Ref.TH.3668-CERN

TOWARDS A DESCRIPTION OF NUCLEAR PHYSICS AT THE QUARK LEVEL *)

A.W. Thomas
CERN - Geneva

C O N T E N T S

- Lecture 1 - FROM QCD TO CHIRAL BAGS
Introduction
The MIT Bag Model
Relevance to Nuclear Physics
Chiral Symmetry in the Bag Model
- Lecture 2 - THE CLOUDY BAG MODEL
Introduction
Pions in the Bag : An Alternate Formulation
The Nucleon
Magnetic Moments of the Nucleon Octet
Pion-Nucleon Scattering
Conclusion
- Lecture 3 - DEEP INELASTIC SCATTERING
Introduction
Scaling
The Quark-Parton Model and the Structure of
the Nucleon
A Constraint on the Pion Field of the Nucleon
The EMC Effect
- References

*) Lectures presented at the International School of Nuclear Physics, 7th Course, Mesons, Isobars, Quarks and Nuclear Excitations, Erice (6-18 April, 1983).

LECTURE 1

FROM QCD TO CHIRAL BAGS

INTRODUCTION

There has been a vast change in our understanding of particle physics over the past 20 years. The classification of almost one hundred "elementary particles" according to the eightfold way was in many ways the first step. It was then natural to propose that the fundamental representation of the symmetry group, $SU(3)$ -flavour, could actually exist (Gell-Mann, 1964; Zweig, 1964). This set into motion a long series of quark searches, in spite of a theoretical problem which such an interpretation posed -- namely the existence of the Δ^{++} . The simple picture of the Δ^{++} as three identical up-quarks, with spin up, in the same spatial orbital, violated the Pauli principle. Thus an extra, apparently arbitrary quantum number called colour was assigned.

From that rather crude state to the present day, where Quantum Chromodynamics (QCD) is regarded almost universally as *the* theory of the strong interactions, there have been a number of astounding experimental discoveries. The deep inelastic scattering experiments at SLAC in the late 60's revealed point-like constituents inside the nucleon through the phenomenon of scaling. (In lecture 3 we shall review the quark-parton description of DIS within the context of the chiral bag models and the nuclear many-body problem.) In addition, the discovery of neutral currents at CERN in 1973 was interpreted as a very strong sign that gauge theories contained some fundamental truth. Further confirmation of both the quark model and the gauge principle has come from the discovery of the J/ψ , the investigation of e^+e^- annihilation into hadrons, and the continued success of the Weinberg-Salam model.

Within the framework of QCD, the apparently arbitrary addition of colour as a property of quarks becomes an essential element. By demanding that the theory should be invariant under arbitrary transformations amongst the colours at each point in space, one arrives at a very powerful dynamical theory. Just as in electromagnetism the gauge principle based on $U(1)$ demands the existence of massless vector photons, in QCD [based on $SU(3)$] we have eight massless gluons which mediate the interaction [see, for example, Marciano and Pagels (1978) and Abers and Lee (1973)].

Our purpose in these lectures is not to discuss the many attempts made to obtain solutions to the QCD equations. In the long term the most likely method for producing accurate numerical predictions for the structure of the proton based on QCD is to use a discrete space-time lattice. However, the present state of the art involves a lattice of 20 points in time and 10 points in each spatial direction -- see Lipps and co-workers (1983). Since the lattice spacing is of order 0.1-0.15 fm a single baryon is not wholly contained, and the answers appear to depend on the presence of the boundary. The calculation of the magnetic moment of the proton on a smaller lattice required some 9 hours of CDC 7600 time (Martinelli and others, 1982). Moreover, in all of these calculations the effects of virtual $q-\bar{q}$ fluctuations (e.g. pionic corrections) had to be omitted.

Clearly, for the next few years at least, these techniques cannot be applied to the nuclear many-body problem. Nevertheless, as we shall argue later in this lecture, there is good reason to believe that an acceptable microscopic description

of nuclear physics should involve the quarks explicitly. The only avenue open therefore is to use a phenomenological model of hadron structure. Of course, whatever the model it should incorporate as much as possible of what we know about QCD.

The first crucial property of QCD is usually referred to as asymptotic freedom. Unlike QED where the renormalized coupling constant $\alpha(Q^2)$ grows as a function of Q^2 ; in QCD it actually decreases logarithmically. Indeed at the one-loop-level we find

$$\alpha_s(Q^2) = \frac{4\pi}{(11 - \frac{2}{3}n_F) \ln(Q^2/\Lambda^2)}, \quad (1.1)$$

with n_F the number of quark flavours and Λ the scale parameter. The proof to all orders relies on renormalization group arguments. This rather weak decrease of the coupling constant at high energy is the ultimate justification of the success of the quark-parton description of DIS.

A second property of QCD which is not yet proven rigorously, but is almost universally accepted is that quarks are confined to colour singlet configurations only. That is if we start with a bound $q\bar{q}$ pair and try to move them apart the energy of the system rises approximately linearly, and in fact they can only be free of each other at finite energy by making another $q\bar{q}$ pair in between, and hence two colour singlet $q\bar{q}$ pairs.

The last property of QCD which will concern us is that at least, for the first three flavours, the quark masses seem to be very light (Pagels, 1975; Gasser and Leutwyler, 1982). In fact, with $(m_u + m_d)/2$ less than 10 MeV, which is tiny compared with typical hadronic masses, QCD has an almost exact chiral symmetry. That is to say, that in the limit $m_u = m_d = 0$ there is no interaction which can mix left-handed with right-handed fermions -- hence the notation $SU(2)_L \times SU(2)_R$. In such a theory the Goldstone theorem then says we have two choices. Either for every eigenstate of the theory there is a degenerate partner of opposite parity, or the symmetry of the vacuum is broken and massless pseudoscalar bosons appear. For $SU(2) \times SU(2)$ this Goldstone boson is the pion.

In order to finish this introduction let us summarize briefly. We would like to ask what are the consequences for nuclear physics of taking QCD as our starting point. Because of its complexity we cannot actually use QCD, but rather phenomenological models which incorporate its main features, namely

- a. confinement,
 - b. asymptotic freedom,
 - c. chiral symmetry.
- (1.2)

In the rest of this lecture we discuss the ambiguities associated with implementing (1.2) in a phenomenological model. We then introduce the essential features of the so-called chiral bag models. In lecture 2 we discuss the properties of one particular model of this type -- the Cloudy Bag Model (CBM). Finally in lecture 3 we return to DIS and show that it can place severe restrictions on the parameters of any chiral bag model (Thomas, 1983a). Furthermore, the modification of the structure function of Fe expected on the basis of the ideas presented here is consistent with the recent observations of the European Muon Collaboration (Aubert and others, 1983).

General Phenomenological Considerations

The earliest and probably the most widely successful QCD motivated phenomenology is the non-relativistic quark model (NRQM) (De Rújula, Georgi, Glashow, 1975; Isgur,

1980). There one deals with a system once removed from QCD in the sense that the u and d masses are of the order of 300 MeV. That is the quarks must be considered as having been dressed by higher order processes. Confinement is incorporated through a phenomenological potential, and the hyperfine q - q interaction arising from one-gluon-exchange is then obtained as an expansion in powers of v/c . By construction the model is not intended to incorporate asymptotic freedom, nor does the notion of chiral symmetry appear naturally.

The MIT bag model was one of several somewhat more ambitious schemes developed in the mid-70's (Chodos and others, 1974a, 1974b). Both confinement and asymptotic freedom were incorporated by letting the quarks (now almost massless and therefore highly relativistic) move freely inside, but only inside, a cavity. That such a model intrinsically destroyed the third of our favourite properties of QCD, namely chiral symmetry, was realized immediately, and some possible extensions were proposed (Chodos and Thorn, 1975; Inoue and Maskawa, 1975). However, the phenomenological consequences of these chiral bag models were not explored for another five years.

In the rest of this lecture we shall summarize the properties of the MIT bag model and show how chiral symmetry is implemented. Since there are strongly felt differences between models, the crucial question we would like to address now is whether QCD offers any more specific guidelines than the global constraints of Eq. (1.2). For example, if the "true" solution resembles a bag, how thick is the surface, where do the Goldstone bosons appear, and so on?

The lattice calculations of Kogut and collaborators (1982) strongly indicate that there is a first order phase transition at the surface of the hadron. For $SU(3)$ -colour this transition occurs in essentially the same place (actually the same temperature) for both the Wilson line and the chiral symmetry breaking parameter $\langle 0 | \bar{\psi} \psi | 0 \rangle$. The rapid variation with temperature near the critical point strongly suggests that the surface region in a phenomenological representation should be rather thin. Of course one must add some cautions in interpreting this calculation. The number of time steps was only 2 and 4, compared with 20 in the work of Lipps and co-workers (1983). In addition fermion loops were neglected, and one cannot draw any conclusions related to the question of where Goldstone bosons may or may not exist.

Shuryak (1983) has presented a rather different picture, in many ways closer to the NRQM. Because of the very large values obtained for the gluon condensate from QCD sum rules, he suggests that there are probably three scales inside hadrons. At very short distances the theory is asymptotically free. As we move further from a quark (few $\frac{1}{10}$ fm) there are very strong fluctuations and chiral symmetry is broken. Finally at larger distances (~ 1 fm) this dressed quark (with a constituent mass) is confined. Within this model the mechanism for dynamical symmetry breaking is a residual instanton induced attraction for any q - $\bar{q}$ pair with quantum numbers of a pseudoscalar meson (t'Hooft, 1976). The resulting (almost) Goldstone bosons then dress each of the "constituent" quarks, and certainly exist at distances much less than those at which quark confinement takes over.

Clearly even this very brief summary indicates that QCD still gives us very little guidance in constructing phenomenological models. There is as yet no appeal to any higher authority, save one, to justify a model. The one exception is of course experiment, and as long as any model satisfies the global constraints of Eq. (1.2), and reproduces all the data for which it is suitable, it has as much right to be believed as any other. Since there will inevitably be more models than correct descriptions of Nature, our job must be to find as many experimental tests as possible to eliminate pretenders.

With these preliminary remarks at last complete, let us briefly review the MIT bag model.

THE MIT BAG MODEL

There have been a number of quite thorough reviews of the MIT bag model over the past seven years. The two most recent by De Tar and Donoghue (1983) and Thomas (1983b) contain full references to earlier work on the MIT bag. Needless to say our conventions are identical with Thomas (1983b), which we shall refer to as I. There is little point in reproducing algebra which already appears in I, so here we simply present the key equations.

In its most common form, the static cavity approximation, massless u and d quarks move freely inside a fixed spherical cavity. They are then described by the free Dirac equation in that region

$$\vec{\alpha} \cdot \vec{p} \psi = i \frac{\partial \psi}{\partial t}, \quad (1.3)$$

which has solutions

$$\psi_{\kappa}^{\mu} = N_{\kappa} \begin{bmatrix} j_{\ell}(Er) \\ -i S_{\kappa} j_{\bar{\ell}}(Er) \vec{\sigma} \cdot \hat{r} \end{bmatrix} \chi_{\kappa}^{\mu} e^{-iEt}. \quad (1.4)$$

Here $\kappa = \ell$ if $j = \ell - 1/2$ and $\kappa = -\ell - 1$ if $j = \ell + 1/2$,

$$|\chi_{\kappa}^{\mu}\rangle = \sum_{m\nu} C_{\ell}^{m\nu} \frac{1}{2} \mu_j |\ell m\rangle |1/2 \nu\rangle, \quad (1.5)$$

$\bar{\ell} = \kappa - 1$ if $\kappa > 0$ and $\bar{\ell} = -\kappa$ if $\kappa < 0$, and finally S_{κ} is the sign of κ . For example, for a quark in an s-state

$$\psi_{n,-1}^{\mu}(\vec{r}) = \frac{N_{n,-1}}{\sqrt{4\pi}} \begin{bmatrix} j_0(Er) \\ i \vec{\sigma} \cdot \hat{r} j_1(Er) \end{bmatrix} \chi_{1/2}^{\mu}, \quad (1.6)$$

with $\chi_{1/2}^{\mu}$ a Pauli spinor.

Of course at this stage any value of the energy is permitted. The imposition of confinement amounts to saying that for $r > R$ the particle moves in a scalar potential well, and so acts as if it had mass $M \rightarrow \infty$. If we write the corresponding solution of the Dirac equation and demand that both its upper and lower components are continuous with (1.4) at $r = R$ we find the following *boundary condition* (see I for details)

$$j_{\ell}(\omega_{n,\kappa}) = -S_{\kappa} j_{\bar{\ell}}(\omega_{n,\kappa}), \quad (1.7)$$

where

$$E_{n,\kappa} = \omega_{n,\kappa}/R. \quad (1.8)$$

The $1s_{1/2}$ and $2s_{1/2}$ levels correspond to $\omega_{1,-1} = 2.04$ and $\omega_{2,-1} = 5.40$, respectively, while the $1p_{1/2}$ level has $\omega_{1,+1} = 3.81$, and so on. The general expression for the $ns_{1/2}$ -level normalization constant is

$$N_{n,-1}^2 = \frac{\omega_{n,-1}^3}{2R^3(\omega_{n,-1} - 1) \sin^2 \omega_{n,-1}}. \quad (1.9)$$

It is interesting that unlike the corresponding situation for the Schrödinger equation it is not the density which vanishes at $r = R$ -- in fact for an s-state:

$$\begin{aligned}\rho(R) &= \bar{q}(R)\gamma^0 q(R) \\ &\propto j_0^2(\omega) + j_1^2(\omega) = 2j_0^2(\omega) .\end{aligned}\quad (1.10)$$

On the other hand, it is easily shown that

$$\hat{r} \cdot \vec{j}(R) = 0 , \quad (1.11)$$

where

$$\vec{j}(\vec{r}) = \bar{q}(\vec{r}) \vec{\gamma} q(\vec{r}) , \quad (1.12)$$

because of the boundary condition (1.7). Thus it is not the density but the flow of colour current normal to the surface of the bag which vanishes.

