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CONVERGENT SELF-ENERGY IN THE CLOUDY BAG MODEL

Gregory A. Crawford^{*)} and Gerald Miller^{*)}
CERN -- Geneva

ABSTRACT

The use of a pion-quark interaction that incorporates the finite extent (in time and space) of the pion leads to convergent self-energies. Furthermore, occupation probabilities for quarks in excited states are found to be small.

^{*)} Permanent address: Dept. of Physics, Univ. of Washington, FM-15, Seattle, Wa 98195, U.S.A.

1. - INTRODUCTION

The Cloudy Bag Model¹⁾ (CBM) is a phenomenological description of hadrons in which the quarks are assumed to be confined in a static spherical cavity that is surrounded by a cloud of pions. It and other chiral bag models [a review is given in Ref. 2)] have had considerable success in reproducing baryonic properties as well as pion-nucleon scattering.

Based on the idea that admixtures of excited states into ground state wave functions should be suppressed by large energy denominators, a useful approximation was to restrict the quark orbitals of the hadronic ground states to the wave function of lowest energy, the $1S_{1/2}$ state. This approximation was called into question by the work of Chin³⁾ who showed that removing this restriction leads to a divergence in the single loop contribution of the self-energy. This problem arises from carrying out the sum over all quantum numbers for the quark in the intermediate state. Chin did not use the CBM, but rather a chiral bag model⁴⁾ in which the pion is excluded from the interior of the bag. However, Oset proved⁵⁾ that the divergence also occurs in the pseudoscalar surface-coupling version of the CBM¹⁾.

In the CBM or in chiral bag models, the pion is treated typically as a fundamental field interacting with pointlike quarks confined to a cavity with a sharp boundary. Several workers^{3),6)} have argued that including a finite size for the pion or a finite surface thickness for the bag boundary can eliminate this divergence. Indeed Thomas⁶⁾ has stated that the divergence is "physical nonsense". This is because⁶⁾ the pion field associated with the transition of a quark from a low energy state to a high energy state is squeezed into a region close to the surface. Indeed the allowed volume is less than the pion size even for excitations of states as low in energy as the $3S_{1/2}$ ⁶⁾.

Nevertheless, it is necessary to study the divergence and show how it can be eliminated. This is because extremely large corrections to self-energies and other observables resulting from quarks in excited states have not been ruled out. Indeed Thomas⁶⁾ obtained only an upper limit on the self-energy and, no quantitative computation of the finite part of the self-energy remaining after elimination of the divergence has been made. Furthermore, one should justify the truncations employed and examine the influence of excited intermediate quark states on physical observables. For example, if excited intermediate states are important for the self-energy, then they might be important for other processes which involve a quark-pion loop. This could make higher order calculations very cumbersome and might change some previous results for which excited intermediate quark states were neglected. In addition, a large pionic contribution to the self-energy would result in a large admixture of pions in the physical nucleon. This could violate

an upper limit recently derived by Thomas⁷⁾ on the fraction on the nucleon momentum which is carried by the pions. Therefore, it is of interest to study some phenomenological models which eliminate this divergence.

In the present note the divergent self-energy terms are evaluated numerically for the CBM. Then a modified π -quark interaction in which the pion's finite extent (in time as well as space) is employed. The motivation for this ansatz is that the internal wave function of an object composed of a relativistic $q\bar{q}$ pair must be a function not only of the relative distance between quarks, but also the relative time. This is a well-known property of solutions of the Bethe-Salpeter equation. As a result, the contribution to the self-energy from excited quark states is found to be finite and relatively small. The modified or "smeared" interaction has other benefits: the resulting expectation values of the pion number operator turn out to be small, with very minor contributions coming from quarks in excited states. The time smearing plays an important role in these results, especially in reducing occupation probabilities for quarks in excited states.

