FURTHER EVIDENCE FOR REFLECTION ASYMMETRY IN THE Ra REGION
THE DECOUPLING PARAMETER AND THE MAGNETIC MOMENT

I. Ragnarsson *)
CERN -- Geneva

A B S T R A C T

Decoupling parameters and magnetic moments are calculated for octupole deformed shapes in the strong coupling approximation. Comparisons to experimental values give strong evidence for stable octupole deformation in ^{227}Ra and especially ^{225}Ra .

*) Permanent address : Department of Mathematical Physics,
Lund Institute of Technology, Box 725, S-220 07 Lund,
Sweden.

The very low-lying $K^\Pi = 0^-$ bands in the Ra/Th region indicate the importance of octupole correlations. These correlations can be understood as deformations of Y_{30} type. There is, however, a long-standing controversy if the nuclei are really permanently deformed or rather vibrational with respect to the octupole degree of freedom. For a long time there were no theoretical calculations giving potential energy minima for non-zero octupole deformation. Recently, however, it was found that for a standard folded-Yukawa single-particle potential combined with the exponential plus Yukawa surface energy, such minima did come out^{1,2)}. Furthermore, somewhat similar results have been obtained within an approach based on the modified oscillator potential³⁾. Also some of the most recent experimental investigations rather suggest stable deformations. Of special importance is that although one has made careful investigations⁴⁾, one has not found any vibrational two-phonon 0^+ state in $^{222,224}\text{Ra}$ at around double the energy of the one-phonon state. The absence of such a two-phonon state should exclude the possibility of a pure vibration.

Recently, rather detailed level schemes of ^{227}Ra 5) and ^{225}Ra 6) have been constructed from experimental studies using a combination of various methods. The ground state spins deduced from these studies have been confirmed in still unpublished fast-beam laser experiments⁷⁾ where also magnetic moments and spectroscopic quadrupole moments of $^{221-229}\text{Ra}$ were measured. Attempts were made to interpret the level schemes within the Nilsson model assuming no octupole deformation but it turned out to be difficult to get a consistent description of the different rotational bands. One notes especially that the measured decoupling factor in the band which seems to be built from the $[631\ 1/2]$ Nilsson orbital is $a = 1.4$ in ^{225}Ra and $a = 0.6$ in ^{227}Ra compared to a theoretical value close to zero or rather slightly negative. It then seems natural to ask what would be the result if the Nilsson wave-functions were calculated for octupole deformed shapes. In the present paper we will investigate if this leads to a better description of the experimental data.

We will confine ourselves to the strong coupling approximation, i.e. the disturbance of the odd particle wave-function due to the motion of the core will be neglected. For reflection symmetric shapes, the odd-nucleus wave-function then takes the form

$$\psi_{IMK}^{\nu} = \sqrt{\frac{2I+1}{16\pi^2}} \left(D_{MK}^I(\omega) \chi_{\nu} + p D_{M-K}^I(\omega) \chi_{\bar{\nu}} \right)$$

where χ_{ν} is a Nilsson wave-function which has a good $\langle j_z \rangle = \Omega$ quantum number. The phase factor p takes different values depending on the definition of the conjugate state $\chi_{\bar{\nu}}$. The symmetry problem in connection with the generalisation to asymmetric shapes is treated by Bohr and Mottelson⁸⁾. For each single-particle orbital one gets two rotational bands having odd and even total parity, $\Pi = +$ and $\Pi = -$. The corresponding wave-function can be written

$$\psi_{IMK}^{\Pi\nu} = \sqrt{\frac{2I+1}{16\pi^2}} \left(D_{MK}^I(\omega) \chi_{\nu} + p \Pi D_{M-K}^I(\omega) \sum_{\pi} (\pi \chi_{\bar{\nu}}^{\pi}) \right)$$

where π is the intrinsic parity which is not pure in the Nilsson wave-function. Instead it can be split up into components having $\pi = +$ and $\pi = -$, $\chi_{\nu} = \chi_{\nu}^{+} + \chi_{\nu}^{-}$. In the present paper we concentrate on the rotational band having lowest energy, the parity Π of which should correspond to the dominating parity in χ_{ν} .