In the general case, where we are not restricted to a static, spherical bag, the condition (1.11) is replaced by

$$n^\mu \bar{q} \gamma_\mu q = 0 \text{ on } S , \quad (1.13)$$

where n^μ is an outward normal to the surface. In fact, in order to guarantee Eq. (1.13) it is sufficient that q satisfy a *linear boundary condition* (see I for details)

$$i\gamma \cdot n q = q \text{ on } S . \quad (1.14)$$

Up to this stage the MIT bag model is equivalent to the confining potential model of Bogolioubov (1967). However, the MIT group realized that this solution was not stable. The reason is simply that as with any gas the relativistic quarks exert a pressure on the surface of the bag and nothing balances this pressure. As a solution the MIT group proposed that because of the different structure of the vacuum inside and outside the bag it should cost energy to make a bag. In its simplest form this energy was taken to be simply $(4\pi/3)R^3 B$, with B a phenomenological constant. Formally the pressure balance is then imposed by demanding that the divergence of the energy-momentum tensor of the bag vanish. This means that

$$\partial_\mu T^{\mu\nu} = (B - P_D) n^\nu \delta_S = 0 , \quad (1.15)$$

with δ_S a surface δ -function. Here P_D is the Dirac pressure of the quark gas, and more explicitly Eq. (1.15) can be seen to be a *non-linear boundary condition*.

$$B = P_D = -\frac{1}{2} n \cdot \partial(\bar{q}q) \text{ on } S . \quad (1.16)$$

Putting all these ingredients together we find that the mass of a nucleon or delta (degenerate in the absence of spin-dependent interactions) is

$$M(R) = 3 \frac{\omega_{1,-1}}{R} + \frac{4\pi}{3} B R^3 . \quad (1.17)$$

It is a simple exercise using Eq. (1.6) to show that Eq. (1.16) is equivalent to the stability condition

$$\frac{\partial M}{\partial R} = 0 . \quad (1.18)$$

Thus B and R are not independent parameters. Indeed, the stable solution satisfies

$$R^4 = \frac{3\omega_{1,-1}}{(4\pi B)} . \quad (1.19)$$

If we then adjust R to fit the average of N and Δ masses we easily find

$$\begin{aligned} R_N &= 1.4 \text{ fm} , \\ B^{1/4} &= 120 \text{ MeV} . \end{aligned} \quad (1.20)$$

Having once determined B in this way, it is supposed to be a universal constant, and through Eq. (1.18) it then determines the radii of all other baryons.

Further Corrections to the Model

What we have described so far might be called the bare bones of the MIT bag model. If it truly represents an approximation to QCD one would expect additional terms in the expression for the mass. For example, after cavity independent mass renormalizations have been performed there will remain a finite, R-dependent piece of the self-energy. Similarly there should be an R-dependent energy associated with zero point motion. Finally, there is almost certainly some correction to the mass because of spurious c.m. motion of the three quarks. All of these (and possibly other) contributions are usually included in a phenomenological contribution to the mass, $-Z_0/R$, with Z_0 constant -- typically $Z_0 \approx (1.3-1.8)$. We refer to I, and to Breit (1982) and Baacke and co-workers (1983) for further discussion of the origin of Z_0 .

For reasons essentially the same as those presented by De Rújula, Georgi and Glashow (1975), it is assumed in the MIT bag model that the long range confining force resulting from QCD is spin-independent. In order to preserve asymptotic freedom in a simple way, the only interaction allowed between quarks inside the bag is one gluon exchange. The colour electric piece of this interaction was dropped in the original MIT work -- see Breit (1982) for a more recent discussion. However the magnetic piece, which gives rise to an interaction of the form

$$\Delta H_{F-S} = -\frac{\alpha_s}{R} \sum_{i < j} \vec{\sigma}_i \cdot \vec{\sigma}_j \lambda_i^a \lambda_j^a m(m_i, m_j, R) , \quad (1.21)$$

is essential if the model is to produce reasonable spectroscopic predictions.

In Eq. (1.21) the function m is the result of a radial integration of the bag model wave functions of quarks i and j , whose masses we now allow to be different from zero. (For example, $m_s \neq 0$ is necessary in order to fit the masses of the strange mesons and baryons.) It is weakly dependent on the bag radius R . Using the fact that the observed mesons and baryons are colour singlets, we find

$$\begin{aligned} \lambda_i^a \lambda_j^a &= -8/3, \text{ baryons} \\ &= -16/3, \text{ mesons} \end{aligned} \quad (1.22)$$

and hence

$$\Delta H_{F-S} \propto \lambda \frac{\alpha_s}{R} \sum_{i < j} \vec{\sigma}_i \cdot \vec{\sigma}_j \tilde{m}(m_i, m_j, R) , \quad (1.23)$$

with

$$\begin{aligned}\lambda &= +1 \text{ baryons} \\ &= +2 \text{ mesons} .\end{aligned}\tag{1.24}$$

Clearly this implies much larger spin-dependent forces for mesons than for baryons. In addition, spin triplet states are energetically unfavoured. For the $N-\Delta$ system the interaction (1.23) moves the N up and the Δ down by equal amounts of about 150 MeV. Indeed this essentially determines $\alpha_s = 2.2$. For future reference we also mention the $\rho-\pi$ pair which would be degenerate at 650 MeV without Eq. (1.23). In its presence the ρ is moved up by about 130 MeV -- a 20% correction, which is probably not too large for first order perturbation theory. On the other hand for the spin-zero pion $\langle \vec{\sigma}_i \cdot \vec{\sigma}_j \rangle$ is negative and three times as big, and hence it gets shifted down by about 400 MeV! Clearly one-gluon-exchange is not going to be a very good approximation in this case and one should consider going to higher order -- see, for example, Goldman and Haymaker (1981), Miransky and Fomin (1981) and Brockmann, Weise and Werner (1983). This observation will be important when we come to consider mechanisms for dynamically breaking chiral symmetry.

Summary

For massless quarks the eigenfunctions of the MIT bag model were given in Eq. (1.4), with the quantized energies determined by Eq. (1.7). (It is a simple extension to write down the corresponding solutions for non-zero mass, for example, for the strange quark with $m_s \sim 200-300$ MeV -- see I.) Including the additional terms mentioned above, the total mass of a bag state is

$$M(R) = \frac{\sum_i \omega_i}{R} + \frac{4\pi}{3} R^3 B - Z_0/R + \Delta H_{F-S} .\tag{1.25}$$

The radius of the state is determined by the stability condition (1.18).

Now it is a very attractive feature of the MIT bag model that it can be summarized in a QCD-like Lagrangian density

$$\begin{aligned}\mathcal{L}_{\text{MIT}} = & \left[i \bar{q} \not{\partial} q - \bar{q} m q - \frac{1}{4} F^{a, \mu\nu} F_{\mu\nu}^a + \right. \\ & \left. + g \bar{q} A_{\mu}^a \gamma^{\mu} \lambda^a q - B \right] \theta_V - \frac{1}{2} \bar{q} q \delta_s .\end{aligned}\tag{1.26}$$

Here θ_V is one inside the cavity and zero outside, and δ_s is a surface δ -function. The equations of motion are obtained by demanding that the corresponding action should be invariant under arbitrary variations in q , $\bar{q}$, A_{μ}^a and the size of the cavity.

It is not our intention to discuss Eq. (1.24) at any length in these lectures except as it illustrates that the model violates chiral symmetry. Even that we postpone for a short time while we make a general point.

RELEVANCE TO NUCLEAR PHYSICS

Even with the refinements to the mass formula the radius of the nucleon in the MIT bag model remains at about 1 fm or a little larger. This immediately raises some questions with regard to our usual perception of nuclear physics in terms of point-like nucleons interacting through two-body potentials. Since the average inter-

nucleon separation is about 2 fm at nuclear matter density (ρ_0) it is clear that the nucleons must overlap with each other a good fraction of the time. How then is it possible that the independent particle shell model works so well, or indeed that within 1% $m_A = A m_N$?

Baym (1979) has presented this problem another way. He considered first some cubic blocks, some Cu and some wooden, arranged on an infinite cubic lattice. When 31% of the blocks are Cu there must be at least one infinite conducting chain. For spherical balls arranged on regular lattices the critical density is $15 \pm 1.5\%$. While finally for conducting blocks alone, arranged at random, when they occupy 34% of space one is guaranteed an infinite conducting network.

By analogy one would expect that when bags touch a colour current could flow. Thus at a density of

$$\rho_c = 0.34 / \left(\frac{4\pi R^3}{3} \right) \text{ nucleons/fm}^3, \quad (1.27)$$

nuclei might begin to show non-trivial collective effects, associated with the linking together of a chain of bags. With $R = 1$ fm ρ_c is one half of nuclear matter density! Of course if R drops to 0.8 fm, $\rho_c = \rho_0$ and so on. Clearly the bag size must be a critical parameter in such a discussion, and there has been considerable controversy over this issue.

Brown and Rho (1979) originally proposed that the coupling of Goldstone pions to the MIT bag -- next section -- might compress it to perhaps $1/10$ th the volume it would otherwise have (~ 0.3 fm). In that case $\rho_c \gg \rho_0$ and quark physics is essentially irrelevant to nuclear physics -- except through the coupling constants and masses of the heavy bosons which then mediate the N-N force.

In our view the controversy over the nucleon bag size is something that can only be resolved experimentally. In lecture 2 we shall present considerable indirect evidence to suggest that $R_N \in (0.8-1.1)$ fm, while in lecture 3 we show that deep-inelastic scattering puts a lower *limit* on R_N of 0.87 ± 0.10 fm (Thomas, 1983a) -- at least in the Cloudy Bag Model. Thus to anticipate a little, the conclusion of these lectures is that it is *wrong* [not just in principle, as stated by Brodsky (1982), but for very simple physical reasons] to think of nuclei as composed of isolated nucleons exchanging mesons. In fact, the nucleon radius is about half of the average internucleon separation in nuclear matter, and if one may speculate a little, this is probably not an accident. Almost certainly the saturation properties of nuclear matter are intimately linked to the overlap of finite size quark bags.

Let us now turn to the discussion of chiral symmetry breaking in the bag model where this controversy had its beginnings.

CHIRAL SYMMETRY IN THE BAG MODEL

Let us return to the Lagrangian density (1.26) describing the MIT bag model, and keep those pieces involving only the quark fields.

$$\mathcal{L} = (i\bar{q}\not{\partial}q - B)\theta_V - \frac{1}{2} \bar{q}q\delta_S. \quad (1.28)$$

We recall that in general for every transformation which leaves $\mathcal{L}$ invariant there is a conserved current. It is a simple exercise for example to show that (1.28) is invariant under rotations of u into d quarks, for example,

$$\begin{aligned} q &\rightarrow q + i\vec{T} \cdot \vec{a} q, \\ \bar{q} &\rightarrow \bar{q} - i\bar{q} \vec{T} \cdot \vec{a}. \end{aligned} \quad (1.29)$$

Therefore there is a conserved vector current

$$I_a^\mu = \bar{q} \gamma^\mu (\tau_a/2) q \theta_V, \quad (1.30)$$

associated with the MIT bag ($\partial_\mu I_a^\mu = 0$).

On the other hand under chiral transformations

$$\begin{aligned} q &\rightarrow q + i \frac{\vec{\tau} \cdot \vec{\epsilon}}{2} \gamma_5 q, \\ \bar{q} &\rightarrow \bar{q} + i \bar{q} \gamma_5 \frac{\vec{\tau} \cdot \vec{\epsilon}}{2}, \end{aligned} \quad (1.31)$$

the piece of Eq. (1.28) involving δ_S is not invariant. Thus the corresponding axial current

$$A_a^\mu = \bar{q} \gamma^\mu \gamma_5 (\tau_a/2) q \theta_V \quad (1.32)$$

is not conserved. In fact, a simple calculation shows that

Fig. 1. Illustration of the intrinsic violation of chiral symmetry in the MIT bag model.

$$\partial_\mu A_a^\mu = - \frac{i}{2} \bar{q} \gamma_5 \tau_a q \delta_S. \quad (1.33)$$

Thus the MIT bag model, while incorporating two of the three basic properties of QCD [Eq. (1.2)] badly violates the third. Furthermore, as illustrated in Fig. 1, this violation is an intrinsic property of the model. Because quarks are confined they must be reflected at the boundary, and when they are reflected their helicity is flipped. Technically the problem is associated with the $\bar{q} q \delta_S$ piece of Eq. (1.28), which is essential in order to get the linear boundary condition (1.14).

As we mentioned already, this problem was recognized almost immediately by Chodos and Thorn (1975) and Inoue and Maskawa (1975). They proposed a solution along the lines of the classical σ -model of Gell-Mann and Lévy (1960). That is, they introduced four new fields ($\sigma, \pi_i, i \in 1, 2, 3$) which had appropriate transformation properties under $SU(2)_L \times SU(2)_R$ so that the new Lagrangian was invariant. The new Lagrangian density replacing Eq. (1.28) is then

$$\begin{aligned} \mathcal{L}' = & (i \bar{q} \not{\partial} q - B) \theta_V \\ & - \frac{1}{2} \bar{q} (\sigma + i \vec{\tau} \cdot \vec{\pi} \gamma_5) q \delta_S + \frac{1}{2} (\partial_\mu \vec{\pi})^2 \\ & + \frac{1}{2} (\partial_\mu \sigma)^2. \end{aligned} \quad (1.34)$$

The appropriate transformations of σ and π under Eqs. (1.29) and (1.31) may be found in I, together with the expressions for the corresponding conserved vector and axial-vector currents.

Notice that in order to preserve the chiral symmetry the new scalar field σ , and pseudoscalar-isovector field π , must be massless. The pion is the obvious candidate as the latter field -- its mass being much less than other hadrons. On the other hand there is no realistic candidate for the σ -field. For this reason it is quite common to eliminate it by a very simple trick. That is, the combination $(\sigma^2 + \pi^2)$ happens to be invariant under the transformations of $SU(2) \times SU(2)$. By setting $(\sigma^2 + \pi^2)$ equal to some constant, f^2 , we can then eliminate all mention of σ in a *chiral invariant way*. The resulting theory is however necessarily non-linear. Indeed there is not one resulting theory but an infinite number, depending on how σ is eliminated. For example, one could eliminate it by substituting

$$\sigma = \sqrt{f^2 - \pi^2} , \quad (1.35)$$

in Eq. (1.34). Alternatively, one could introduce a new pion field, ϕ , through the definitions

$$\begin{aligned} \sigma &= f \cos (\phi/f) \\ \pi &= f \hat{\phi} \sin (\phi/f) , \end{aligned} \quad (1.36)$$

and so on.

To first order in the pion field all of these interactions lead to a surface coupling of the pion field to the confined quarks of the following form

$$H_{\text{int}} = - \frac{i}{2f} \bar{q} \gamma_5 \phi q \delta_s . \quad (1.37)$$

However, in higher order, they can be quite different and it is essentially an empirical problem to find which best represents Nature.

LECTURE 2

THE CLOUDY BAG MODEL

INTRODUCTION

In the last lecture we briefly showed that one could formally restore chiral symmetry to the MIT bag model by coupling an apparently elementary pion field to the surface of the bag. At first sight this seems even further removed from our ideal starting point, QCD. In reality this is not the case. On very general grounds we have already argued that QCD must produce very light pseudoscalar mesons in order to break the symmetry of the vacuum. The effects associated with such bosons can never be obtained in perturbation theory in terms of the original quark fields. Of course, it is truly an idealization to treat the pion as pointlike. This should not be taken to mean that one should expect to see pointlike spin-0 objects in deep-inelastic scattering (DIS) on nucleons. Instead it limits the range of momentum transfer (Thomas, 1983c; Gasser and Leutwyler, 1982) within which the model can be used without some more sophistication -- see lecture 3 for an example of how to extend the model for DIS.