2. - POINT PIONS

There are two versions of the CBM. In the earliest¹⁾ or surface coupling version, the pion quark interaction is a pseudoscalar coupling localized at the bag surface. The other version⁸⁾ uses a pion-quark pseudovector coupling throughout the volume of the bag. The πNN couplings in these two theories are the same; however, the couplings to excited nucleon states are different. The Lagrangian densities to order $1/f^2$ for these two theories are

$$\begin{aligned} \mathcal{L}^S(x) = & (i\bar{q}(x)\not{\partial}q(x) - B)\theta_V(x) - \frac{1}{2}\bar{q}(x)q(x)\Delta_S(x) \\ & - \frac{i}{2f}\bar{q}(x)\underline{\tau}\cdot\phi(x)\underline{\gamma}_5q(x)\Delta_S + \frac{1}{2}(\partial_\mu\phi(x))^2 - \frac{1}{2}m_\pi^2\phi^2(x), \end{aligned} \quad (1)$$

$$\begin{aligned} \mathcal{L}^V(x) = & (i\bar{q}(x)\not{\partial}q(x) - B)\theta_V(x) - \frac{1}{2}\bar{q}(x)q(x)\Delta_S(x) \\ & - \frac{\theta_V(x)}{2f}\bar{q}(x)\underline{\gamma}^\mu\underline{\gamma}_5\underline{\tau}q(x)\partial_\mu\phi + \frac{1}{2}(\partial_\mu\phi(x))^2 - \frac{1}{2}m_\pi^2\phi^2(x), \end{aligned} \quad (2)$$

where θ_V is one inside the bag and zero outside. Δ_S is a surface delta function. B is the bag energy density, f is the pion decay constant (93 MeV), and $q(x)$, $\phi(x)$ are the quark and pion fields. The equations of motion and boundary conditions are well described elsewhere²⁾. We only note here that for

the surface coupling version the linear boundary condition for the quark fields involves the pion field operator; while for the volume coupling version the linear boundary condition is the same as that for the MIT bag model. We quantize our quark field operators with a complete set of wave functions which satisfy the MIT linear boundary condition. Thus our quark field operator does not satisfy the linear boundary condition of the surface coupling Lagrangian. This is one reason for preferring the volume coupling version; other reasons can be found in the literature^{8),9)}.

The calculation of the self-energies for these models is similar to the one presented by Chin and only the result is given here. For the surface coupling version the self-energy of the nucleon is the following (not including the self-energy, E^S , due to pion exchange between different quarks in the nucleon)

$$E^S = -\frac{9}{16\pi^2} \frac{1}{f^2 R^3} \left(\frac{\chi_{1-1}}{\chi_{1-1}-1} \right) \left(\frac{1}{j_0(\chi_{1-1})} \right)^2 \sum_{\nu\kappa} |\kappa| \left(\frac{\chi_{\nu\kappa}}{\chi_{\nu\kappa}+\kappa} \right) \cdot \left(\frac{1}{j_\ell(\chi_{\nu\kappa})} \right)^2 \int_0^\infty \frac{z^2 dz}{\omega_z} \left(\frac{1}{(\chi_{\nu\kappa}+\omega_z-\chi_{1-1})} [j_0(\chi_{1-1}) j_\ell(\chi_{\nu\kappa}) j_{\bar{\ell}}(z)]^2 \right. \\ \left. - \frac{1}{(\chi_{\nu\kappa}+\omega_z+\chi_{1-1})} [j_0(\chi_{1-1}) j_\ell(\chi_{\nu\kappa}) j_{\bar{\ell}}(z)]^2 \right) \quad (3)$$

where R is the bag radius, $\chi_{\nu\kappa}$ is the dimensionless MIT quark eigenvalue for radial quantum number ν and angular quantum number κ , ℓ is the angular momentum of the upper component of the Dirac spinor and $\bar{\ell}$ is the angular momentum of the lower component. If j is the total angular momentum of the Dirac spinor, we have

$$j = |\kappa| - \frac{1}{2} \quad (4)$$

$$\ell = j + \frac{1}{2} \text{sgn}(\kappa) \quad (5)$$

$$\bar{\ell} = j - \frac{1}{2} \text{sgn}(\kappa). \quad (6)$$

The function j_ℓ is the spherical Bessel function of order ℓ and

$$\omega_z = (z^2 + m_\pi^2 R^2)^{1/2}. \quad (7)$$

m_π is the pion mass.