In the symmetric case, the decoupling parameter is calculated from the matrix element of j_{+} between the χ_{ν} and $\chi_{\bar{\nu}}$ states. It is different from zero only for orbitals having $\Omega = 1/2$. If χ_{ν} is expressed in an $|N\ell\Lambda\Sigma\rangle$ basis,

$$\chi_{\nu} = \sum_{N\ell\Lambda\Sigma} a_{N\ell\Lambda\Sigma}^{(\nu)} |N\ell\Lambda\Sigma\rangle$$

it takes the form⁹⁾

$$a = \sum_{N,\ell} \pi \left[(a_{N\ell 0 \frac{1}{2}})^2 + 2 \sqrt{\ell(\ell+1)} a_{N\ell 0 \frac{1}{2}} a_{N\ell 1 - \frac{1}{2}} \right]$$

where the intrinsic parity π is often written as $\pi = (-)^{\ell}$.

The generalisation to reflection asymmetry is now straightforward. The decoupling parameter should be calculated from the matrix element of j_{+} between χ_{ν} and $\Pi \sum_{\pi} (\pi \chi_{\bar{\nu}}^{\pi})$ leading to the formula

$$a = \Pi \sum_{N,\ell} \left[(a_{N\ell 0 \frac{1}{2}})^2 + 2 \sqrt{\ell(\ell+1)} a_{N\ell 0 \frac{1}{2}} a_{N\ell 1 - \frac{1}{2}} \right]$$

independent of the intrinsic parity of the orbital but with a different sign for the two bands with positive and negative total parity (cf. ref. 8)). For reflection symmetry, the intrinsic parity

is equal to the total parity. Thus the decoupling parameter formula has the correct limiting behaviour when the reflection asymmetry disappears.

We will now also give the formula for the magnetic moment for one-particle states of rotationally symmetric nuclei, i.e. states having $\Omega = K$. The formula for reflection symmetry can be found in the literature⁹⁾

$$\begin{aligned} \mu = & \frac{1}{I+1} \left\{ (g_S - g_\ell) \frac{K}{2} \sum_{N,\ell} \left((a_{N\ell\Omega-\frac{1}{2}\frac{1}{2}})^2 - (a_{N\ell\Omega+\frac{1}{2}-\frac{1}{2}})^2 \right) \right. \\ & + (g_\ell - g_R) \Omega K + g_R I(I+1) \\ & + \delta_{K\frac{1}{2}} \left((g_S - g_\ell) \pi(-)^{I-\frac{1}{2}} \frac{1}{2} (I+\frac{1}{2}) \sum_{N,\ell} (a_{N\ell 0\frac{1}{2}})^2 \right. \\ & \left. \left. + (g_\ell - g_R) (-)^{I-\frac{1}{2}} \frac{1}{2} (I+\frac{1}{2}) a \right) \right\} \end{aligned}$$

The gyromagnetic factors are given by g_S , g_ℓ and g_R while a is the decoupling parameter for which we have already given the formula. Furthermore, in the term which comes from the coupling between χ_ν and $\Pi_\pi(\pi\chi_\nu^\pi)$, we have replaced the intrinsic parity $\pi = (-)^{\ell}$ by the total parity Π . This makes the formula valid also in the reflection asymmetric case.