Unfortunately, once we have agreed to construct a chiral symmetric bag model with quarks and pions, the controversy begins. In order to illustrate the difficulty consider the particular realization following from Eqs. (1.34) and (1.36). [For full details see Thomas (1983b) -- referred to as I.] The full non-linear Lagrangian density is

$$\begin{aligned} \mathcal{L}_{\text{CBM}}(x) = & (i\bar{q}\not{\partial}q - B)\theta_V \\ & - \frac{1}{2} \bar{q} \exp(i\vec{\tau} \cdot \vec{\phi} \gamma_5 / f) q \delta_S \\ & + \frac{1}{2} (D_\mu \phi)^2, \end{aligned} \quad (2.1)$$

where $D_\mu \phi$ is the covariant derivative

$$\begin{aligned} D_\mu \phi &= \partial_\mu \phi - [1 - j_0(\phi/f)] \hat{\phi} \times (\partial_\mu \phi \times \hat{\phi}), \\ &\equiv (\partial_\mu \phi) \hat{\phi} + f \sin(\phi/f) \partial_\mu \hat{\phi}, \end{aligned} \quad (2.2)$$

and as usual

$$\phi = |\phi|; \quad \hat{\phi} = \phi/\phi. \quad (2.3)$$

The conserved axial current associated with this is

$$\begin{aligned} A^\mu &= \bar{q} \gamma^\mu \gamma_5 \vec{\tau} / 2q\theta_V - f \hat{\phi} (\partial^\mu \phi) \\ &- f \frac{\sin(2\phi/f)}{(2\phi/f)} \hat{\phi} \times (\partial^\mu \phi \times \hat{\phi}), \end{aligned} \quad (2.4)$$

from which by expanding to lowest order in ϕ we recognize f as the pion decay constant, $f = 93 \text{ MeV}$.

Now the two major models, the Little Bag Model (LBM) and the Cloudy Bag Model (CBM) differ in two essential respects. In the former it is *assumed* (mainly with a view to avoiding the difficulties in nuclear physics to which we referred last lecture) that:

LBM(a) the pion is strictly forbidden from the interior of the bag, so that there is a clear separation of phases;
and LBM(b) the non-linear coupling of the pion is essential, and compresses the bag to perhaps $1/10$ th of the volume that it has in the MIT bag model.

On the other hand, in the CBM it is *assumed* that:

CBM(a) to a first approximation the pion should be allowed to propagate freely inside the bag; and CBM(b) the size of the bag is determined by non-linear effects in QCD unrelated to the pion, whose effects can then be treated as a weak perturbation on the MIT bag model.

Note that whatever may be said later to motivate either choice, both models are truly phenomenological and involve working hypotheses which can only be accepted or discarded on the basis of comparison with experimental data. As we explained in lecture 1, the present state of theoretical QCD cannot eliminate one model or the other.

From a very practical point of view the difference is that in the LBM one keeps all the non-linearity of Eq. (2.1) and multiplies the pion kinetic energy term by $\theta_{\bar{v}}$ (which is zero inside and one outside the bag). In the CBM there is no $\theta_{\bar{v}}$ and we expand systematically in powers of ϕ/f . Clearly the latter is far easier to use in practical calculations. Indeed very few LBM calculations have actually retained condition LBM(a), excluding the pion -- see Chin (1982) for a rare example. Furthermore, the full non-linearity has only been kept in unphysical examples like the hedgehog which conserves neither isospin, charge nor angular momentum.

We shall leave the description of the merits of the LBM to its major proponents (Brown and Rho, 1979; Brown, Rho and Vento, 1979, 1980; Brown, 1982; Vento and others, 1980), and concentrate on the CBM from this point.

Motivation

The primary motivation for the working assumption CBM(b) was the success of the MIT bag model itself. Given the fact that this model incorporates two of the three properties (1.2), and reproduces a lot of data, it would be almost indecent if it turned out to be completely wrong. Thus, preferring to believe that Nature is not devious, we have made the minimal extension of the bag model necessary to guarantee chiral symmetry. The pionic effects are treated as quantum fluctuations about the stable MIT bag.

The first assumption, CBM(a) has proven most controversial even though it is a technical matter. If we view the MIT bag as an approximate classical solution corresponding to a saddle point in a path integral formulation of QCD, it is clear that there will be quantum corrections, and that the expansion functions for these fluctuations should be calculated in some effective potential. The LBM assumes that this effective potential gives the pion an infinite mass inside the bag, while the CBM assumes that the effective potential is negligible. In practice the former is very difficult to handle because the expansion functions are complicated. Indeed, in many applications of the LBM the exclusion of the pion from the bag interior has been abandoned in the numerical work.

The first advantage of the CBM is therefore its simplicity of application. One simply expands in plane waves. A somewhat deeper reason for this choice is that it automatically preserves the correct prediction of g_A in the MIT bag model. To see why, consider Eq. (2.4) for the axial current to lowest order in ϕ

$$\vec{A}_i^{\text{CBM}} = \vec{q}\gamma_5 \frac{\tau_i}{2} q\theta_V - f\vec{\nabla}\phi_i. \quad (2.5)$$

If we take the matrix element in a dressed nucleon and integrate over all space, we know that

$$\int d^3x \vec{A}_i(x) = g_A \frac{\vec{\sigma}\tau_i}{2}, \quad (2.6)$$

with g_A the axial charge (1.26 experimentally).

We can, of course, replace the integral of $\vec{\nabla}\phi$ by a surface integral which vanishes. Thus in the CBM g_A is given only by the confined quarks in the MIT bag, and hence $g_A = 1.09$ (or 1.31 with centre-of-mass corrections) -- see Chodos and others (1974b) and Donoghue and Johnson (1980). If on the other hand we exclude the pion from the bag interior,

$$\vec{A}_i^{\text{LBM}} = \vec{q}\gamma_5 \frac{\tau_i}{2} q\theta_V - f\vec{\nabla}\phi_i\theta_{\vec{V}}, \quad (2.7)$$

then the integral over the pion field reduces to an integral of the pion source over the bag surface, which in turn is related to the axial current of the quarks. Overall the effect is to augment g_A in the LBM by about 50% (Jaffe, 1979). Thus $g_A = 1.63$ in the LBM (1.85 with c.m. corrections), and a major improvement of the bag over the NRQM (which gives 1.66) is lost. The essential point is that there should be no discontinuity in ϕ .

More recently a compromise has been proposed by Vento (1983) and by Chin and Miller (1983), wherein the pion is excluded from a smaller cavity [of radius R_{ch} , $\xi = R_{ch}/R \in (0,1)$] inside the bag. In this case one finds

$$g_A = g_A^{\text{MIT}} (1 + \xi^3/2), \quad (2.8)$$

and the agreement with experiment is not badly affected provided $\xi \lesssim 0.8$. Once again for this model the numerical calculations are difficult. Indeed, the πNN form-factor was not given correctly until this year [Guichon, Miller and Thomas (1983)]. Moreover, since most physical quantities involve an integral in which the pion field is weighted by a power of r , the numerical differences between the CBM and this hybrid model should be no greater than for g_A -- and hence unimportant for $\xi \lesssim 0.8$.

To summarize, we stress that the motivation for the CBM is intimately connected with the phenomenological success of the MIT bag model. We have made the minimal changes necessary to guarantee essential symmetry properties, and the agreement with a fundamental piece of data -- namely the axial charge of the nucleon.

PIONS IN THE BAG: AN ALTERNATE FORMULATION

Up to this point we have avoided epistemological discussion, preferring to stay close to data in formulating the CBM. However, it must be said that there are theoretical indications that pions should exist inside the bag. First, the fixed bag surface is a mathematical idealization and it is certain that the actual sur-

face would be deformed when hit by a pion, which would therefore penetrate. Secondly, it has been suggested that iterated one-gluon exchange (which is extremely attractive for the pion) could lead to dynamical symmetry breaking -- Goldman and Haymaker (1981) and Miransky and Fomin (1981). Such a mechanism could lead to coherent $q\bar{q}$ -pairs with pion quantum numbers inside a nucleon bag. The effects of such pairs could never be obtained in perturbation theory!

Finally we recall Shuryak's (1983) argument that chiral symmetry breaking could occur on a much smaller scale than confinement. The pion would then be produced by dynamical symmetry breaking as a result of instanton effects deep inside the bag. At first sight this picture may seem almost orthogonal to the CBM, but appearances can be deceptive. In fact the CBM with its surface coupling can be transformed to look very much like the picture proposed by Shuryak.

In order to see this we follow the discussion of Thomas (1981) -- see I for more algebraic detail, and Szymacha and Tatur (1981) for a similar model. The idea is to define a new, dressed, quark field q_w as

$$q_w = Sq = \exp(i\vec{\tau} \cdot \vec{\phi} \gamma_5 / 2f) q . \quad (2.9)$$

Then it is straightforward to show that the surface coupling in Eq. (2.1) becomes simply $-1/2 \bar{q}_w q_w \delta_s$. On the other hand, the kinetic energy term written in terms of the new fields ($q = S^\dagger q_w$, $\bar{q} = \bar{q}_w S^\dagger$) is more complicated. After quite a bit of algebra (see I) we obtain

$$i\bar{q}\not{\partial}q\theta_V = i\bar{q}_w\not{\partial}q_w\theta_V + \frac{1}{2f} \bar{q}_w \gamma^\mu \gamma_5 \vec{\tau} q_w \theta_V \cdot D_\mu \vec{\phi} , \quad (2.10)$$

where $D_\mu \vec{\phi}$ is the covariant derivative given in Eq. (2.2), and the corresponding derivative on the new quark fields $\not{\partial}$ is

$$\not{\partial} q_w = \not{\partial} q_w - i \frac{[\cos(\phi/f) - 1]}{2} \vec{\tau} \cdot (\hat{\phi} \times \not{\partial} \hat{\phi}) q_w . \quad (2.11)$$

The new Lagrangian density is therefore

$$\begin{aligned} \mathcal{L}'_{CBM}(x) = & (i\bar{q}_w\not{\partial}q_w - B)\theta_V - \frac{1}{2} \bar{q}_w q_w \delta_s \\ & + \frac{1}{2} (D_\mu \vec{\phi})^2 + \frac{1}{2f} \bar{q}_w \gamma^\mu \gamma_5 \vec{\tau} q_w \theta_V \cdot (D_\mu \vec{\phi}) . \end{aligned} \quad (2.12)$$

To order (ϕ/f) this is simply the MIT bag model for the new fields q_w , with pseudo-vector coupling to the pion field throughout the volume of the bag

$$H_{PV} = \frac{1}{2f} \int d^3x A_{quarks}^\mu(\underline{x}) \partial_\mu \phi(\underline{x}) . \quad (2.13)$$

For a transition in which the quark does not change its radial orbit (e.g. $N \rightarrow N\pi$, $\Delta \rightarrow N\pi$, etc.) this gives rise to exactly the same form-factor as the surface coupling Eq. (1.37), namely

$$u(kR) = 3j_1(kR)/kR . \quad (2.14)$$

The strength of the coupling as $k \rightarrow 0$ is obviously given by the axial charge [the volume integral of $\vec{A}(x)$] divided by $2f$. That is,

$$\frac{g}{2m_N} = \frac{g_A^{\text{MIT}}}{2f}$$

$$m_N g_A^{\text{MIT}} = fg, \quad (2.15)$$

which is the Goldberger-Treiman relation.

Notice that in any chiral bag model the πNN form-factor arises because the pion couples to an extended hadron, of size R , which provides a natural high momentum cut-off. Of course the actual form (2.14), with its oscillatory behaviour at large k , arises because of the fixed bag surface. As shown in Fig. 2 it can be very well approximated by a Gaussian (for $kR \lesssim 3$)

$$u(kR) \approx \exp(-0.106 k^2 R^2), \quad (2.16)$$

Fig. 2. The $NN\pi$ vertex function in the CBM, compared with the "best fit" exponential form-factor given in Eq. (2.) -- from Thberge and Thomas (1983).

which gives the same answers to within 1% for all static properties, and may even be more realistic at large values of the pion momentum. Finally, we must point out that the form-factors arising from (2.13) and (1.37) differ in the case where the quark orbital changes as a result of pion absorption or emission. There is some indirect evidence [Eisenberg and Kalbermann (1983)] that Eq. (2.13) may be better phenomenologically.

If we go to next order in ϕ/f , the covariant derivative on q_w gives rise to an interaction

$$H_s = \frac{1}{4f^2} \int d^3x \bar{q}_w \gamma^\mu \tau_w q_w \theta_V \cdot (\phi \times \partial_\mu \phi) , \quad (2.17)$$

which can scatter pions from the bag. In particular, for pion-bag scattering at threshold ($\vec{k} = 0$), only $\partial_0 \phi \equiv \pi$ is non-zero, and the interaction takes the form

$$H_s = I_t \cdot \hat{i}_\pi / 2f^2 . \quad (2.18)$$

Here I_t is the isospin of the target. Equation (2.18) is in fact the Weinberg-Tomozawa result for pion-nucleon scattering. It is quite a general result following from current algebra, and the fact that we obtain it is an indirect indication that the CBM satisfies correct commutation relations. What is remarkable, however, is how easily the result is obtained from the new Lagrangian density. It is very difficult to see how to extract it from the original Lagrangian density (2.1).

There has so far been no application of Eq. (2.17) to πN s-wave scattering at higher energies, although this should definitely be tried. The $SU(3)_L \times SU(3)_R$ generalization of Eq. (2.12) has been applied to the coupled $\bar{K}p - \Sigma\pi$ system at threshold -- that is, in the region of the $\Lambda'(1405)$ -- although so far the results are only preliminary [Barrett and co-workers (1983)].

It is important to stress that both q and q_w *cannot* satisfy canonical commutation relations. That is one can choose either Eq. (2.1) or (2.12) as the starting point but not both. Nevertheless to order ϕ/f they are identical in many applications, and presumably one could think of the q_w as in some sense a "quasi-quark". In this regard it is remarkable how similar the new model is to that proposed by Shuryak (1983). In both cases the quarks can be thought of as dressed by higher order effects, and the pion field couples to these quarks throughout the confining volume. While we find Eq. (2.12) more attractive in many ways, it was discovered later than Eq. (2.1), and most of the applications we discuss in the rest of this lecture use the latter. Fortunately the (limited) equivalence at $O(\phi/f)$ means that most of our results would not be altered by changing to Eq. (2.12).

THE NUCLEON

Our approach to calculating the pionic corrections to the MIT bag model of the nucleon was described at length in I. Therefore we shall just summarize the key results and report only recent results in more detail.

