusses the NN system as six antisymmetrized quarks confined in some spherical region. The quarks are coupled to the quantum numbers of the given state. A useful feature of the work of Ref. 17) is that the normalizations of the six-quark wave functions are determined from knowledge of the NN wave functions for $r \geq r_0$.

The value of r_0 is related to the nucleon size. For $r_0 = 0$ fm the nucleons are point-like whereas large values of r_0 correspond to large nucleons.

To discuss the computation of observables, it is necessary to begin with the Ericson-Rosa-Clot formula for the value of η as computed in conventional potential models. (If the S and D state radial wave functions are proportional

(as $r \rightarrow \infty$) to $A_S e^{-\alpha r}$ and $A_D e^{-\alpha r}$ then $\eta \equiv A_D/A_S$.) The potential model value (defined as η^P) is given by¹²⁾

$$\eta^P = \frac{M}{N\sqrt{2\alpha}} \int_0^\infty r \mathcal{J}_2(r) V_{20}(r) u(r) dr \quad (1)$$

where $u(r)$ is the S state wave function. $V_{20}(r)$ is the tensor force that connects the S and D states. The function $\mathcal{J}_2(r)$ is the $L = 2$ solution of the homogeneous Schrödinger equation that is regular at the origin. M is the nuclear mass, $\alpha^2 = MB$ where B is the deuteron binding energy and, N is the well-determined¹²⁾ asymptotic S state normalization. Within the framework of potential models Eq. (1) is an exact expression.

In our model which includes both quark and nucleonic contributions, we have

$$\eta = \eta^N + \eta^Q \quad (2)$$

in which the superscript denotes the nucleonic (N) or quark (Q) contributions. Since we only use the nucleon wave function for $r \geq r_0$ we have

$$\eta^N = \frac{M}{N\sqrt{2\alpha}} \int_{r_0}^\infty r \mathcal{J}_2(r) V_{20}(r) u(r) dr \quad (3)$$

so that, in our calculation, cutting off the integral for $r < r_0$ plays the role of a form factor. Since we are concerned with the changes brought about by using this "form factor", it is convenient to define a quantity $\Delta\eta$, with

$$\Delta\eta = \frac{M}{N\sqrt{2\alpha}} \int_0^{r_0} r \mathcal{J}_2(r) V_{20}(r) u(r) dr \quad (4)$$

$\Delta\eta$ represents the contribution lost due to our cut-off procedure. This is to be eventually compensated by quark effects. In computations of (4) we take $u(r)$ from the Paris potential, $V_{20}(r)$ and $\mathcal{J}_2(r)$ are computed with point OPEP as in Ref. 12).

In our model, the short distance parts of the $S(u)$ and $D(\mathcal{J}_2)$ wave functions are treated as bags of six quarks. Tensor interactions between the quarks cause the transition between S and D states. The resulting expression for the quark contributions is

$$\eta^Q = \frac{M}{N\sqrt{2\alpha}} \sqrt{P_{S_q}} \sqrt{\tilde{P}_D} \langle D_{6q} | V_{20}^\pi + V_{20}^G | S_{6q} \rangle \quad (5)$$

Here $|S_{6q}\rangle$ is the antisymmetrized state formed by placing six quarks in the lowest harmonic oscillator S state. The non-relativistic quark model¹⁹⁾ is employed. The six quarks are coupled to the correct spin (1) and isospin (0) of the deuteron. The harmonic oscillator parameter b (the wave function of a single quark is proportional to $e^{-\frac{1}{2}r^2/b^2}$) is given by $b = r_0/\sqrt{3}$ which is obtained from the requirement that the average distance between quarks in the state $|S_{6q}\rangle$ be r_0 . (The distances between quarks should not be too large since we wish to use a single confinement region when all the quarks are close together.) The state $|D_{6q}\rangle$ is generated by the action of a Galilean invariant operator on the state $|S_{6q}\rangle$:

$$|D_{6q}\rangle = \frac{1}{N_D} \sum_{i < j} r_{ij}^2 Y^2(\hat{r}_{ij}) |S_{6q}\rangle \quad (6)$$