The E_{4+}/E_{2+} ratios in ^{224}Ra , ^{226}Ra and ^{228}Ra are 2.97, 3.12 and 3.14, respectively. These high values suggest that the neighbouring odd nuclei could be treated as good rotors and that the strong coupling approximation should be appropriate. Furthermore, if it is assumed that ^{229}Ra has a pure $\Omega = K$ value, the spectroscopic quadrupole moment measurements⁷⁾ show that this is the case also for ^{227}Ra . The $2^+ \rightarrow 0^+$ transition probabilities have been measured for the even nuclei¹⁰⁾. If the quadrupole deformation parameter, ϵ , is calculated from the corresponding quadrupole moments by use of the formula

$$Q = \frac{4}{5} Z A^{2/3} r_0^2 \epsilon (1 + \frac{1}{2}\epsilon); \quad r_0 = 1.2 \text{ fm},$$

one obtains $\epsilon = 0.16$, 0.18 and 0.19 for ^{224}Ra , ^{226}Ra and ^{228}Ra , respectively. In the formula, the small influence on the quadrupole moment from octupole and hexadecapole deformations has not been accounted for.

In the present calculations, we have chosen to fix the quadrupole deformation at $\epsilon = 0.18$. Furthermore, it is well known that ϵ_4 is rather large and negative in this region. Guided by some recent Strutinsky type calculations^{2,11)}, we have chosen $\epsilon_4 = -0.059$ (for $\epsilon = 0.18$). The ratio of ϵ_5 to ϵ_3 has been fixed as in ref.²⁾, $\epsilon_5 = -0.9\epsilon_3$. Finally, we have put $\epsilon_6 = 0$. What concerns the modified oscillator parameters, we have chosen those fitted for $A = 225$ (ref.¹²⁾, however, with the modification that for $N \geq 7$, the ℓ^2 coupling term, μ , has been reduced by a factor 0.73¹³⁾. This gives a subshell ordering with $j_{15/2}$ above $i_{11/2}$ in agreement with experiment. We have verified that reasonable changes in the deformations (ϵ , ϵ_4 , ϵ_5 and ϵ_6) or the potential parameters (κ and μ) do not modify our conclusions in any important way. The numerical calculations are carried out in a stretched basis⁹⁾. Then, as is usually done, we calculate for example the decoupling parameter as the matrix element of the stretched j_+ operator, $(j_+)_t$. For the small deformations considered here, the difference between $(j_+)_t$ and j_+ is negligible.

The single-particle orbitals are shown as functions of ϵ_3 in fig. 1 (for $\epsilon = 0.18$). To make it easier to trace the origin of the different orbitals, the plot has for $\epsilon_3 = 0$ been continued to spherical shape. One notes that the different orbitals get quite mixed in parity already for $\epsilon_3 = 0.04$. The calculated²⁾ ϵ_3 equilibrium deformations are generally around $\epsilon_3 = 0.08 - 0.10$. For such deformations there are only very few orbitals which are pure in parity to more than 70%.

The ground state spin of $^{225}_{88}\text{Ra}_{137}$ is 1/2, the parity is positive and a decoupling parameter of 1.4 is deduced from the corresponding rotational band⁶⁾. The best candidate for a Nilsson assignment at $\epsilon_3 = 0$ is [631 1/2] but this orbital is calculated to be too high in energy and it furthermore has a theoretical decoupling parameter which is close to zero or rather slightly negative (similar calculated properties for [631 1/2] are obtained in the Woods-Saxon calculations of ref.¹⁴⁾). It is now very interesting that as a function of ϵ_3 , the orbital labelled [631 1/2] slopes steeply downwards and, as seen in fig. 2, gets a positive decoupling parameter. The experimental value, $a = 1.4$, is reached at $\epsilon_3 \approx 0.10$ which is close to the calculated equilibrium deformation. As also seen in fig. 2, the measured magnetic moment⁷⁾, $\mu \approx -0.74$, is almost perfectly reproduced at $\epsilon_3 = 0.10$ but the dependence on ϵ_3 is rather weak. Finally, this orbital comes

quite close to the Fermi surface at $\epsilon_3 = 0.10$ but is still a few hundred keV too high in energy.