Briefly the idea is to expand Eq. (2.1) to $O(\phi/f)$, in which case it breaks into three terms corresponding to the usual MIT bag, a free pion field, and an interaction term. From $\mathcal{L}(x)$ we can construct $T^{00}(x)$ and hence the Hamiltonian, which has the form

$$H = H_{MIT} + H_\pi + H_{int} . \quad (2.18)$$

If for the present we work only in the P-space of (non-exotic) three-quark baryon bags -- one can do the same thing for mesons other than the pion -- we find (in the notation of I)

$$H_{int} \rightarrow PH_{int}P = (2\pi)^{-3/2} \sum_{\alpha, \beta, i} \int d^3k (v_{ki}^{\beta\alpha} a_{\beta}^{\dagger} a_{\alpha} + \text{h.c.}) . \quad (2.19)$$

Here $\alpha(\alpha^+)$ is a destruction (creation) operator for the non-exotic bag state $|\alpha\rangle$, a_{ki} destroys a pion of momentum k and isospin i , and the coefficient is given by the matrix element of Eq. (1.37) between bag states $|\alpha\rangle$ and $|\beta\rangle$. For example, using bag model SU(6) wave functions we find

$$\begin{aligned} v_{ki}^{NN} &= i(4\pi/2w_k)^{1/2} (f_{NN\pi}^{(0)}/m_\pi) u(k) \vec{\sigma} \cdot \vec{k} \tau_i, \\ v_{ki}^{\Delta N} &= i(4\pi/2w_k)^{1/2} (f_{\Delta N\pi}^{(0)}/m_\pi) u(k) \vec{S} \cdot \vec{k} T_i, \\ v_{ki}^{\Delta\Delta} &= i(4\pi/2w_k)^{1/2} (f_{\Delta\Delta\pi}^{(0)}/m_\pi) u(k) \vec{S} \cdot \vec{k} T_i. \end{aligned} \quad (2.20)$$

Here $\vec{S}$ and T are transition spin and isospin operators describing a change from spin $1/2$ to $3/2$. The vertex function $u(k)$ was given in Eq. (2.14) and the bare baryon-pion couplings are in the ratios

$$f_{NN\pi}^{(0)} : f_{\Delta N\pi}^{(0)} : f_{\Delta\Delta\pi}^{(0)} = 1 : \sqrt{72/25} : 4/5. \quad (2.21)$$

The physical nucleon $|\tilde{N}\rangle$ is then the solution of the equation

$$H|\tilde{N}\rangle = m_N |\tilde{N}\rangle, \quad (2.22)$$

which formally can be expanded in terms of bare bag states (e.g. $|N\rangle$, $|\Delta\rangle$) in the form

$$|\tilde{N}\rangle = Z^{1/2} |N\rangle + c |N\pi\rangle + c' |\Delta\pi\rangle + \dots \quad (2.23)$$

Of course such an expansion is only useful if it converges relatively quickly and that is indeed the case for the CBM. If we restrict the bare bag states to N and Δ , it has been shown by Dodd, Alvarez-Estrada and Thomas (1981) that the probability of finding n pions in $|\tilde{N}\rangle$ is bounded by $\Lambda^n/n!$, with $\Lambda = 0.68$ for the MIT bag radius. (At the CBM radius determined from pion nucleon scattering, $R = 0.82$ fm [Thomas, Thberge and Miller (1981)] we find $\Lambda = 0.9$.) Thus $P_1 \leq 68\%$, $P_2 \leq 23\%$, and P_3 is bounded by 5%. Furthermore, the average number of pions is bounded by Λ . Explicit calculation in the case $R = 0.82$ yields $P_1 = 35\%$, $P_2 \sim 5\%$, and $P_3 < 1\%$, so it is clear the bound could be improved even more. Nevertheless, the convergence properties of the CBM are remarkable. The pion cloud cover around the nucleon is more like a halo!

The Nucleon Self-Energy

With confidence in the convergence of Eq. (2.23) we can evaluate pionic self-energy corrections to the mass of the nucleon -- see Fig. 3. The first term of Fig. 3 was considered explicitly by Gasser and Leutwyler in their "improved chiral perturbation theory". Certainly we agree very strongly with their idea that highly excited intermediate states should be cut out in order not to double count higher order QCD effects. (The recent storm in a tea-cup over the so-called divergence of the nucleon self-energy seems to us to be misguided. However, it is true that the CBM gave no recipe for cutting off those contributions involving intermediate states with one quark highly excited. It is precisely because of this ambiguity that we placed little emphasis on spectroscopy as a way to determine bag size.) On the other hand, the Δ intermediate state does not involve a change of orbital or angular momentum quantum number and should certainly be calculated at the same level as the N intermediate state.

The total contribution of Fig. 3 is about -200 MeV at $R = 1$ fm and about -400 MeV at 0.82 fm. (In general it behaves as $\sim R^{-3.5}$ when form-factor and pion mass

Fig. 3. Pionic self-energy contribution to the mass of the nucleon involving intermediate nucleon and delta bag states [from Thberge and Thomas (1983)].

effects are included.) Most important from the point of view of the comparison with QCD, the corresponding pionic self-energy for the Δ is somewhat smaller. Thus part of the N - Δ splitting which was attributed to gluon-exchange in the MIT bag model actually comes from the pionic or chiral corrections, and one can use a value of $\alpha_c \sim 1.5$ rather than 2.2 -- see I for details. While still very large, at least the change is in the right direction.

The Renormalized $NN\pi$ Coupling Constant

Because the Goldberger-Treiman relation (2.15) implies that the bare πNN coupling constant is rather close to the observed renormalized coupling constant, it is important to consider this renormalization in the CBM. This involves computing the wave function renormalization [Z in Eq. (2.23)], and the vertex renormalization shown in Fig. 4. In the Chew-Low model the latter does not compensate for the reduction caused by Z and $(f(r)/f(0))^2$ is about $1/3$ [Henley and Thirring (1962)].

Fig. 4. Vertex renormalization of the $NN\pi$ coupling constant in the CBM [from Thberge and Thomas (1983)].

In the CBM, however, the presence of the explicit Δ (in Fig. 4 in particular), combined with the softer form-factor, means that the renormalization is very small for bag radii near 1 fm. To illustrate this we show in Fig. 5 the unrenormalized $NN\pi$ coupling constant f_0^2 , required to reproduce the observed, renormalized coupling constant for a given bag radius. For $R \gtrsim 0.8$ fm the bare and renormalized coupling constants are within 10% of each other! Thus conventional strong interactions are amenable to low order perturbation theory!

Fig. 5. The unrenormalized $NN\pi$ coupling constant needed to reproduce the observed renormalized coupling as a function of the bag radius in the CBM [from Thberge, Miller and Thomas (1982)].

Electromagnetic Properties

Given the rapid convergence of the perturbation expansion (2.23), it is natural to use the model to calculate the electromagnetic properties of the nucleon [DeTar (1981a, 1981b); Thomas, Thberge and Miller (1981); Thberge, Miller and Thomas (1982)]. Here we shall concentrate on the magnetic moment of the nucleon in the CBM. Figure 6 shows the diagrams which must be evaluated.

It is important to notice that the only parameter in these calculations is the bag radius. (That could also be fixed once we have a consistent scheme for cutting-off highly excited intermediate states in the evaluation of the pionic self-energy -- as discussed earlier.) We typically show results for R between 0.7 fm, where first order perturbation theory is not so reliable, up to 1.1 fm.

One final caution concerns the centre-of-mass problem. As shown by Carlson and Chachkhunashvili (1981), this correction is very sensitive to the quark wave functions in the surface of the bag. Even the sign is not agreed upon. Nevertheless, with some slight smoothing of the bag wave functions in the surface, their results were close to those of Donoghue and Johnson (1980). In order to avoid the controversy we show the CBM predictions for μ_p and μ_n both with (solid) and without the Donoghue-Johnson correction. [For a thorough discussion of the more sophisticated question of the cancellation between retardation and spin-precession we refer to the work of Guichon (1983).]

In order to put the results shown in Fig. 7 in perspective, we note that the original MIT bag gave $\mu_p \sim 1.9 \mu_N$ and $\mu_n \sim -1.26 \mu_N$ without c.m. corrections ($2.24 \mu_N$ and $-1.49 \mu_N$, respectively with the c.m. correction). Expressed as a ratio to the experimental values (as in Fig. 7) this would be (0.68, 0.66) [and (0.80, 0.78)]

Fig. 6. Contribution to the magnetic moment of the nucleon from (a) the quark current, (b) the pion current with intermediate nucleon present, (c) the pion current with intermediate delta present [from Thberge, Miller and Thomas (1982)].

with c.m. corrections]. Furthermore, there are corrections due to the creation of virtual $q\bar{q}$ pairs in the bag whose size is generally found to be of order $(0.2-0.3)\mu_N$, but which are not yet generally agreed upon -- e.g. see Maxwell and Vento (1983) and Donoghue and Golowich (1977). Thus at the present stage of the theory, agreement with data at the 10% level should be considered quite good.

Clearly the results shown in Fig. 7 are a *significant quantitative improvement* over the original bag model. The best agreement with data is undoubtedly found when the c.m. correction is included -- although this is no argument in favour of including it. It may be instead that there is other physics at the 10% level -- for example, a small residual rho-meson coupling.

It is remarkable that the results for both μ_p and μ_n in the CBM are quite insensitive to variations in R , the bag radius. As R decreases the pionic corrections increase to compensate the reduction in the bare bag contribution (both Z and μ^{MIT} decrease as R decreases). This illustrates a point which we have made often, but

Fig. 7. Predictions of the CBM for the magnetic moments of the proton and neutron, as a function of the bag radius, with (solid) and without (dash) centre-of-mass corrections [Théberge and Thomas (1983)].

which has not been widely appreciated. That is, the pionic corrections are *not* purely isovector because of the coupling to the extended core while the pion is in the air! One must calculate all of the terms shown in Fig. 6 in order to obtain a reliable result.

The Axial Form-Factor

As we explained in detail at the beginning of this lecture, in the CBM only the quarks contribute to the axial (as opposed to the induced pseudoscalar) current of the bag. Thus, unlike the electromagnetic properties for which there are pionic contributions, the axial form-factor is a direct measure of the quark distribution in the nucleon. At present the data on $g_A(q^2)$ comes from two sources, the reaction $\nu_\mu + n \rightarrow \mu^- + p$ and pion electroproduction -- see Amaldi, Fubini and Furlan (1979). It is usually represented as a dipole

$$g_A(q^2) = (1 + q^2/m_A^2)^{-2} , \quad (2.24)$$

with $m_A = 0.95 \pm 0.14$ GeV.

If we calculate $g_A(q^2)$ for the CBM we find this corresponds to a bag radius $R = 1.16 \pm 0.20$ fm [Guichon, Miller and Thomas (1983)]. Clearly there should be corrections to this value arising from c.m. and recoil effects, but as a first estimate this strongly suggests a bag size similar to that expected in the original MIT bag model.

One can also investigate $g_A(q^2)$ in the hybrid model discussed earlier, where the pion is excluded from a region $r < R_{ch}$ inside the bag [i.e. $\xi = R_{ch}/R \in (0,1)$]. For $\xi \neq 0$ the pion also contributes to $g_A(q^2)$. However, as shown by Guichon, Miller and Thomas (1983) the slope of $g_A(q^2)$ changes by less than 10% over the whole range of values of ξ . Thus the result $R = 1.16 \pm 0.20$ fm is a general result for all chiral bag models.

Finally we note that we can also calculate the πNN form-factor in the hybrid model. If we parametrize $g_A(q^2)$ as $[1 - q^2 r_A^2/6]$, and $g_{\pi NN}(q^2)$ as $[1 - q^2 r_\pi^2/6]$, then for all values of ξ $r_\pi > r_A$. That is, the πNN form-factor in all chiral bag models is softer than $g_A(q^2)$. In the CBM, where $\xi = 0$, $g_{\pi NN}(q^2)$ would correspond to a dipole of mass 0.90 ± 0.14 GeV ($r_A/r_\pi \sim 0.9$), which is very soft. For ξ in the range 0.0 to 0.8 this hardly changes, but in the range 0.8 to 1.0 $g_{\pi NN}(q^2)$ becomes rapidly softer, with r_A/r_π dropping to 0.65 and the corresponding dipole mass to about 0.76 GeV!

Clearly it would be very valuable to have more precise data for $g_A(q^2)$. Nevertheless, even at the present accuracy, we regard the arguments which we have just reviewed as the most direct indication (apart from the discussion of DIS in lecture 3) that the nucleon bag is of the order of 1 fm in radius.

MAGNETIC MOMENTS OF THE NUCLEON OCTET

As a further test of the CBM one can also calculate the pion couplings to the hyperons (Λ , Σ , Σ^* , etc.). Then we can calculate the properties of the hyperons using exactly the same perturbative expansion as for the nucleon, e.g.

$$|\tilde{\Lambda}\rangle = \sqrt{Z_\Lambda} |\Lambda\rangle + c |\Sigma\pi\rangle + c' |\Sigma^*\pi\rangle. \quad (2.25)$$

Just as in the case of the nucleon, we include the coupling to those baryons where no orbital or angular momentum excitation is involved. For example, for the Σ we include $\pi\Lambda$, $\pi\Sigma$ and $\pi\Sigma^*$ as possible coupled channels.

The corresponding probabilities of finding a bare three-quark bag in the physical baryon are shown in Fig. 8 as a function of the bag radius. Even for bag radii near 1 fm there is a sizeable pionic component in the physical Λ and Σ . It will be very interesting to see how this modifies the description of hyperon decays in the bag model.

As a further indication of the size of pionic corrections, in Fig. 9 we show the ratio of the "bare bag mass $m_{0\Lambda}$ ", to the observed baryon mass m_Λ , as a function of the radius of the bag. Our operational definition of $m_{0\Lambda}$ is the physical mass minus the pionic self-energy (Fig. 2). Even at $R = 1$ fm the pionic self-energy of the Λ is of the order of -100 MeV. We shall mention this a little later in connection with the existence of possible exotic (six-quark) bound states.

Finally in Fig. 10 we show the results obtained in the CBM [Théberge and Thomas (1982, 1983)] for the magnetic moments of the Σ^+ and Σ^- hyperons. This calculation involved no free parameters because once the strange quark mass is specified (we used 144 MeV), all the $\pi YY'$ and $\gamma YY'$ couplings are uniquely determined. (There are however a lot of graphs -- see Fig. 6, and insert all possible Y or Y^* intermediate

Fig. 8. Probability of finding a bare bag in the dressed baryon, as a function of bag radius [Théberge and Thomas (1983)].

Fig. 9 Ratio of the bare mass to the observed mass of the nucleon octet. The bare mass is defined to be m_A minus the second order pionic self-energy [from Théberge and Thomas (1983)].

Fig. 10. Ratio of calculated to experimental magnetic moment as a function of bag radius. The Σ^- magnetic moment data has since been improved -- see the text [Thberge and Thomas (1983)].

states.) The experimental values were taken to be $\mu(\Sigma^+) = 2.33 \pm 0.13 \mu_N$ and $\mu(\Sigma^-) = -0.89 \pm 0.14$. More recently the Σ^- measurement using exotic atoms has become much more accurate [Roberts and Welsh (1983)], and the best value is now $\mu(\Sigma^-) = -1.10 \pm 0.03 \mu_N$. With this improved value, both the Σ^+ and Σ^- magnetic moments calculated in the CBM are in excellent agreement with the data. (Again the solid curve includes the Donoghue-Johnson c.m. correction, which is not as important here as for the proton.)

It is worth while to stress that the charged- Σ magnetic moments provide an important test of pionic corrections. Because of the near degeneracy of the Σ^0 and Λ , the pion contribution is twice as big as it might otherwise be (i.e. $\Sigma^- \rightarrow \Lambda \pi^-$ and $\Sigma^- \rightarrow \Sigma^0 \pi^-$). This was first pointed out by Eeg and Pilkuhn (1978), but they used very hard form-factors and hence obtained a value of $\mu(\Sigma^-) = -1.66 \mu_N$ which is far too large. In the CBM with $R = 1$ fm, $m_\pi = 279$ MeV (as in the original MIT work) we find $\mu(\Sigma^-) = -1.08 \mu_N$ [Thberge and Thomas (1982)] whereas the MIT bag model gave $\mu(\Sigma^-) = -0.80 \mu_N$ (including the c.m. correction).