in which magnetic quantum numbers and a C-G coefficient are eliminated for simplicity of presentation. The factor N_D insures that the state $|D_{6q}\rangle$ is normalized to unity as is the state $|S_{6q}\rangle$. The use of Eq. (6) to generate $|D_{6q}\rangle$ insures that no spurious excitation of the centre-of-mass wave function is included. Our use of $|S_{6q}\rangle$ and $|D_{6q}\rangle$ corresponds to reasonable choices, which should allow us to make realistic but rough estimates of quark effects. Of course, further developments in the physics of confinement could cause our results to be altered. The terms P_{S_q} and $\tilde{P}_D$ are the "probabilities" for the six quark state to exist in the deuteron. These are determined by conservation of the probability current^{17),19)}. For example, in the Paris potential the total probability to have an S state is 0.94. This is unchanged if the total probability is divided into two parts: a nucleonic one, P_N , and a six quark P_{6q} term so that

$$0.94 = P_N + P_{6q} \quad (7)$$

In our hybrid treatment $u(r)$ is unmodified for $r \geq r_0$ so that

$$P_N = \int_{r_0}^{\infty} u^2(r) dr \quad (8)$$

which via Eq. (1) implies

$$P_{6q} = 0.94 - P_N = \int_0^{r_0} u^2(r) dr \quad (9)$$

In a similar fashion $\tilde{P}_D$ is given by

$$\tilde{P}_D = \int_0^{r_0} r^2 \tilde{\gamma}_2^2(r) dr \quad (10)$$

where $\tilde{\gamma}_2(r)$ must be used since it is that wave function that appears in Eq. (1).

It is worthwhile to comment on Eq. (10). Whereas $u(r)$ is a normalizable bound state wave function so that probability is a well-defined concept, $\tilde{\gamma}_2(r)$ is not normalizable¹²⁾ even though it is a regular solution (at negative energy) of the Schrödinger equation. However, solutions of Schrödinger's equation satisfy the constraints of conservation of probability current. Thus^{17),19)}, it is only the knowledge of the probability current into a spherical region bounded by $r = r_0$ which is required to determine the "probability", $\tilde{P}_D$, contained within $r \leq r_0$. Furthermore, any interactions that lead to the same values of $\tilde{\gamma}_2(r)$ and $\tilde{\gamma}_2'(r)$ at $r = r_0$ must have the same value of $\tilde{P}_D$ [Eq. (10)]. This is the same as the treatment of scattering wave functions which has been discussed in detail in Refs. 17) and 19). One form of the result is that $\tilde{P}_D$ is simply the integral of $\tilde{\gamma}_2^2(r)$ over the inner region ($r \leq r_0$). This is analogous to Eq. (9), but now $\tilde{P}_D$ has units of volume, as shown in Eq. (10).

To compute the value of Eq. (10), $\tilde{\gamma}_2(r)$ is taken from the Reid soft-core potential since short range attractive effects of the spin orbit potential are relevant. The results for $\tilde{P}_D$ would be the same if the Paris potential were employed.

The S and D six quark states are connected by one pion and one gluon forces:

$$V_{20}^{\pi} = \sum_{i < j} \frac{g^2}{4\pi} m_{\pi} (\vec{\sigma}_i \cdot \hat{r}_{ij} \vec{\sigma}_j \cdot \hat{r}_{ij} - \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{3}) \chi_2(m_{\pi} r_{ij}) \vec{\tau}_i \cdot \vec{\tau}_j \quad (11)$$

$$V_{20}^G = \sum_{i < j} -\frac{3}{4} \alpha_s m (\vec{\sigma}_i \cdot \hat{r}_{ij} \vec{\sigma}_j \cdot \hat{r}_{ij} - \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{3}) \frac{1}{(m r)^3} \vec{\lambda}_i \cdot \vec{\lambda}_j \quad (12)$$

in which the sum on i, j is over quarks. The pion-quark coupling constant is determined so that the pion-nucleon coupling constant is reproduced: $f^2/4\pi (5/3)^2 = 0.08$. The function $Y_2(x)$ is the usual one: $Y_2(x) = e^{-x}/x(1+3/x+3/x^2)$. Standard values of the light quark mass²⁰⁾, $m = 336$ MeV and, $\alpha_s (= 0.9)$ ²¹⁾ are used. Should better values of α_s and m be obtained later, our results can be rescaled. The operators $\vec{\tau}_i \cdot \vec{\tau}_j$ ($\vec{\lambda}_i \cdot \vec{\lambda}_j$) are the usual isospin (colour spin) ones.