The increase with ϵ_3 of the decoupling parameter for the "[631 1/2]" orbital can be understood from the coupling to the negative parity orbitals, [770 1/2] and [761 1/2]. The negative parity band of both these orbitals have a negative decoupling parameter as seen in fig. 2. Consequently, when for $\epsilon_3 \neq 0$ they contribute to build a band of positive parity, their contribution to the decoupling parameter will be positive. This seems to explain the rather drastic increase with ϵ_3 for the decoupling parameter of the orbital labelled [631 1/2].

In ^{225}Ra , one has also observed the [501 1/2] band head and rather tentatively a rotational band with a decoupling parameter, $a = 1.21$. As seen in fig. 2, the calculated decoupling parameter for [501 1/2] is almost unchanged over the whole range of ϵ_3 considered and the experimental value is reproduced independently of ϵ_3 . In this connection it is interesting to note that the two orbitals labelled [640 1/2] and [501 1/2] at $\epsilon_3 = 0$ interact and exchange character at $\epsilon_3 \approx 0.09$.

In ^{227}Ra , the ground state spin is $3/2^+$ with a magnetic moment, $\mu \approx -0.40$ (refs. ^{5,7}). For $\epsilon_3 = 0$, the only $\Omega^\pi = 3/2^+$ orbital which is close to the Fermi surface is [631 3/2]. Its theoretical magnetic moment is around 0.4 n.m. as seen in fig. 3. For $\epsilon_3 \neq 0$, the calculated magnetic moment decreases although it never becomes small enough. The remaining discrepancy might be due to the neglect of the Coriolis coupling. It turns out that, at least for $\epsilon_3 = 0$, the inclusion of this coupling leads to a small decrease of the magnetic moment¹⁵⁾.

A band similar to the $K^\pi = 1/2^+$ ground band in ^{225}Ra is observed also in ^{227}Ra , however, at an excitation energy of 121 keV and with a decoupling parameter, $a \approx 0.6$. Again, this value can be reproduced for $\epsilon_3 \neq 0$, namely at $\epsilon_3 = 0.05 - 0.06$ which value could be compared to $\epsilon_3 \approx 0.10$ in ^{225}Ra . The [501 1/2] band is observed also in ^{227}Ra with a tentative decoupling parameter, $a = 0.87$. As for ^{227}Ra , this value is consistent with the calculations but does not really tell anything about octupole deformation.

Let us also mention that the measured ground state spin of ^{229}Ra is the same as in ^{229}Th , namely $I = 5/2$, and also the magnetic moments of these two nuclei are very similar. The properties of ^{229}Th are nicely described in Woods-Saxon calculations without invoking any octupole shapes¹⁴⁾. These data are somewhat more difficult

to understand in the Nilsson model as the wave-functions of the two orbitals, [633 5/2] and [622 5/2], are rather different compared to the Woods-Saxon results of ref.¹⁴⁾. Indeed, because the $h_{9/2}$ sub-shell comes below $i_{11/2}$ (see fig. 1), i.e. the energy of the j -shells does not decrease monotonically with j , the asymptotic quantum numbers are unusually mixed in this mass region for small ϵ . The ϵ -value where the asymptotic quantum numbers become reasonably pure depends on the distance between $h_{9/2}$ and $i_{11/2}$. In the present case where $h_{9/2}$ and $i_{11/2}$ are quite far apart, this happens for rather large ϵ -values, ($\epsilon > 0.20$) while in for example ref.¹⁴⁾ where $h_{9/2}$ and $i_{11/2}$ are very close, the asymptotic classification seems to become appropriate for smaller ϵ -values. It is difficult to tell which spacing is more reasonable to use for the Ra isotopes. For the present arguments it is important to note that independently of this spacing, it seems for $\epsilon_3 = 0$ impossible to find a positive parity $\Omega = 1/2$ which can explain the measured decoupling parameter in especially ^{225}Ra , $a = 1.4$. Note also that in the present study, we use the asymptotic quantum numbers as defined in fig. 1 independently of the variation of the wave functions due to deformation changes.