PION-NUCLEON SCATTERING

The first major achievement of the CBM was to provide a unified framework within which to understand the P_{33} resonance. Before the analysis of Thberge, Thomas and Miller (1980), we had either the Chew-Wick description entirely in terms of the πNN coupling, or the quark model within which the three-quark Δ decayed to $N\pi$. In the CBM both $NN\pi$ and $\Delta N\pi$ couplings appear from the same Hamiltonian, with the same form-factor and related coupling constants. Without the pion-bag coupling all of the baryons N , Δ , R (Roper) and so on would be stable. However, once we impose chiral symmetry, and hence the coupling of pions, only the nucleon and the other members of the nucleon octet remain stable. All of the others become unstable and cannot be treated with any accuracy without explicit consideration of the channels to which they can decay. Thus the CBM treatment of the Δ is a prototype for understanding all of the unstable resonances within a bag model framework. Similar analyses have been made for the Roper resonance (Rinat, 1982; Nogami and Ohtsuka, 1982; Brown, Durso and Johnson, 1983).

Since the CBM of the P_{33} resonance is now quite ancient and was reviewed extensively in I we shall not repeat ourselves.

Perhaps the only point which should be emphasized is that contrary to claims in the literature the ratio $f_{\Delta N\pi}:f_{NN\pi} = (72/25)^{1/2}$ does explain the observed width of the delta provided one includes *all* the relevant diagrams [Thberge, Thomas and Miller (1980); Thomas, Thberge and Miller (1981); Niskanen (1981)]. This is of some relevance to other lecturers at this school when fixing their parameters for the calculation of (e.g.) the suppression of Gamow-Teller strength.

EXOTICS

One of the more exciting possibilities raised by the MIT bag model was that there might be stable, exotic states. For example, it was suggested that the so-called H-dibaryon (a $\Lambda\Lambda$ state) might be bound by (50-80) MeV -- Jaffe (1977). In view of the relatively large self-energy corrections associated with pions for single hadrons (see Fig. 9), it is reasonable to ask how those corrections affect the masses of exotic states.

In order to check this in a scheme consistent with the philosophy of the CBM, Mulders and Thomas (1982) refitted the usual hadron spectrum with the phenomenological form

$$E(R) = E_Q + E_V + E_M + E_p . \quad (2.26)$$

Here E_Q , E_V , and E_M are, respectively, the standard kinematic energy, volume and colour magnetic contributions to the bag energy. The last term, E_p , is a phenomenological representation of the pion self-energy which has the form

$$E_p = \frac{-1}{pR^3} \sum_{i,j} (\vec{\sigma}_i) \cdot (\vec{\sigma}_j) . \quad (2.27)$$

The spin-isospin structure corresponds to keeping only the lowest orbital in the intermediate state, and treating all such states as degenerate. Finally, p is a phenomenological constant.

There were several notable features associated with the best fit parameters. As expected from the earlier discussion of the $N\Delta$ splitting the colour coupling constant was reduced by about 35%. The strange quark mass also came down to 218 MeV (from 279 MeV) -- a little closer to the usual current algebra value of 150 MeV.

Lastly, we observe that, although treated as an adjustable parameter, the value of p agreed very well with that calculated for a nucleon in the chiral bag models.

For the non-strange, $B = 2$, exotic bag states, the pionic corrections had little effect. In 3S_0 and 1S_0 the bag masses were 2.18 and 2.24 GeV, respectively (c.f. 2.16 and 2.23 in the original MIT bag model). Since these lie well above the appropriate thresholds they will be quite broad, and should not have dramatic experimental consequences.

On the other hand, for the doubly strange H-dibaryon the change is dramatic. The combination of decreased colour attraction (smaller α_c), and the R^{-3} dependence of the pionic self-energy result in a larger mass for the H -- 2.22 instead of 2.15 GeV. Thus the H is almost certainly unbound, and it is no longer a mystery why recent searches have failed to reveal it.

CONCLUSION

In this lecture we first reviewed the similarities and the essential differences of various chiral bag models. We then showed how the CBM could be transformed to look very much like the picture of hadron structure proposed by Shuryak. Within that framework we saw that the model incorporated both the Weinberg-Tomozawa result for low-energy pion-nucleon scattering, and the Goldberger-Treiman relation for the bare πNN coupling constant. Next we examined the structure of the nucleon in the CBM, showing that there are in fact very few pions in the cloud, that the renormalization of the πNN coupling constant is small, and that both its magnetic moments and axial form-factor are well described for a bag radius in the range 0.8 to 1.1 fm. An extension to the properties of the stable hyperons was also very successful.

We briefly reviewed the application of the CBM to pion-nucleon scattering in the region of the $\Delta(1231)$. Finally we discussed recent results for multiquark bag states. Because it would go beyond the scope of these lectures, we did not discuss one of the more exciting applications of the CBM, namely an estimate of the effect on the lifetime of the proton in the simplest SU(5) Grand Unified Theory of imposing chiral symmetry (Thomas and McKellar, 1983). In addition, we avoided any discussion of the intermediate range N-N force in the quark model, since that was discussed by Faessler at this school -- see also I. We will however raise this matter in the next lecture in connection with the EMC effect.

LECTURE 3

DEEP INELASTIC SCATTERING

INTRODUCTION

In a certain sense in this lecture we come full circle. Early in lecture 1 we referred to the fact that DIS provided the first hard evidence that there were point-like constituents, with the properties expected of quarks, inside hadrons. We then showed how phenomenological quark models suggest that we might have to take a new view of the microscopic structure of the nucleus. Indeed, if the confinement radius for the valence quarks in a nucleon is of order 1 fm, rather than the classical model of point-like nucleons interacting through potentials generated by heavy meson exchange, one would expect to find two (or more) nucleons overlapping, and therefore sharing their valence quarks a large fraction of the time. [Many authors have suggested ideas like this in the past few years, as examples and sources of further references we cite Bergström and Fredriksson (1980), Pirner and Vary (1981), Miller (1983) and Thomas (1983b).]

If such ideas make any sense one might expect to be able to test them experimentally using DIS. Therefore in this lecture we shall briefly review the ideas of scaling in DIS. This is already well treated in text books (Feynman, 1972; Close, 1979), and we shall merely summarize the main ideas. We then apply these ideas to the nucleon itself, to see whether DIS can test or constrain the chiral bag models. In fact this could be regarded as something of a challenge in view of statements like the following (Vento, 1983). "The other version of the model includes in addition ... a pion like degree of freedom in the interior of the bag ... from an experimental point of view such a model is in clear contradiction with the fact that short distance probes do not see any pseudoscalar constituent objects. Due to these inconsistencies the latter model would not be of any relevance if it were not because it seems to reproduce a fair amount of data." We shall show that while DIS gives not one jot of evidence about whether pions exist inside or outside of bags, it does put severe constraints on the form-factor at the $NN\pi$ vertex. Within the CBM it places a *lower limit* on the radius of the nucleon bag of 0.87 ± 0.10 fm (Thomas, 1983a).

Thus we almost certainly live in a universe where nucleons overlap frequently in nuclei. A smart theorist might have even expected the distribution of valence quarks in a nucleus to be modified from that in a free nucleon. Unfortunately in this case the experimenters were smarter (or luckier). The European Muon Collaboration has recently published fairly convincing evidence for just such a modification. As the finale to this set of lectures we discuss the EMC effect and speculate on how we might interpret it.

SCALING

Consider the scattering of an electron or muon of initial energy E in the laboratory to a final energy E' and scattering angle θ . According to convention we define the energy transfer $(E - E')$ to be ν , and $Q^2 = -q^2$, where q is the four-momentum transfer to the target ($Q^2 > 0$ for space-like momentum transfer). If the

lepton scatters elastically from a target of initial momentum p [$p = (m_t, \vec{0})$ in the laboratory], then

$$\begin{aligned} m_t^2 &= (p + q)^2 \\ &= m_t^2 + 2m_t \nu - Q^2 \end{aligned}$$

and hence

$$Q^2 = 2m_t \nu . \quad (3.1)$$

This is the crucial kinematic relation behind all scaling arguments, because if a target is elementary, elastic scattering is the only channel available in such a collision. It then follows that Q^2 and ν must be related by Eq. (3.1) in scattering from an elementary target.

In an electromagnetic interaction of an electron or muon, the most general form for the cross-section is

$$\sigma \sim \frac{\alpha^2}{Q^4} L_{\mu\nu} W^{\mu\nu} , \quad (3.2)$$

where $L_{\mu\nu}$ describes the lepton photon vertex and $W^{\mu\nu}$ the photon target vertex. (The factor α^2 then just tells us the charge and Q^{-4} comes from the photon propagator.) The lepton tensor is simply

$$\begin{aligned} L_{\mu\nu} &= \frac{1}{2} \text{Tr}[(K' + m)\gamma_\mu(K + m)\gamma_\nu] \\ &= 2[k'_\mu k_\nu + k_\mu k'_\nu - g_{\mu\nu}(k \cdot k' - m^2)] , \end{aligned} \quad (3.3)$$

where k and k' are the initial and final lepton four-momentum ($q = k - k'$). It is a purely kinematic factor.

On the other hand, for a general hadronic target $W^{\mu\nu}$ is defined as

$$\begin{aligned} W^{\mu\nu} &= \frac{1}{2} \sum_n \langle p | J^{\mu+} | n \rangle \delta^{(4)}(p + q - p_n) \\ &\quad \times \langle n | J^\nu | p \rangle , \end{aligned} \quad (3.4)$$

where the sum over n includes all possible hadronic final states. Using just the principles of gauge invariance and parity conservation one can show that the most general form allowed for $W^{\mu\nu}$ is

$$\begin{aligned} W^{\mu\nu} &= W_1(\nu, Q^2) [q^\mu q^\nu / q^2 - g^{\mu\nu}] \\ &\quad + \frac{W_2(\nu, Q^2)}{m_t^2} [(p^\mu - p \cdot q q^\mu / q^2)(p^\nu - p \cdot q q^\nu / q^2)] . \end{aligned} \quad (3.5)$$

Using Eqs. (3.2), (3.3) and (3.4) one can show that in general

$$\frac{d^2\sigma}{dE' d\theta} = \frac{4E'^2 \alpha^2}{Q^4} [\sin^2 \theta / 2 \, 2W_1 + \cos^2 \theta / 2 \, W_2] . \quad (3.6)$$

The arbitrary functions W_1 and W_2 , which have dimensions of $(\text{energy})^{-1}$ contain all the information we can obtain about the structure of the target from such a reaction. They are called the structure functions.

It is actually more convenient for many purposes to consider the dimensionless structure functions

$$\begin{aligned} F_1 &= m_t W_1, \\ F_2 &= \nu W_2. \end{aligned} \quad (3.7)$$

As a simple example we take the case of electron scattering from a structureless target of mass m_t . Then from the arguments presented at the beginning of this section we know that F_1 and F_2 cannot depend on Q^2 and ν independently. In fact, reference to your favourite text-book on Quantum Electrodynamics reveals that

$$\begin{aligned} 2F_1 &= \delta(1 - Q^2/2m_t\nu), \\ F_2 &= \delta(1 - Q^2/2m_t\nu). \end{aligned} \quad (3.8)$$

That is, Q^2 and ν may vary independently, but F_1 and F_2 are unaltered if the dimensionless combination x ,

$$x = Q^2/2m_t\nu \quad (3.9)$$

is the same. This phenomenon is known as scaling.

Although it is not often stressed, scaling is a fairly common phenomenon. For a very beautiful discussion we refer to the text of Close (1979). Here we content ourselves with a single example, namely electron proton elastic scattering. Once again by referring to standard texts one can find the result

$$2F_1(\nu, Q^2) = G_M(Q^2) \delta(1 - Q^2/2m_p\nu), \quad (3.10)$$

where m_p is the proton mass and $G_M(Q^2)$ its magnetic form-factor. This has the general structure $F_1 = F(Q^2)f(x)$. Clearly if one is in a range of Q^2 where $F(Q^2)$ is essentially constant, we will see scaling. In the proton example this would correspond to $1/Q \gg$ proton radius (i.e. $Q^2 \lesssim 0.01 \text{ GeV}^2$).

In order to see how DIS can reveal the substructure of a target, let us consider electron scattering from ^{56}Fe , which for the present we think of as 56 nucleons. Elastic scattering on ^{56}Fe will occur at $x_{56} = Q^2/2m(^{56}\text{Fe})\nu = 1$. On the other hand, elastic scattering on a nucleon in ^{56}Fe (ignoring small binding and Fermi motion corrections) would occur at $x_N = Q^2/2m_N\nu = 1$, which means $x_{56} \sim 1/56$.

Following the argument we gave above we would then expect that for $1/Q \gg$ size of ^{56}Fe the structure function of Fe would look like $\delta(1 - x_{56})$. That is it would exhibit scaling in terms of a structureless particle of mass $56m_N$. On the other hand, if we raise Q^2 to the region where the elastic scattering from Fe is small, but the proton form-factor is still near unity, the structure function would exhibit scaling and have the form $\delta(1/56 - x_{56})$. [This is simply quasi-elastic scattering, $(e, e'p)$.] Finally, if a nucleon consisted of three point-like quarks, we would expect that for $1/Q \ll$ size of the proton (and $\nu \gg$ excitation energy of the proton) the structure function would peak at $x_{56} = 1/168$, or $x_N = 1/3$.

Of course in any field theory no particle is ever exactly point-like, because its effective charge varies with Q^2 . However, for QCD the logarithmic decrease in the coupling constant -- see Eq. (1.1) -- means that at the quark level scaling violations happen slowly. Over quite a large range of Q^2 one can in fact write the structure function of the nucleon as a function of x alone.

THE QUARK-PARTON MODEL AND THE STRUCTURE FUNCTION OF THE NUCLEON

For the rest of this lecture, in the interest of simplicity, we shall ignore the evolution of the structure function of the nucleon with Q^2 which must happen in QCD (Altarelli and Parisi, 1977). What remains in this approximation is usually called the naive quark parton model. Its basic premises are (i) that DIS from a hadron can be described entirely in terms of the interaction with point-like, spin- $\frac{1}{2}$ constituents which are on-mass-shell, and (ii) this interaction can be treated in impulse approximation. The justification of point (i) in terms of the quark model is clear, and we will assign these constituents the charges and weak couplings expected of quarks. The second assumption is meant to incorporate asymptotic freedom, so that during the collision the quarks are free. It is only after the collision, when the outgoing lepton is far away, that the effects of confining forces determine the hadronic final state.

It is conceptually easier to think of the DIS process in the limit where both Q^2 and ν go to infinity (with $x = Q^2/2m_N\nu$ fixed). In that case one can view the reaction in an infinite momentum frame, where the nucleon has a momentum far larger than any of its constituents when it is at rest.

In this frame the nucleon four-momentum is simply $(P, 0_\perp; P)$, and each constituent has a momentum $(x_i P, 0_\perp; x_i P)$, with $x_i \geq 0$ and $\sum_i x_i = 1$. At any finite momentum transfer there will be corrections of order P^{-1} from the transverse momentum of the mass of the partons. Again we ignore these complications in the interest of simplicity.