The self-energy of the volume coupling version is given by Eq. (3) if one replaces the term

$$j_0(\chi_{1-1}) j_\ell(\chi_{\nu\kappa}) j_{\bar{\ell}}(z) \quad (8)$$

with

$$j_0(\chi_{1-1}) j_\ell(\chi_{\nu\kappa}) j_{\bar{\ell}}(\bar{z}) + \frac{1}{2} \int_0^1 p^2 dp (\chi_{\nu\kappa} - \chi_{1-1} - \omega_z) \\ \times \left\{ j_1(\chi_{1-1} p) j_\ell(\chi_{\nu\kappa} p) + \text{sgn}(\kappa) j_0(\chi_{1-1} p) j_{\bar{\ell}}(\chi_{\nu\kappa} p) \right\} j_{\bar{\ell}}(z p) \quad (9)$$

and one also replaces

$$j_0(\chi_{1-1}) j_\ell(\chi_{\nu\kappa}) j_\ell(z) \quad (10)$$

with

$$j_0(\chi_{1-1}) j_\ell(\chi_{\nu\kappa}) j_\ell(z) + \frac{1}{2} \int_0^1 p^2 dp (\chi_{\nu\kappa} + \chi_{1-1} - \omega_z) \\ \times \left\{ j_0(\chi_{1-1} p) j_\ell(\chi_{\nu\kappa} p) + \text{sgn}(\kappa) j_1(\chi_{1-1} p) j_\ell(\chi_{\nu\kappa} p) \right\} j_\ell(z p). \quad (11)$$

One can easily show that for the lowest positive energy $1S_{1/2}$ state ($\kappa = -1$, $\nu = 1$) the contribution to the self-energy is the same for both versions. This is because the use of $\nu = 1$, $\kappa = -1$ along with $\ell = 0$ and $\bar{\ell} = 1$ causes the bracketed term of (9) to vanish. For other intermediate states, especially those of negative energy, the two models give different results.

The results of numerical calculations for these self-energies indicate a linear divergence in both cases; see Fig. 1. The bag radius, R , is taken as 0.8 fm. The divergence has a negative sign for the surface coupling version and has a positive sign for the volume coupling version; one should recall that the positive and negative energy intermediate states contribute to the self-energy with opposite signs and their cancellation causes an otherwise quadratic divergence to become linear. In the model with the volume-interaction, the coupling to negative energy states is stronger than for positive energy ones. Thus in that version the self-energy is positive. This can be seen, in more detail, by comparing the terms of Eqs. (8) and (10) with those of (9) and (11).

4. - PION FINITE SIZE

Let us examine a phenomenological model which eliminates this divergence. There are at least two possible changes we could implement: 1) including a finite size for the pion, and 2) including a finite surface thickness for the bag. We choose to include a finite size for the pion. This is easy to implement and has already been shown by de Kam and Pirner¹⁰⁾ to stabilize the bag against collapse. Since we are interested in examining the minimal effect from smearing the interaction, we choose our size parameter to be the small size of the $q\bar{q}$ core of the pion (0.17 fm), as determined by Brodsky and Le Page¹¹⁾, rather than the experimental charge radius. This is because it seems reasonable (on the basis of single gluon exchange) to anticipate that quarks couple more strongly to the $q\bar{q}$ core than to the more complicated configurations of the pion. Since the pion is a composite object containing relativistic quarks, we smear the interaction in both space and time¹²⁾. Because only one input parameter is known we may choose the exact form of the interaction by the criterion of simplicity. The interaction Lagrangian densities for the two versions are modified to be

$$\tilde{\mathcal{L}}_I^S(x) = \frac{-i}{2f} \int d^3\eta d\tau f(|\eta|^2) g(\tau^2) \Delta_S \left\{ \bar{q}(\vec{x}, t+\tau) \gamma_5 \tau \cdot \underline{\phi}(\vec{x}+\vec{\eta}, t) \right. \\ \left. \times q(\vec{x}, t+\tau) \right\} \Delta_S(x) \quad (12)$$

and

$$\tilde{\mathcal{L}}_I^V(x) = -\frac{\Theta_V(x)}{2f} \int d^3\eta d\tau \bar{q}(\vec{x}, t+\tau) \gamma^\mu \gamma_5 \tau \cdot q(\vec{x}, t+\tau) \\ \times \left\{ \partial_\mu \phi(\vec{x}+\vec{\eta}, t) f(|\eta|^2) g(\tau^2) \right\} \quad (13)$$

with

$$f(|\eta|^2) = \exp(-\eta^2/\Delta^2) / \pi^{3/2} \Delta^3 \quad (14)$$

$$g(\tau^2) = \exp(-\tau^2/T^2) / \pi^{1/2} T \quad (15)$$

and

$$\Delta = T = \sqrt{2} \cdot 0.17 \text{ fm} \quad (16)$$

where the $\sqrt{2}$ arises from the need to treat Δ and T as corresponding to a relative co-ordinate of the $q\bar{q}$ system. With the Lagrangians of (12) and (13) one can retain chiral symmetry although the axial vector and axial charge operators become non-local because of the non-local interaction.