In the potential energy calculations of ref.²⁾, the minimum for $\epsilon_3 \neq 0$ is best developed in ^{222}Ra . It is then found at a smaller ϵ_3 -value and gets more shallow with increasing mass. In ^{228}Ra there is a very shallow minimum which has completely disappeared in ^{230}Ra . This trend as a function of neutron number is supported by the measured excitation energies of the $K^\pi = 0^-$ bands for Ra and Th nuclei. The lowest excitation energies are found for $N = 134$ and $N = 136$. The present study supports this picture with increasing importance of the octupole degree of freedom with decreasing neutron number for $N = 141, 139$ and 137 Ra isotopes. The large ϵ_3 -value which the present calculations suggest for ^{225}Ra , $\epsilon_3 \approx 0.10$, is a strong evidence for permanent octupole deformation. It is difficult to imagine how such a large effective ϵ_3 -value could result from a vibration in ϵ_3 . From the arguments above, one would expect also the lighter isotopes to have permanent octupole deformations. However, they have not been considered here because of both experimental and theoretical reasons. Thus, except for the ground states, their spectra are uncertain and, furthermore, it is questionable if the strong coupling approximation is valid for them.

In summary, we have given formulas for the decoupling parameter and the magnetic moment for nuclei having a stable octupole deformation. Numerical calculations show that the experimental spectra of ^{225}Ra and ^{227}Ra are much better described for $\epsilon_3 \neq 0$ than for $\epsilon_3 = 0$. This is especially the case for the decoupling parameter of the low-lying $K^\pi = 1/2^+$ band in $^{225,227}\text{Ra}$ ("631 1/2") which seems impossible to reproduce for reflection symmetric shapes. However, also the measured magnetic moments and excitation energies support the conclusion that $\epsilon_3 \neq 0$. In further studies it would be especially interesting to get a better understanding of the negative parity orbitals emerging from $j_{15/2}$. In the theoretical description of these states, the coupling between different orbitals due to the Coriolis interaction would probably not be negligible. It would also be interesting to follow the different rotational bands to higher spins. This could give information about the spin dependence of the octupole correlations. Another more straightforward extension of the present calculations would be to calculate spectroscopic factors for $\epsilon_3 \neq 0$ and make comparisons with experiments. Finally, of course, search for parity doublets and possible E3 transitions is of great interest as has already been pointed out several times.

After these calculations were completed, I have learned that somewhat similar calculations, using decoupling parameters and magnetic moments as probes for octupole deformation in Ra nuclei, have been carried out by Leander and Sheline¹⁶⁾.

I am grateful to the authors of ref.⁷⁾ for permission to quote their experimental data prior to publication. I especially want to thank C. Ekström for several clarifying discussions. Some valuable comments from I. Hamamoto and W. Nazarewicz are acknowledged.

References

1. P. Möller and J.R. Nix, Nucl. Phys. A361 (1981) 117
2. G.A. Leander, R.K. Sheline, P. Möller, P. Olanders, I. Ragnarsson and A.J. Sierk, Nucl. Phys. A388 (1982) 452
3. A. Gyurkovich, A. Sobiczewski, B. Nerlo-Pomorska and K. Pomorski, Phys. Lett. 105B (1981) 95
4. W. Kurcewicz et al., Nucl. Phys. A289 (1977) 1
W. Kurcewicz, E. Ruchowska, N. Kaffrell, T. Björnstad and G. Nyman, Nucl. Phys. A356 (1981) 15
5. T. von Egidy et al., Nucl. Phys. A365 (1981) 26
6. K. Nybø et al., subm. to Nucl. Phys.
7. S.A. Ahmad, C. Ekström, W. Klempt, R. Neugart, E.W. Otten and K. Wendt, Proc. 6th Int. Conf. on Hyperfine Interactions, Groningen, Netherlands, July 1983, to be publ. in Hyp. Int.
8. A. Bohr and B.R. Mottelson, Nuclear Structure, vol. II (W.A. Benjamin Inc., New York, 1975)
9. S.G. Nilsson, Mat. Fys. Medd. Dan. Vid. Selsk. 29 (1955) No 16
10. P.H. Stelson and L. Grodzins, Nucl. Data A1 (1965) 21
11. I. Ragnarsson and R.K. Sheline, Physica Scripta, to be publ.
12. S.G. Nilsson et al., Nucl. Phys. A131 (1969) 1
13. C.G. Andersson et al., Nucl. Phys. A309 (1978) 141
14. R.R. Chasman, I. Ahmad, A.M. Friedman and J.R. Erskine, Rev. Mod. Phys. 49 (1977) 833