In this simple picture all that happens in DIS muon scattering, for example, is that the exchanged photon, with definite x , can only be absorbed on a parton with $x_i = x$ [because of Eq. (3.1) and the assumption that partons are *structureless*]. Thus DIS of muons measures directly the product $q_i^2 f_i(x)$, where q_i is the charge of the quark, and $f_i(x) dx$ is the number of quarks of flavour i carrying a momentum between xP and $(x + dx)P$ in an infinite momentum frame. It follows that the entire information about the structure of a nucleon which is provided by DIS can be summarized by the twelve functions $\{u^P(x), d^P(x) \dots \bar{s}^P(x), u^n(x), d^n(x) \dots \bar{s}^n(x)\}$ -- the superscript indicates either neutron or proton.

On the basis of charge symmetry one usually reduces this to six independent quark distributions

$$\begin{aligned} u^P(x) &= d^n(x) = u(x) , \\ d^P(x) &= u^n(x) = d(x) , \end{aligned} \tag{3.11}$$

and so on, through $s(x)$, $\bar{u}(x)$, $\bar{d}(x)$ and $\bar{s}(x)$. Clearly by integrating these distributions weighted by x we can find the total fraction of the momentum of a nucleon (in an infinite momentum frame!) carried by each parton. These are labelled by capital letters $U, D \dots \bar{S}$ in an obvious notation

$$\begin{aligned} U &= \int_0^1 u(x) \times dx , \\ D &= \int_0^1 d(x) \times dx , \dots \\ \bar{S} &= \int_0^1 \bar{s}(x) \times dx . \end{aligned} \tag{3.12}$$

If this description is to make sense one would expect these momentum fractions to satisfy a sum rule, namely

$$U + D + S + \bar{U} + \bar{D} + \bar{S} = 1 - G, \quad (3.13)$$

where

$$G = \int_0^1 g(x) \times dx \quad (3.14)$$

is the fraction of the momentum of the nucleon carried by gluons.

Of course G cannot be measured directly in DIS of leptons for the obvious reason that gluons have no charge for electroweak interactions. On the other hand, the measurement of the momentum dependence of three independent structure functions, such as $F_2(x, Q^2)$, $F_3(x, Q^2)$ and $\bar{q}(x, Q^2)$, using beams of ν and $\bar{\nu}$ (see below) does allow the determination of $g(x, Q^2)$ through the Altarelli-Parisi equations -- see, for example, Bergsma and collaborators (1983). Alternatively, one could try to extract $g(x)$ from multi-jet events at the ISR, or better the SPPS.

However, at the moment this game is played the other way. That is one takes a $g(x)$, albeit indirectly determined, and uses perturbative QCD to try to understand the observations in the colliding beam experiments.

A second, more direct, sum rule can be derived simply by conserving baryon number. Because each quark has $B = 1/3$ the total excess of quarks over anti-quarks should be three. In fact we would expect two u-quarks and one d-quark. These are usually called valence quarks. Of course in a DIS experiment one cannot tell if a given u quark (say) is a valence quark or part of the sea of virtual $q-\bar{q}$ pairs present in any hadron. Operationally one *defines* the valence distributions $u_V(x)$ and $d_V(x)$ as

$$\begin{aligned} u_V(x) &= u(x) - \bar{u}(x), \\ d_V(x) &= d(x) - \bar{d}(x). \end{aligned} \quad (3.15)$$

Then the sum rule giving the baryon number of the nucleon is

$$\begin{aligned} \int_0^1 u_V(x) dx + \int_0^1 d_V(x) dx \\ = \int_0^1 [u(x) - \bar{u}(x)] + [d(x) - \bar{d}(x)] dx \\ = 3. \end{aligned} \quad (3.16)$$

In fact, if one allows for QCD corrections to order (α_s/π) , the theoretical expectation is about 2.8, and the experimental data gives 2.56 ± 0.41 (statistics) ± 0.10 (systematics) (Bergsma and collaborators, 1983).

In terms of the quark distribution functions, the structure functions F_1 and F_2 defined earlier can be written down directly

$$2xF_1^{e,\mu}(x) = F_2^{e,\mu}(x) = \sum_i q_i^2 f_i(x) \times. \quad (3.17)$$

For example, for muon scattering on the proton we have

$$F_{2p}^{\mu}(x) = \frac{4x}{q} [u(x) + \bar{u}(x)] + \frac{x}{q} [d(x) + \bar{d}(x)] + \frac{x}{q} [s(x) + \bar{s}(x)] , \quad (3.18)$$

while for the neutron [see Eq. (3.11)]

$$F_{2n}^{\mu}(x) = \frac{x}{q} [u(x) + \bar{u}(x)] + \frac{4x}{q} [d(x) + \bar{d}(x)] + \frac{x}{q} [s(x) + \bar{s}(x)] . \quad (3.19)$$

Clearly one cannot use an electromagnetic probe to separate the quark and antiquark distributions. For that the weak interaction is perfect, because it couples only left-handed particles and right-handed antiparticles.

Consider the reaction

$$\nu_{\mu} + d \rightarrow \mu^{-} + u , \quad (3.20a)$$

which involves only left-handed particles. In the c.m. system of the ν and d there is therefore no angular momentum and the angular distribution is isotropic. On the other hand for a neutrino with an antiquark the angular distribution is proportional to $|d_{11}^1(\theta^*)|^2$, or $(1 + \cos \theta^*)^2$ -- where θ^* is the scattering angle in the ν - $\bar{q}$ c.m. system. Thus there is a clear experimental signature for the antiquark distribution. What is actually measured is conventionally labelled $\bar{q}^{\bar{\nu}}(x)$, defined as

$$\bar{q}^{\bar{\nu}}(x) = x[\bar{u}(x) + \bar{d}(x) + 2\bar{s}(x)] . \quad (3.20)$$

This is directly obtained from the isotropic piece of the $\bar{\nu}_{\mu} \rightarrow \mu^{+}$ cross-section on an isoscalar target.

In Fig. 11 we show the results for $\bar{q}^{\bar{\nu}}$ obtained by the CERN-Dortmund-Heidelberg-Saclay (CDHS) group using the ν_{μ} and $\bar{\nu}_{\mu}$ beams from the SPS at CERN. The shape is qualitatively consistent with the form $(1-x)^7$ expected on the basis of the Drell-Yan-West relation [see, for example, Close (1979) or Brodsky (1982)]. For a more accurate parametrization we refer to the original papers (Abramowicz and co-workers, 1982, 1983). Notice that the sea distribution essentially vanishes for $x > 0.3$. In addition, the fact that $\bar{q}^{\bar{\nu}}(x)$ does not vanish at $x = 0$, together with Eq. (3.20), implies that the actual number of sea quarks is logarithmically divergent. The momentum carried by the sea is however finite. In fact, if we define $\bar{Q}^{\bar{\nu}}$ [by analogy with Eq. (3.12)] as the integral of $\bar{q}^{\bar{\nu}}$ we obtain $5.95 \pm 0.4\%$ [at $Q^2 = 5 \text{ GeV}^2$ (Eisele, 1982)].

We also show in Fig. 11 the comparison between the structure function F_2 extracted from neutrino reactions (on an iron target)

$$2xF_1(x) = F_2(x) = \{u + d + s + \bar{u} + \bar{d} + \bar{s}\} , \quad (3.21)$$

with the electromagnetic structure functions ($F_{2n}^{\mu} + F_{2p}^{\mu}$). From Eqs. (3.18) and (3.19) we find

$$(F_{2n}^{\mu} + F_{2p}^{\mu}) \equiv F_2^{\mu N} = \frac{5}{18} \{u + d + \bar{u} + \bar{d} + \frac{2}{5} (s + \bar{s})\} . \quad (3.22)$$

Thus to within a small correction from the strange part of the sea the quark model predicts

$$F_2^{\mu N}(x) = \frac{5}{18} F_2(x) . \quad (3.23)$$

As we see from the figure, Eq. (3.23) is remarkably well confirmed.

Fig. 11. Results of the CDHS group (Abramowicz and co-workers, 1983) for the structure functions F_2 and F_3 (using neutrino beams), and the antiquark distribution $\bar{q}^v$ defined in Eq. (3.20).

Finally we observe that unlike the electromagnetic case, where parity conservation restricted us to two structure functions, for weak interactions there is a third, called $F_3(x)$. For an isoscalar nucleus the sum F_3^v and $F_3^{\bar{v}}$, denoted $F_3(x)$, is exactly the distribution of valence quarks $[u_v(x) + d_v(x)]$. This is also shown in Fig. 11. It is clear that the valence quarks dominate over the sea for x beyond about 0.1. We shall recall this again soon in connection with the EMC effect.

The final piece of information which we need to pin down the properties of the nucleon is the ratio of strange to non-strange quarks in the sea. Fortunately there is a direct measurement of this available through the reaction

$$\bar{\nu}_\mu + \bar{s} \rightarrow \mu^+ + \bar{c} \rightarrow \mu^- , \quad (3.24)$$

with its unique di-muon signal. Again we show the results of the CDHS group (Fig. 12). The shape of $\bar{s}(x)$ is identical with that of $\bar{q}^v(x)$ within the experimental errors. The actual magnitude of the strange sea is best determined from the *shape* of the ν_μ and $\bar{\nu}_\mu$ x distributions, as described by Abramowicz and co-workers (1982). In fact what they found is

$$\frac{2S}{(\bar{U} + \bar{D})} = 0.52 \pm 0.09 . \quad (3.25)$$

Fig. 12. Comparison of the shape of the $\bar{s}$ distribution in Fe with that of $\bar{q}$ described earlier -- from Abramowicz and co-workers (1983).

It seems to be an almost universal assumption^{*)} that $s(x) = \bar{s}(x)$ and hence $S = \bar{S}$, so we shall also use S and $\bar{S}$ interchangeably.

Summary

We have seen how to determine the momentum distributions of all the types of parton in the nucleon. Briefly we can summarize the data (at $Q^2 \approx 5$ GeV²) as^{**)}

$$G \approx 54\% , \quad (3.26a)$$

$$Q + \bar{Q} \approx 46\% , \quad (3.26b)$$

$$\bar{Q}\bar{V} = 5.95 \pm 0.4\% , \quad (3.26c)$$

$$\bar{S} = 1.0 \pm 0.3\% , \quad (3.26d)$$

$$(\bar{U} + \bar{D})/2 = 1.95 \pm 0.3\% , \quad (3.26e)$$

$$Q - \bar{Q} \approx 36\% . \quad (3.26f)$$

^{*)} Certainly wrong in principle, but the error is probably not large.

^{**)} I benefited a great deal from discussions with F. Eisele concerning the CDHS data.

That is, the glue carries a little more than half the momentum of the nucleon [Eq. (3.26a)], the valence quarks a little more than one third [Eq. (3.26f)], and the sea about 10% [Eqs. (3.26d) and (3.26e)]. Finally we note the obvious breaking of SU(3)-flavour [SU(3)_F] by the sea -- there are on average four times as many non-strange as strange quarks [c.f. twice as many in the SU(3)_F limit]. To some extent this was expected because of the higher mass of the strange quark. In the next section we shall show that this symmetry breaking can be used to constrain the strength of the pion field about the nucleon in any chiral bag model.

A CONSTRAINT ON THE PION FIELD OF THE NUCLEON

Let us begin with the observation by Sullivan (1972) that there is a contribution to the structure function of the nucleon from the process illustrated in Fig. 13. The virtual photon (γ^*) finds a quark in the pion with fraction x of the momentum of the original nucleon. Naively, at Q^2 of order 10 GeV^2 one would expect to probe distances of order $\frac{1}{20}$ th of a Fermi or less. Thus the dressing of the nucleon by pions, which is a relatively long range effect, would not be expected to play a role. The resolution of this apparent contradiction is that one is probing the short distance structure of the pion which is described by the pion's own structure function ($F_{2\pi}$) -- the latter evaluated at x/y , with y the fraction of the momentum of the nucleon carried by the pion in the infinite momentum frame.

Fig. 13. The contribution of the pion to the structure function of the nucleon.

Putting all this together we find the contribution of Fig. 13, $\delta F_{2N}(x)$, is given by the expression

$$\delta F_{2N}(x) = \int_x^1 f(y) F_{2\pi}(x/y) dy. \quad (3.27)$$

The function $f(y)$ is the number density of pions carrying fraction y of the momentum of the nucleon. It is calculated very simply from the lower portion of Fig. 13 in terms of the πNN coupling constant ($g = 13.5$), and the high-momentum cut-off at the πNN vertex, $F(t)$.

$$f(y) = \frac{3g^2}{16\pi^2} \int_{\frac{m_N^2 y^2}{1-y}}^{\infty} \frac{dt \, t |F(t)|^2}{(t + m_\pi^2)^2} \quad (3.28)$$

Here $t = (\vec{q}^2 - q^0{}^2)$ is minus the four-momentum of the exchanged pion, and the lower limit on t in Eq. (3.28) follows from demanding that the final nucleon be on mass shell -- see Sullivan (1972). [If instead of a nucleon one had some resonance in the final state, the lower limit on the integral over t would become even higher. This would reduce the corresponding $f(y)$ and force it to peak at smaller y . From the form of Eq. (3.27) this moves the corresponding contribution to smaller x . For this reason the region $x < 0.05$ is quite complicated to analyse. However, for $x > 0.05$ numerical calculations show that the $N \rightarrow N\pi$ process is dominant, and we shall treat only that -- again see Sullivan (1972).]

In order to estimate $f(y)$ we took a simple Gaussian for $F(t)$ (actually it is a Gaussian in $\vec{q}^2$, where $\vec{q}^2 \gg q^0{}^2$ for Fig. 13)

$$F(t) = \exp \left[-\lambda(t + m_\pi^2)/m_\pi^2 \right] \quad (3.29)$$

From our discussion of the CBM, see Fig. 2, we recall that Eq. (3.29) is an excellent approximation the πNN form-factor provided $\lambda = 0.106 \, m_\pi^2 R^2$. Of course the CBM form-factor is based on the static, spherical cavity approximation to the MIT bag model, and therefore applies only for spacelike momenta. Fortunately, as we have already pointed out, $q^0 \sim -\vec{q}^2/2m_N$ if the final nucleon is on-shell, and hence $|q^0| \ll |\vec{q}|$. Thus it is meaningful to talk about the CBM prediction for the πNN form-factor in the present context.

In Fig. 14 we show the function $f(y)$ which results from evaluating Eq. (3.28) using Eq. (3.29), for several values of λ (or R). There are two essential features to which we would like to draw attention. First, $f(y)$ has its maximum value at $y \approx 0.25$ for all reasonable values of λ . Second, the value at the peak increases rapidly as λ decreases -- that is, as the form-factor becomes harder.

Returning to Eq. (3.27), we see that the pion structure function is evaluated at x/y . As usual we expect that the valence component of the pion should dominate for $x/y > 0.1$. Since y is typically 0.25, this implies that the pionic contribution to the nucleon structure function for $x > 0.03$ involves only non-strange quarks. Thus, if the pion is an important component of nucleon structure, it should contribute to breaking the SU(3) flavour symmetry of the sea. Of course, as we mentioned earlier, it is generally expected that SU(3)_F will be broken because of the larger strange quark mass, and it would be unreasonable to attribute the entire excess of non-strange sea quarks to the pion. Nevertheless, it seems quite reasonable to use any evidence for SU(3)_F breaking to impose a limit on the pionic contribution to the nucleon structure function.