In (12) and (13), a quark at $(\vec{x}, t+T)$ interacts with a pion that is spread out in space and time. The quantity $\phi(\vec{y}, t)$ is now a field operator for the π centre-of-mass (and time). In both versions the quarks are confined to the bag, but the pion can exist inside or outside (or partly inside and outside) the bag.

With the modifications (12) and (13) in the interaction the change in the quark-quark-pion interaction is quite simple. The result is expressed in its most compact form as a replacement of the energy denominators of Eq. (3):

$$\frac{1}{\chi_{\nu K} + \omega_2 \pm \chi_{1-1}} \rightarrow \frac{1}{\chi_{\nu K} + \omega_2 \pm \chi_{1-1}} \exp\left(-\frac{z^2 \Delta^2 m_\pi^2}{2}\right) \cdot \exp\left(-\frac{(\chi_{\nu K} \pm \chi_{1-1})^2 T^2}{2 R^2}\right). \quad (17)$$

It is useful to note that the exponential factor involving $(\chi_{\nu K} \pm \chi_{1-1})^2$ causes large reductions in the contributions of the negative energy states. Thus, the time smearing is very important. For example, the effects of exciting the negative energy state of highest energy is reduced by a factor of about two, due to the time smearing alone.

The implementation of (17) leads to a self-energy that is both finite and relatively small, as shown in Fig. 1 and the Table. The self-energy from the $1S_{1/2}$ states listed in the table includes the self-energy from pion exchange between different quarks. As indicated in the Table, the total contribution to the self-energy from all the states above the $1S_{1/2}$ state is of roughly the same size as the contribution from the $1S_{1/2}$ state. Such corrections are not very large since there are many uncertainties in the parameters entering in a chiral bag model computation of the nucleon mass.

In order to assess the importance of excited quark states on computable observables such as magnetic moments or charge radii we compute the probabilities for finding a pion along with a quark in a $1S_{1/2}$ or excited state. The results are shown in the Table and in Fig. 2. Clearly, the excited states are not occupied frequently. The probability of finding an excited quark state in the physical nucleon is between 10% and 25%, depending on the model and the bag radius used. The most important excited state in the surface coupling version is the $1P_{3/2}$ state, while the most important excited state for the volume coupling version is the $1P_{1/2}$ state. Each of these excited states has less than 5% probability in the physical nucleon. The probabilities for the negative energy states are very small. This is because of the exponential factors of Eq. (17) which are particularly small for negative energy intermediate states.

5. - SUMMARY AND OUTLOOK

The self-energy diverges linearly in two versions of the CBM^{1),8)}. If a finite pion size effect is phenomenologically included in a minimal fashion, then the self-energy is finite and not large. Also, the probability of finding an excited state in the physical nucleon is between 10% and 25% with no single excited state contributing more than 5%. The use of smearing in both space and time is crucial in obtaining these results. If other effects such as the bag surface thickness are included the coupling to excited quark states should be suppressed further. These results seem to be reasonable after the fact justification for ignoring excited states in previous calculations. It also seems that the effects from excited states are reasonably small but dependent on the specific form and parameters involved. Thus it may be premature to include excited states in most calculations unless one can arrive at a better motivated model for pion size and surface thickness effects than that used here.

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Table Caption: Self-energies and occupation probabilities for theories with pions of finite extent.

	Surface coupling		Volume coupling	
	R = 0.8 fm	R = 1.0 fm	R = 0.8 fm	R = 1.0 fm
Self-energy from $1S_{1/2}$ state	244	125	244	125
Self-energy from higher excited states	250	128	261	134
Probability of $(1S_{1/2})^3$ and <u>no</u> pions	0.55	0.71	0.50	0.66
Probability of $(S_{1/2})^3$ and <u>one</u> pion	0.28	0.18	0.25	0.17
Probability of an excited state plus one pion	0.17	0.11	0.25	0.17

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H.J. Pirner.

FIGURE CAPTIONS

Fig. 1 : Negative of the self-energy versus $|\kappa|$. Contributions of the lowest 10 values of ν are summed for each value of κ . Only results from terms involving a single quark are shown.

Fig. 2 : Probability to have a pion plus a quark in a given state. (The $1S_{1/2}$ is the one at about 500 MeV, with a bag radius of 0.8 fm.)

Fig. 1

Fig. 2