15. C. Ekström, priv. comm.

16. R.K. Sheline and G.A. Leander, subm. to Phys. Lett.,
G.A. Leander and R.K. Sheline, to be subm. to Nucl. Phys.

Figure captions.

Fig. 1 Single-neutron orbitals as given by the $A = 225$ modified oscillator parameters, however with the modification that the λ^2 -strength parameter has been reduced by a factor 0.73 for the oscillator shells with $N \geq 7$. In the left part, starting from spherical shape, the orbitals are exhibited as functions of quadrupole deformation up to $\epsilon = 0.18$. A hexadecapole Y_{40} deformation has been implicitly included where $\epsilon_4(\epsilon)$ has been chosen in a way which should be appropriate for $N = 130 - 140$ Ra isotopes. In the right part, ϵ and ϵ_4 are fixed at 0.18 and -0.059, respectively, and the orbitals are drawn as functions of ϵ_3 . The orbitals are drawn with solid or dashed lines depending on whether the dominating intrinsic parity is positive or negative. Furthermore, the percentage of positive intrinsic parity is given for a few ϵ_3 values. The large parity mixing means that the asymptotic classification completely loses its relevance for $\epsilon_3 \neq 0$. Still, also for $\epsilon_3 \neq 0$, we sometimes use this classification as indicated for [631 1/2] and [631 3/2] but then mainly as a label without really bothering about the wave-function.

Fig. 2 Calculated decoupling parameter and magnetic moment as functions of ϵ_3 for some $\Omega = 1/2$ neutron orbitals in the $N = 130 - 140$ region. The orbitals are labelled with the standard asymptotic quantum numbers for $\epsilon_3 = 0$. The parameters are the same as in fig. 1. The decoupling parameter is only drawn for the total parity which coincides with the dominating intrinsic parity; full lines are used for positive parity and dashed lines for negative parity. The decoupling parameter for the alternative total parity has the same absolute value but a different sign. Note that the two orbitals "640 1/2" and "501 1/2" interact and exchange character for $\epsilon_3 \approx 0.09$. Similarly, the "761 1/2" orbital gets a dominating positive parity for $\epsilon_3 = 0.06 - 0.07$ (cf. fig. 1). For the magnetic moment, thicker lines are used for that total parity which coincides with the dominating intrinsic parity but also the other total parity is drawn. The gyromagnetic factors which have been used are $g_s = 0.6 g_s^{\text{free}}$, $g_\lambda = 0$ and $g_R = 0.388$, the latter value taken from Z/A for ^{227}Ra . Some experimental values are drawn as wavy lines and are placed to indicate possible octupole deformations.

Fig. 3 Calculated magnetic moments for some $\Omega = 3/2$ orbitals drawn in a similar way as in fig. 2. For $\Omega \neq 1/2$, the magnetic moment is independent of the total parity. Two values of g_s are compared, $g_s = 0.6 g_s^{\text{free}}$ and $g_s = g_s^{\text{free}}$. The other parameters are the same as in fig. 2. The measured magnetic moment of the "[631 3/2]" ground state in ^{227}Ra is drawn with a wavy line.

FIG. 1

$$\epsilon = 0.18, \quad \epsilon_4 = -0.059, \quad \epsilon_5 = -0.162 \epsilon_3$$

FIG. 2

$$\epsilon = 0.18, \quad \epsilon_4 = -0.059, \quad \epsilon_5 = -0.162 \epsilon_3$$

FIG. 3