Integrating Eq. (3.27) over x , we find that

$$\int_0^1 \delta F_{2N}^u(x) \, dx = \left[\int_0^1 F_{2\pi}^u(\xi) \, d\xi \right] \left[\int_0^1 dy \, y \, f(y) \right] \quad (3.30)$$

Thus, if we knew the structure function of the pion, we could use the measured excess of u - and d -quarks in the sea to put an upper limit on the average momentum fraction of the nucleon carried by the pion ($\langle y \rangle_\pi$) -- the latter being given by the

Fig. 14. The probability $f(y)$ of finding a pion carrying a fraction y of the momentum of the nucleon, for several values of the cut-off parameter λ (or bag radius R) -- from Thomas (1983).

second bracket on the right of Eq. (3.30). Let us call the excess of non-strange over strange quarks in the sea $q^{(2)}(x)$. [We conservatively assume that $(\bar{u} - \bar{s}) = (\bar{d} - \bar{s}) = (u - u_v - s) = (d - d_v - s) = q^{(2)}$, even though there is some evidence from the Gottfried sum rule that $\bar{d} > \bar{u}$. If this were used we would get an even lower upper bound on $\langle y \rangle_\pi$.] Then the contribution to the integral over $F_{2N}(x)$ from the excess of non-strange sea quarks, which we shall call $F_{2N, \text{excess}}$, is found, from Eq. (3.17), to be

$$F_{2N, \text{excess}} = \int_0^1 \left(\frac{4}{9} + \frac{4}{9} + \frac{1}{9} + \frac{1}{9} \right) q^{(2)}(x) \times dx$$

$$= \frac{10}{9} \left\{ \frac{\bar{U} + \bar{D}}{2} - \bar{S} \right\}. \quad (3.31)$$

Thus our bound on $\langle y \rangle_\pi$ is, from Eq. (3.30) and the argument following it

$$\langle y \rangle_\pi \equiv \int_0^1 f(y) y dy \leq \frac{\frac{10}{9} \left\{ \frac{\bar{u} + \bar{d}}{2} - \bar{s} \right\}}{\int_0^1 F_{2\pi}(\xi) d\xi}. \quad (3.32)$$

Determination of the structure function of the pion

Because the pion is unstable one cannot measure its structure function directly using lepton DIS. However, there is another technique available, the Drell-Yan process. In a collision of a high energy pion with a nucleon, an antiquark of the former can annihilate on a quark of the latter to give a massive photon which then decays to $\mu^+\mu^-$. The invariant mass distribution of $\mu^+\mu^-$ pairs produced this way gives a measure of the product of the quark distributions in the pion and the nucleon (see, for example, Kenyon, 1982). Since we know the structure function of the nucleon we can extract that of the pion.

The results obtained by the NA3 group at CERN are shown in Fig. 15. Just to show that the method works we also show (Fig. 16) the proton structure function extracted by the same group from the process $q\bar{q} \rightarrow \mu^+\mu^-$ in $p\bar{p}$ collisions. The result agrees very well with the proton structure function (at the same $|Q^2|$) known from DIS up to a renormalization factor of 2.3. This can be explained in terms of the next to leading log corrections in perturbative QCD. The main correction seems to come from the larger phase space in the Drell-Yan process ($q^2 = -Q^2 \sim (4-8.5) \text{ GeV}^2$ in UA3) compared with DIS ($Q^2 = -q^2 \sim 5 \text{ GeV}^2$).

Fig. 15. The structure function of the pion extracted by the UA3 group at CERN from the Drell-Yan process (Badier and others, 1983).

From the results quoted in the very readable review by Kenyon (1982), and also from Badier and co-workers (1983), we find (at $|Q^2| \sim 5 \text{ GeV}^2$)

$$\int_0^1 F_{2\pi}(\xi) d\xi = 0.015 \pm 0.004 . \quad (3.33)$$

Fig. 16. Comparison of the structure function of the proton extracted from the Drell-Yan process in $\bar{p}p$ collisions (points), with that from DIS (dashed line) -- from Kenyon (1982).

Summary

But for one small problem we could put together Eqs. (3.33), (3.32) and (3.26) to find an upper limit on $\langle y \rangle_\pi$ of $7 \pm 4\%$. The little problem is known as the EMC effect, and will be the subject of the last part of this lecture. For our present purposes the essential point is that the sea per nucleon in Fe is enhanced by between 20 and 40% over that for a free nucleon (see Fig. 18). Unfortunately, all of the CDHS neutrino data, on which Eq. (3.26) was based, was taken on Fe.

The most naive correction for the EMC effect would simply lower the whole sea by 15%, leading to a non-strange excess in a free nucleon $[(\bar{U} + \bar{D})/2 - \bar{S}]_N = 0.67 \pm 0.4\%$. On the other hand, some of the proposed explanations, including the one we describe in the next section (Llewellyn Smith, 1982, 1983; Ericson and Thomas, 1983), would lead to an enhancement of only the non-strange part of the sea in a nucleus. In that case the 30% correction would be applied only to $(\bar{U} + \bar{D})/2$, leading to an excess of non-strange sea quarks of about $0.4 \pm 0.4\%$ in a free nucleon. Then the upper limit allowed for the fraction of the momentum of the nucleon carried by pions $\langle y \rangle_\pi$ would be $3 \pm 3\%$.

From this brief analysis it should be clear that there is not yet a very reliable, direct, experimental determination of the non-strange excess. Nevertheless, the qualitative consistency of the numbers with those of Field and Feynman (FF, 1977) is remarkable. Using the DIS data from SLAC together with general theoretical constraints (including the Gottfried sum rule) they concluded that $(\bar{U} + \bar{D})/2 = 1.8\%$ and $\bar{S} = 1.1\%$ in a free nucleon. These numbers are very close to those obtained by the CDHS group, and may even be more accurate, despite the fact that they were obtained indirectly. Because of the uncertainty over the EMC correction, we actually used the FF value for $[(\bar{U} + \bar{D})/2 - \bar{S}]_N = 0.7\%$. This puts an upper limit on $\langle y \rangle_\pi$ of (Thomas, 1983a)

$$\langle y \rangle_\pi \leq 5 \pm 1.5\% , \quad (3.34)$$

[from Eqs. (3.32) and (3.33)]. We regard this as somewhat more conservative than the value $3 \pm 3\%$ obtained from the CDHS data.

Fig. 17. The average fraction of the nucleon's momentum carried by the pion as a function of λ (or bag radius R). The shaded area represents the upper bound obtained from SU(3)_F breaking in the nucleon's sea (Thomas, 1983a).

In Fig. 17 we compare this upper limit on $\langle y \rangle_\pi$ (shaded area) with the theoretical value (obtained numerically) for a range of values of λ . This analysis clearly puts *quite a severe lower limit* on the range of values allowed, namely $\lambda \geq 0.039 \pm_{-0.006}^{+0.012}$. Also shown in the figure is the corresponding value of the bag radius in the CBM. The lower bound implied by Eq. (3.34) is $R \geq 0.87 \pm 0.10$ fm. Of course, we must add the caution that the static bag model has many defects, and insistence on a precise value of R would be meaningless. (For example, the bag should have finite surface thickness, and this together with c.m. and recoil corrections could change the exact relationship between R and λ .) Nevertheless, this lower bound is a very strong indication that the essential assumption of the CBM [CBM(b) in the notation of lecture 2] is correct. For the general reasons outlined in lecture 1 we must therefore allow the possibility of a rather new picture of the nucleus -- one in which we find "nucleons" sharing their quarks a considerable fraction of the time. One of the most natural explanations for the EMC effect involves just this! It is therefore quite appropriate to devote the last part of this series of lectures to that dramatic discovery.

THE EMC EFFECT

Figure 18 shows the ratio of the structure function per nucleon in Fe to that in D, as a function of $x = Q^2/2m_N v$. Because of its low density we shall henceforth drop the distinction between data on D and free nucleons. (An analysis at the level of

Fig. 18. The structure function of a "nucleon" in Fe, relative to that in D, as measured by the European Muon Collaboration (Aubert and others, 1983).

one or two percent would have to take the difference into account.) There is a clear softening of the structure function in Fe, with the sea region ($x < 0.3$) enhanced by about 15%, and the valence region depressed by about the same fraction. The data in the region $0.3 < x < 0.65$ was confirmed by a SLAC group (Bodek and others, 1983) using background subtraction data almost ten years old. That data also extended the range of x to about 0.9 where the ratio does climb back above unity.

In view of the enormous range of Q^2 covered by the two experiments ($3 \leq Q^2 \leq 170 \text{ GeV}^2$), and the observation that the effect shows no Q^2 -dependence within the experimental errors, this is almost certainly not a higher twist effect. In order to put this result in the right perspective, we show in Fig. 19 the theoretical predictions for the ratio, made before the data were taken. The variation in the predictions is certainly an overestimate of the theoretical uncertainties involved in the Fermi-averaging procedure. We believe that the dot-dash curve is probably the most reasonable. However, even using that curve, the discrepancy between theory and experiment in the valence region ($x \sim 0.6$) is as much as 30%. That is an enormous change.

Fig. 19. Predictions for the ratio of the structure function per nucleon in Fe to that in D, predicted by various Fermi-averaging prescriptions (again from the compilation of Aubert and co-workers, 1983).

The first theoretical paper to discuss the EMC data was by Jaffe (1983), and it went right to the heart of the matter. Using the MIT bag model, he showed that the softening of the valence distribution could be qualitatively understood if a given quark spent a relatively large fraction of its time in a six quark state -- say (10-20)%. [A similar number was obtained by Pirner and Vary (1983).] Although Jaffe used the bag model, his result is quite general. According to the Drell-Yan-West relation the large x behaviour for an $(N + 1)$ quark bag is

$$F_{N+1}(\tilde{x}) \sim (1 - \tilde{x})^{2N-1} \quad (3.35)$$

where $\tilde{x} = Q^2/2m_{N+1}v \in (0,1)$. In terms of the experimentalists' x we then find the ratio of the structure functions for a six-quark and a three-quark bag to be

$$\frac{F_{6q}(x)}{F_{3q}(x)} \sim \frac{\left(1 - \frac{x}{2}\right)^9}{(1 - x)^3}, \quad (3.36)$$

(where we assumed that $m_6 \sim 2m_3$). It is a trivial exercise to show that this ratio has a minimum value at $x = 0.5$ -- just like the data.

One of the simplest ways to approach the many-body problem for composite nucleons is a boundary condition model. That is, one would assume that for a centre-to-centre separation $r > b$ the bags can be treated as distinct hadrons. However, for $r \leq b$, when the overlap is large (we expect $b \sim R$), we might treat the system as a six-quark state. For $b \sim R_{CBM} \sim 1$ fm the probability of finding a given quark in a six-quark state is indeed of order 20%, even in light nuclei (Greben and Thomas, 1983; Pirner and Vary, 1983).

It is unfortunately more difficult to make predictions about the sea in a six-quark bag. Of course one could always choose it to fit the EMC data, but it is probably fair to say that the multi-quark-bag idea does not provide a natural explanation of the EMC effect in the region $x < 0.3$.

In order to understand the suggestion of Llewellyn-Smith (1982, 1983) for the region $x < 0.3$, we return to Eq. (3.27) giving the contribution of the pion cloud to the structure function of the nucleon. Forgetting about shadowing corrections and so forth, which could alter the predictions for x very near zero, we see that Eq. (3.27) implies

$$\delta F_{2N}(0)/F_{2N}(0) \approx \delta F_{2N}(0)/F_{2\pi}(0) = \int_0^1 f(y) dy. \quad (3.37)$$

Thus for a free nucleon the fractional contribution to the structure function at $x = 0$ from the process shown in Fig. 13 is simply the number of pions (in the infinite momentum frame). It is then clear, that if in a nucleus the number of pions/nucleon was enhanced by about 0.15, one would have a qualitative explanation for the EMC data for $x < 0.3$.

In order to check this quantitatively, Ericson and Thomas (1983) calculated the modified momentum distribution $f(y)$ of pions in Fe. The mechanism for changing the distribution is illustrated in Fig. 20. We see that after being emitted by one nucleon a pion can interact with other nucleons in the medium, making nucleon particle-hole, or delta particle-hole excitations [Figs. 20(b) and 20(c), respectively]. When iterated in the RPA such processes could by themselves produce pion

Fig. 20. Illustration of (a) the basic pion contribution to the nucleon structure function (the $\gamma^*\pi$ vertex involves the structure function of the pion itself); (b) and (c) other coherent processes involving pion rescattering in the nucleus which lead to enhancement for $|\vec{q}| \sim 300\text{--}400$ MeV/c; (d) and (e) the phenomenological short-range repulsion which damps the enhancement arising from (b) and (c) -- from Ericson and Thomas (1983).

condensation at $[q^0 = 0, |\vec{q}| \sim 400 \text{ MeV/c}]$, at nuclear matter density (Migdal, 1972, 1978; Meyer-ter-Vehn, 1981). Since pion condensation is not observed there must be some suppression of the attraction, and this is often described phenomenologically by a short-range repulsive interaction g' (the Landau-Migdal parameter).

There has been a great deal of phenomenological work to determine the values of the three parameters $g'_{NN}, g'_{N\Delta}$ [Fig. 20(d) and 20(e), respectively] and $g'_{\Delta\Delta}$. Much of this school has dealt with the controversy over the role of $g'_{N\Delta}$ in the suppression of Gamow-Teller strength, and we shall not add to the confusion. Suffice it to say that as a first estimate $g'_{NN} = g'_{N\Delta} = g'_{\Delta\Delta} \equiv g' \sim 0.7$ is fairly conservative -- although some theorists argue that $g'_{N\Delta}$ could be as small as 0.4. For a discussion of this formalism as it relates to the calculation of $\tilde{f}(y)$ we refer to the lectures of M. Ericson.

Briefly, Ericson and Thomas use the approximation of treating Fe as nuclear matter with $k_F = 1.30$ fm (Smith and Moniz, 1972), which should not be unreasonable for inclusive processes at momentum transfer of 400 MeV/c. They use $g' = 0.7$ for the reasons just mentioned, and solve for the nuclear response in RPA. Before discussing the results, the only other choice of parameter which should be mentioned is the $NN\pi$ (and $\Delta N\pi$) form-factor. We argued in the last section that for a *free nucleon* the $NN\pi$ form-factor corresponded to $R \geq 0.87 \pm 0.10$ fm. However, there is a great deal of phenomenological evidence for a much harder form-factor in a conventional description of the N-N interaction (see the lectures of T.E.O. Ericson). The resolution of this may be related to the simple fact that the CBM form-factor only modifies the OPE interaction between two nucleons when they start to overlap. However, at that stage the OPE interaction in the CBM would get additional non-local contributions from exchange terms. Thus there is no reason why the $NN\pi$ form-factor for a free nucleon, calculated in the CBM, should be directly applicable to

the OPE N-N interaction. In order to be consistent with the usual Landau-Migdal phenomenology we therefore choose a dipole form-factor of mass 1.67 GeV (monopole of 1.2 GeV) in all exchanges involving two baryons, but at the last vertex, after which the pion is smashed by the photon, we use the CBM with $R \sim 0.8$ fm. (In fact, the standard curve was defined before our lower limit was proven, and involved $R = 0.7$ fm. We now consider this to be a little small.)