The computation of the matrix elements appearing in Eq. (5) is straightforward but lengthy. Since standard techniques²²⁾ are used, we simply state the results

$$\langle D_{6q} | V_{20}^{\pi} | S_{6q} \rangle = 3.96 I_{\pi}^{\Delta d} \quad (13)$$

$$\langle D_{6q} | V_{20}^G | S_{6q} \rangle = -0.815 I_G^{\Delta d} \quad (14)$$

where

$$I_{\pi}^{\Delta d} = \frac{f^2}{4\pi} m_{\pi} \int_0^{\infty} r^2 Y_2(m_{\pi} r \sqrt{2}) R_{\ell}^{1\Delta}(r) R_{\ell}^{1d}(r) dr \quad (15)$$

and

$$I_G^{\Delta d} = -\frac{3}{4} \alpha_s m \int_0^{\infty} r^2 \frac{1}{(m r \sqrt{2})^3} R_{\ell}^{1\Delta}(r) R_{\ell}^{1d}(r) dr \quad (16)$$

in which $R_{\ell}^{1s}(r)$ and $R_{\ell}^{1d}(r)$ are harmonic oscillator wave functions of lowest energy with $\ell = 0$ and 2 .

3. - NUMERICAL RESULTS

The important question is whether or not the loss, $\Delta\eta$, due to "form factor" effects can be compensated for by η^Q . This question can be settled independently of the starting value η^P and independently of the current experimental value of η . One can simply compare $\Delta\eta$ with η^Q for each value of r_0 . The results, shown in Fig. 2, are expressed as in terms of percentage of the value $0.0271 \equiv \eta^{\text{exp}}$. First consider the larger graph. The solid curve

shows $\Delta\eta/\eta^{\text{exp}}$ as a function of r_0 . For example, with $r_0 = 1.2$ fm about 10% of the calculated values of η_p is lost. It is this loss which is to be compensated by quark interaction effects. Indeed, as the long-dashed curve $(\eta^Q/\eta^{\text{exp}})$ shows, much of the loss due to the r_0 cut-off is regained from π and gluon tensor interactions between quarks.

The short-dashed curve gives η^Q/η^{exp} neglecting the effects of gluons. Except for large values of r_0 , for which the π mass becomes significant in reducing the value of I_{π}^{sd} , the gluon exchange term is small. This is in contrast with the Δ -N splitting (caused by the spin-spin interaction) in which the gluonic effect is two to three times larger than the pionic effect^{11),23)}. For the tensor force to be effective here the space and spin wave functions of two quarks must be symmetric. Hence the colour isospin wave function must be antisymmetric. For six quarks and three colours, there are more pairs with a symmetric colour wave function, hence antisymmetric isospin. This favours the $\vec{\tau}_i \cdot \vec{\tau}_j$ interaction over the $\vec{\lambda}_i \cdot \vec{\lambda}_j$ term.

The entire effect of using a model with nucleon and explicit quark degrees of freedom can be examined by considering the difference $(\eta^N - \eta^Q)/\eta^{\text{exp}}$, which is the contribution that is lost. This is shown in the figure insert in Fig. 2. One can have $r_0 > 1.5$ fm and still have a loss of η less than 10 per cent. Thus, one obtains weaker limits on r_0 than are implied in Refs. 12) and 14). To obtain a largest acceptable value of r_0 , it is useful to define a maximum allowed value of the loss in η . We take this to be four per cent. There are three reasons focusing this instead of the quoted experimental error. 1) The Paris potential has a four per cent error with respect to η^{exp} in its value of η_p . 2) Data from as late as 1979 and 1982 [see Ref. 12)] have an error of about four or five per cent. 3) The theoretical value for the OPEP contribution is 0.02762¹²⁾. If we start with a value of 0.02762 a four per cent reduction would leave the number comparable with the current experiments. Thus, we take 4% as an allowable error. This is shown as a horizontal line in the inset. The intersection of this line with the solid curve defines an upper limit on r_0 with

$$r_0 \leq 1.25 \text{ fm} \quad (17)$$

Thus for values of r_0 less than 1.25 fm the co-ordinate space cut-off has no significant consequences for η . The consequences for the deuteron quadrupole moment are discussed below.