Finally we show in Fig. 21 the predictions of Ericson and Thomas for the enhancement of the pionic part of the sea in Fe. Although there is considerable sensitivity to parameters it is clear that the EMC data is consistent with the much searched for enhancement of the nuclear, pionic response function in the region $|\vec{q}| \sim 400$ MeV/c (Meyer-ter-Vehn, 1981; Oset, Toki and Weise, 1982). What may be most interesting to a medium-energy physics audience is the fact that *lowering* $g'_{N\Delta}$ below 0.6 would give far too much enhancement!

Fig. 21. The fractional increase in the ratio of the structure function in Fe compared with D, as a function of x ($= Q^2/2m_N v$), caused by the multinucleon pion emission graphs of Figs. 3(b)-(e). The data are from the European Muon Collaboration, and the shaded area indicates possible systematic errors. The standard input (solid curve) for Fe is $k_F = 1.30$ fm $^{-1}$, $g'_{NN} = g'_{N\Delta} = g'_{\Delta\Delta} = 0.7$, a bag radius of 0.7 fm in $F(q^2)$, and $\Gamma(q^2)$ is a dipole of mass 1.67 GeV. We show in the other curves the effect of altering any single one of these parameters (Ericson and Thomas, 1983).

Obviously, at the present stage of the analysis, we can draw no firm conclusions. On the experimental side we need further studies of the systematic behaviour of the effect -- with A and Q^2 . (Our model predicts that the enhancement of the sea should vary as the density, k_F^3 -- so that in ^{12}C , $k_F = 1.1 \text{ fm}^{-1}$ in Fig. 21, the effect is cut almost in half.) One would like to know whether it is only the non-strange part of the sea which is enhanced (as our model predicts), and so on. A series of follow-up experiments is planned at both CERN and SLAC.

From the theoretical point of view we have the uncertainties in correcting for Fermi motion, illustrated in Fig. 19. In addition, as observed by Llewellyn-Smith (1982, 1983), an enhancement of the sea almost certainly implies a softening of the valence distribution -- again difficult to calculate reliably. Finally, since April when the only theoretical papers of which we were aware were those described above, there have been at least eight more -- Nachtmann and Pirner (1983), Close, Roberts and Ross (1983), Faissner and Kim (1983), Date (1983), Carlson and Havens (1983), Szwed (1983), Furmanski and Krzywicki (1983) and finally Friman, Pandharipande and Wiringa (1983). It will be a long time before the constraints of theoretical consistency and new data allow us to choose definitively which explanation is correct.

Let us close by simply observing that the EMC effect is qualitatively in agreement with our naive expectations for the structure of the nucleus based on the CBM. For the present the explanations of the change of the sea as an enhancement in the number of pions, and the change in the valence distribution in terms of overlapping bags, are quite separate. Nevertheless it may prove possible to explain the parameter g' in terms of the dynamics of overlapping bags. Success in that effort would go a long way towards unifying the two ideas. We have a lot of hard work ahead. However, the reward, a new and deeper understanding of the structure of the nucleus than we have ever possessed, is surely worth the effort.

Acknowledgements

In addition to gratefully acknowledging the criticism and stimulation of those who have collaborated with me in the development of the CBM, in connection with these lectures I would particularly like to thank M. Ericson, F. Eisele, E. Gabathuler and C. Llewellyn Smith. I am indebted to N. Henderson for her rapid and accurate preparation of this manuscript. Finally, I would like to thank Sir Denys Wilkinson for the opportunity to participate in a very stimulating meeting, in a truly impressive location.

REFERENCES

- Abers, E.S. and Lee, B.W. (1973). Gauge Theories, *Phys. Rep.* 9C, 1-141.
- Abromowicz, H. and co-workers in the CDHS collaboration (1982). *Zeit Phys.* C15, 19.
- Abromowicz, H. and co-workers (1983). *Zeit Phys.* C17, 283.
- Altarelli, G. and Parisi, G. (1977). *Nucl. Phys.* B126, 298.
- Amaldi, E., Fubini, S. and Furlan, G. (1979). Pion-electroproduction at low energy and hadron form factors, *Springer Tracts in Modern Physics*, Vol. 83, Springer-Verlag, Berlin.
- Aubert, J.J. and co-workers in the European Muon Collaboration (1983). *Phys. Lett.* 123B, 275-278.
- Baacke, J., Igarashi, Y., Kasperidus, G. and Usler, H. (1983). *CERN preprint* TH-3509 and erratum.
- Badier, J. and co-workers in the NA3 collaboration (1983). *CERN preprint* EP/83- .
- Barrett, R.C., Rosenthal, A.S., Thomas, A.W. and Veit, E.A. (1983). To be published.
- Baym, G. (1979). *Physica* 96A, 113.
- Bergsma, F. and co-workers in the CHARM collaboration (1983). *Phys. Lett.* 123B, 269.
- Bergström, L. and Fredriksson, S. (1980). *Rev. Mod. Phys.* 52, 675.
- Bodek, A. and co-workers (1983). *Phys. Rev. Lett.* 50, 1431.
- Bogolioubov, P.N. (1967). *Ann. Inst. Henri Poincaré* 8, 163.
- Breit, J.D. (1982). *Columbia University Preprint* CU-TP-229.
- Brockmann, R., Weise, W. and Werner, E. (1983). *Phys. Lett.* 122B, 201.
- Brodsky, S.J. (1982). Quantum Chromodynamics at Nuclear Dimensions. In D.C. Fries and B. Zeitnitz (Eds.), *Quarks and Nuclear Forces*, Springer-Verlag, Berlin, Chap. 3, pp. 81-140.
- Brown, G.E. (1982). *Nucl. Phys.* A374, 63c.
- Brown, G.E., Durso, J.W. and Johnson, M.B. (1983). *Nucl. Phys.* A397, 447.
- Brown, G.E. and Rho, M. (1979). *Phys. Lett.* 82B, 177.
- Brown, G.E., Rho, M. and Vento, V. (1979). *Phys. Lett.* 84B, 383.
- Brown, G.E., Rho, M. and Vento, V. (1980). *Phys. Lett.* 97B, 423.
- Carlson, C.E. and Chachkhunashvili, M. (1981). *Nordita preprint*.
- Carlson, C.E. and Havens, T.J. (1983). *Stony Brook preprint*.
- Chin, S.A. (1982). *Nucl. Phys.* A382, 355.
- Chin, S.A. and Miller, G.A. (1983). *Phys. Lett.* 121B, 232.
- Chodos, A. and Thorn, C.B. (1975). *Phys. Rev.* D12, 2733.
- Chodos, A., Jaffe, R.L., Thorn, C.B. and Weisskopf, V. (1974a). *Phys. Rev.* D9, 3471.
- Chodos, A., Jaffe, R.L., Johnson, K. and Thorn, C.B. (1974b). *Phys. Rev.* D10, 2599.
- Close, F. (1979). *An Introduction to Quarks and Partons*, Academic Press, New York.
- Close, F.E., Roberts, R.G. and Ross, G.G. (1983). *Rutherford preprint* RL-83-051.
- Date, S. (1983). *Waseda University preprint*, WU-HEP-83-4.
- De Rújula, A., Georgi, H. and Glashow, S.L. (1975). *Phys. Rev.* D12, 147.
- De Tar, C. (1981a). *Phys. Rev.* D24, 752.
- De Tar, C. (1981b). *Phys. Rev.* D24, 762.
- De Tar, C.E. and Donoghue, J.F. (1983). *Ann. Rev. Nucl. and Part. Science* 33 (to appear).
- Dodd, L.R., Thomas, A.W. and Alvarez-Estrada, R.F. (1981). *Phys. Rev.* D24, 1961.
- Donoghue, J. and Golowich, E. (1977). *Phys. Rev.* D15, 3421.
- Donoghue, J.F. and Johnson, K. (1980). *Phys. Rev.* D21, 1975.
- Eeg, J.O. and Pilkuhn, H. (1978). *Zeit. Phys.* A287, 407.
- Eisele, F. (1982). *Structure Functions*. In P. Petiau and M. Porneuf (Eds.), *Proc. 21st Int. Conf. on High Energy Physics*. J. de Phys., Vol. 12 (supplement), p. C3-337; and private communication.
- Eisenberg, J. and Kalbermann, G. (1983). *Tel Aviv University preprint*, to appear in *Phys. Rev.*
- Ericson, M. and Thomas, A.W. (1983). *CERN preprint* TH-3553, to appear in *Phys. Lett.*

- Faissner, H. and Kim, B.R. (1983). *Aachen preprint*.
- Feynman, R.P. (1972). *Photon Hadron Interactions*. W.A. Benjamin Inc., Reading, Mass.
- Field, R.D. and Feynman, R.P. (1977). *Phys. Rev.* D15, 2590.
- Friman, B.L., Pandharipande, V.R. and Wiringa, R.B. (1983). *University of Illinois preprint*, ILL-(NU)-83-15.
- Furmanski, W. and Krzywicki, A. (1983). *Orsay preprint*, LPHE, Orsay 83/11.
- Gasser, J. and Leutwyler, H. (1982). Quark Masses, *Phys. Rep.* 87, 77.
- Gell-Mann, M. (1964). *Phys. Lett.* 8, 214.
- Gell-Mann, M. and Lévy, M. (1960). *Nuovo Cimento* 16, 53.
- Goldman, T.J. and Haymaker, R.W. (1981). *Phys. Rev.* D24, 724.
- Greben, J.M. and Thomas, A.W. (1983), to be published.
- Guichon, P.A.M. (1983). *CERN preprint* TH-3576.
- Guichon, P.A.M., Miller, G.A. and Thomas, A.W. (1983). *Phys. Lett.* 124B, 109.
- Henley, E.M. and Thirring, W. (1962). *Elementary Quantum Field Theory*, McGraw-Hill, New York.
- Inoue, T. and Maskawa, T. (1975). *Prog. Theor. Phys.* 54, 1833.
- Isgur, N. (1980). *Proceedings of BARYON 80*, University of Toronto Press, Toronto.
- Jaffe, R.L. (1979). *Lectures at the Erice School*, Ettore Majorana.
- Jaffe, R.L. (1983). *Phys. Rev. Lett.* 38, 195.
- Kenyon, I.R. (1982). *CERN preprint* EP/82-81, to be published in *Rep. Prog. Phys.*
- Kogut, J., Stone, M., Wyld, H.W., Gibbs, W.R., Shigemitsu, J., Shenker, S.H. and Sinclair, D.K. (1982). *University of Illinois preprint* ILL-(TH)-82-39.
- Lipps, H., Martinelli, G., Petronzio, R. and Rapuano, F. (1983). *CERN preprint* TH-3548.
- Llewellyn Smith, C.H. (1982). *In Proceedings of the SPS Workshop*, CERN.
- Llewellyn Smith, C.H. (1983). *Oxford preprint* 18-83, to appear in *Phys. Lett.*
- Marciano, W. and Pagels, H. (1978). Quantum Chromodynamics. *Phys. Rep.* 36C, 138-276.
- Martinelli, G., Parisi, G., Petronzio, R. and Rapuano, F. (1982). *CERN preprint* TH-3334.
- Maxwell, O.V. and Vento, V. (1983). *Saclay preprint*, DPh-T/81-76 (revised).
- Meyer-ter-Vehn, J. (1981). *Phys. Rep.* 74, 324.
- Migdal, A.B. (1972). *Sov. Phys. JETP* 34, 1185.
- Migdal, A.B. (1978). *Rev. Mod. Phys.* 50, 107.
- Miller, G.A. (1983). *CERN preprint* TH-3516, to be published in *Proc. 1982 Indiana University Cyclotron Facility Workshop*.
- Miransky, V. and Fomin, P. (1981) *Phys. Lett.* 105B, 387.
- Mulders, P.J. and Thomas, A.W. (1982). *CERN preprint* TH-3443.
- Nachtmann, O. and Pirner, H.J. (1983). *Heidelberg University preprint*, HD-THEP-83-8.
- Niskanen, J.A. (1981). *Phys. Lett.* 107B, 344.
- Nogami, Y. and Ohtsuka, N. (1982). *Phys. Rev.* D26, 261.
- Oset, E., Toki, H. and Weise, W. (1982). *Phys. Rep.* 83, 281.
- Pagels, H. (1975). Departures from Chiral Symmetry. *Phys. Rep.* 16C, 220-311.
- Pirner, H.J. and Vary, J.P. (1981). *Phys. Rev. Lett.* 46, 1376.
- Pirner, H.J. and Vary, J.P. (1983). *Heidelberg University preprint*, UNI-HD-83-02.
- Rinat, A.S. (1982). *Nucl. Phys.* A377, 341.
- Roberts, L. and Welsh, R. (1983), private communication.
- Shuryak, E.V. (1983). *Non-Perturbative Phenomena in QCD Vacuum, Hadrons, and Quark-Gluon Plasma*, CERN report 83-01.
- Smith, R.A. and Moniz, E.J. (1972). *Nucl. Phys.* B43, 605.
- Sullivan, J.D. (1972). *Phys. Rev.* D5, 1732.
- Szwed, J. (1983). *Jagellonian University preprint*, TPJU-1/83.
- Szymacha, A. and Tatur, S. (1981). *Zeit. Phys.* C7, 311.
- Théberge, S., Miller, G.A. and Thomas, A.W. (1982). *Can. J. Phys.* 60, 59.
- Théberge, S. and Thomas, A.W. (1982). *Phys. Rev.* D25, 284.
- Théberge, S. and Thomas, A.W. (1983). *Nucl. Phys.* A393, 252.

- Thomas, A.W. (1981). *J. Phys.* G7, L283.
- Thomas, A.W. (1983a). *CERN preprint* TH-3553 (to appear in *Phys. Lett.*).
- Thomas, A.W. (1983b). Chiral symmetry and the bag model: a new starting point for nuclear physics. In J. Negele and E.W. Vogt (Eds.), *Advances in Nuclear Physics*, Plenum, New York, 13, pp. 1-137.
- Thomas, A.W. (1983c). Recent Developments in the Cloudy Bag Model. In J. Meyer (Ed.), *7^e Session D'Etudes Biennale de Physique Nucleaire*, Université Claude Bernard, Lyon, pp. C16.1-C16.28.
- Thomas, A.W. and McKellar, B.H.J. (1983). *CERN preprint* TH-3376, to appear in *Nucl. Phys.* B.
- Thomas, A.W., Thèberge, S. and Miller, G.A. (1981). *Phys. Rev.* D24, 216.
- t'Hooft, G. (1976). *Phys. Rev.* D14, 3432.
- Vento, V. (1983). *Phys. Lett.* 121B, 370.
- Vento, V., Rho, M., Nyman, E.M., Jun, J.H. and Brown, G.E. (1980). *Nucl. Phys.* A345, 413.
- Zweig, G. (1964). *CERN preprint* TH-412.