4. - LIMIT ON NUCLEON BAG SIZE

It is tempting to convert Eq. (17) into a limit on the radius of a nucleon bag, R . This is difficult since we employ harmonic oscillator wave functions and since we deal with a six quark system. We nevertheless attempt an estimate of an upper limit on R . First we convert the limit on r_0 to one for a six quark bag of radius R_6 . This is done by requiring that the root mean square radius of the harmonic oscillator wave function be the same as that of a quark wave function confined in a MIT bag⁶⁾ of radius R_6 . This gives $b^2 = R_6^2/3$ which when combined with $b^2 = r_0^2/3$ gives

$$r_0 = R_6 \quad (18)$$

Finally, one must relate R_6 to R , this can be done using the non-linear boundary condition (NLBC) which says $R_6 \approx (2)^{1/4}R$. We then have using Eq. (17)

$$R \leq 1.05 f_m \quad (19)$$

Thus, even fairly large bags are not ruled out. However, caution must be exercised regarding the use (19), since it is not at all clear that the NLBC can be used here.

5. - THE QUADRUPOLE MOMENT

It is interesting to see if the hybrid model has any consequences for the quadrupole moment. The importance of using the same procedure for computing η and the deuteron quadrupole moment Q has been stressed in Ref. 12). However, no major constraint on our theory is introduced by considering the quadrupole moment. To see this, recall that in potential models the dominant term of Q is due to the overlap between r^2 and the product of S and D state (w) wave functions. Since $w(r) \sim r^3$ for small r , very small changes in Q are generated by cutting the integral off at a lower limit of r_0 . For example, for $r_0 = 1.5$ fm there is only about a four per cent loss in the computation of Q .

Furthermore, for that (very large) value of r_0 the quarks in the states S_{6q} and D_{6q} give a corresponding contribution of about 3.2% so that the computed error in Q is less than one per cent. Even smaller changes in Q are obtained for smaller values of r . For example, with $r = 1.2$ fm, the error is less than half a per cent. The uncertainty in Q (mainly due to exchange current corrections) is about one or two per cent. Thus, considering the quadrupole moment does not change the upper limits of our model reported

in Eqs. (17) and (19). These results that relative changes of Q are smaller than those of η depend on our assertion that the six quark wave functions corresponding to $w(r)$ and $\tilde{g}_2(r)$ are both embodied in $|D_{6q}\rangle$.

6. - SUMMARY

The use of large bags is not ruled out in computations of η . It is necessary, however, to include the explicit effects of tensor interactions between quarks, and to use antisymmetrized quark wave functions. If this is done, results very similar to those obtained from point-like OPEP are obtained. This is not surprising because our calculations rely heavily on probability current conservation and, the idea that physical effects are continuous whether described in terms of explicit quarks or in terms of two baryons. For these reasons, we believe that the qualitative statements at the beginning of this paragraph will hold even after more detailed and accurate treatments of quark effects become available.

For nucleonic bag radii of about 1 fm the quark contribution to η is about six per cent. This is rather large, considering that we are examining an asymptotic property of the most loosely bound nucleus. This result may further indicate that quark effects could be very significant in more tightly bound nuclei.

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FIGURE CAPTIONS

- Fig. 1 : One pion exchange between quarks.
- a) The direct term.
 - b) the exchange term caused by antisymmetrization of the quark wave function. This term is absent in typical OPEP calculations using form factors.
- Fig. 2 : Changes in contributions to η/η^{exp} . $\Delta\eta$ is the amount lost due to the co-ordinate space cut-off. η^Q is the amount gained by pion and gluon exchanges between quarks (long-dashed curve). If gluon exchange is omitted, η^Q/η^{exp} given by the short-dashed curve. The percentage loss in computations of η , $(\Delta\eta - \eta^Q)/\eta^{\text{exp}}$, is shown in the upper curve inset in the figure.

a)

b)

Fig. 1

Fig. 2