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THE ROLE OF TWO PARTICLES-TWO HOLES EXCITATIONS IN THE
SPIN - ISOSPIN NUCLEAR RESPONSE

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ABSTRACT

We investigate the role of the $2p - 2h$ states in the spin-isospin nuclear response function. This is done in the frame of a microscopic approach which includes the meson exchange currents and the nucleon-nucleon correlations.

We first test our theory on the transverse response in the inclusive deepinelastic electron scattering, where we achieve a satisfactory agreement with the data for values of the momentum transfer ranging from 1 to 2 fm^{-1} .

We next explore the p-wave pion-nucleus absorptive optical potential. We find that a strong (~ 3) Lorentz-Lorenz-Ericson-Ericson quenching factor is needed to reproduce in our framework the phenomenological optical potential deduced from π -mesic atoms data.

We also examine the real photon absorption cross-section accounting rather satisfactorily for its behaviour, in particular for the Pauli blocking at small frequencies.

Finally, we elucidate the conditions for the existence of a connection between the magnetic photon absorption and the p-wave pion absorption in nuclei.

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1. Introduction

The nuclear response to a spin-isospin sensitive probe has been widely explored in the nucleonic sector where the associated physics is well described in terms of particle-hole excitations, if the influence of the Δ -sector is also accounted for. The latter manifests itself, for example, in the quenching of the low energy magnetic and Gamow-Teller transitions and can possibly be included in an RPA scheme.

As one proceeds to explore the nuclear response in the region of deeper inelasticity the particle-hole frame becomes more and more inadequate and the necessity to enlarge the quantum mechanical Hilbert space to include two particle - two hole excitations (making allowance for the Δ degree of freedom) becomes progressively apparent.

This necessity is illustrated, for instance, by the success of the quasi-deuteron model in describing the absorption of both real and virtual photons of energy larger than, say, 100 MeV and of physical pions near threshold.

Indeed when the probe, as in the above mentioned instances, brings into the system large energy and little momentum then the conservation laws can be fulfilled only through the excitation of two particle -two hole quantum states. These are also naturally excited via the meson exchange currents (MEC) because of their two-body nature.

Much work has already been carried out on these problems and here we mention some recent ones. Donnelly and Van Orden⁽¹⁾ have investigated the influence of MEC in deep inelastic electron scattering, using a chiral invariant Lagrangian originally proposed by Peccei⁽²⁾.

The same process has been explored by Laget ⁽³⁾ with a different approach based on a refined version of the quasi-deuteron model.

Wakamatsu and Matsumoto ⁽⁴⁾ also address the questions why the quasi-deuteron model works well in interpreting the high energy photonuclear reactions and why the (γ, pp) cross-section is greatly suppressed as compared with the (γ, pn) one.

Shimizu and Faessler ⁽⁵⁾ have microscopically evaluated in a two particle -two hole basis the imaginary part of the pion-nucleus optical potential for threshold pions.

In this paper we further pursue the program of dealing microscopically with these topics within a unified scheme, which includes the Feynman diagrams describing both the MEC and the nucleonic correlations.

In order to benefit from the advantage of translational invariance, we examine infinite nuclear matter: this should be adequate since in general space confinement affects substantially only the low energy nuclear properties.

As far as the experiment is concerned we focus our attention on the analysis of the transverse nuclear response as it has been measured with inelastic electron scattering on ^{56}Fe at Bates ⁽⁶⁾ and with real photon absorption on ^{208}Pb at Saclay ⁽⁷⁾.

We also reexamine the problem of p-wave pion absorption on nuclei, already treated in detail by Shimizu and Faessler ⁽⁵⁾. One of the reasons for this investigation lies in the attempt of establishing a link between pion and photon

absorption. This is along the line of thought of T.E.O. Ericson and Bernabeu ⁽⁸⁾ with their concept of axial locality and of M. Ericson ⁽⁹⁾ when she connected pion absorption and Gamow-Teller strength.

We have pursued this idea in the frame of perturbation theory which allows to calculate both the p-wave pion absorption on nuclei at threshold and the nuclear magnetic response in deep inelastic electron scattering with the same diagrams, involving the Δ degree of freedom and the nucleon-nucleon correlations.

On this basis one envisages the possibility of relating processes referring to different probes and kinematical domains in such a way that the knowledge of one of them entails the knowledge of the other.

However, while the microscopic model is able to achieve a quantitative agreement with the experiment in (e, e') scattering at large momenta, it can reproduce the p-wave pion absorptive potential only if a large Lorentz-Lorenz-Ericson-Ericson (LLEE) effect is present. This could be properly dealt with a random phase approximation based on an (1 particle-1 hole, 2 particles-2 holes) irreducible kernel. Thus the connection between pions and photons absorption turns out to be a subtle one.

The present article is organized as follows.

In Section 2 we shortly review, for completeness, the nuclear response to an electromagnetic probe within the frame of particle-hole and Δ -hole excitations.

In Section 3 we revisit the Feynman diagrams of the MEC and their many-body interpretations.

In Section 4 we discuss the two particle -two hole correlation diagrams, their non-relativistic reduction and the associated polarization propagator.

In Section 5 we compare our theory with the data of deep inelastic electron scattering.

In Section 6 we analyze the p-wave pion-nucleus optical potential at threshold and the data of photon absorption on ^{208}Pb .

In the last Section the results of our investigation are summarized and discussed.

2. Nuclear response in the one particle - one hole sector

The vector current of the nucleon, the one which couples to the electromagnetic field, reads

$$J_{\mu}(\vec{p}_1', \vec{p}_1) = \frac{1}{V} \bar{U}(p_1', s_1') \chi_{s_1'}^{\dagger} T_{\mu} U(p_1, s_1) \chi_{s_1} \quad (2.1)$$

where

$$T_{\mu} = \frac{e}{2} (1 + \tau_3) \gamma_{\mu} - i q^{\alpha} \sigma_{\alpha\mu} \frac{\mu_0}{2} [(1 + \tau_3) \mu_p + (1 - \tau_3) \mu_n] \quad (2.1')$$

is the γNN vertex. In the above the U 's are the nucleonic spinors, V the normalization volume, μ_0 the Bohr nuclear magneton, $\mu_p = 1.79$ and $\mu_n = -1.91$ the anomalous proton and neutron magnetic moments, s and $\mathcal{S}$ the spin and isospin variables and

$$\sigma_{\alpha\mu} = \frac{i}{2} (\gamma_{\alpha} \gamma_{\mu} - \gamma_{\mu} \gamma_{\alpha}) \quad (2.2)$$

is the usual antisymmetric combination of Dirac matrices.

The standard non-relativistic reduction (expansion in the inverse of the nucleonic mass M) yields, to leading order,

$$J_0(\vec{p}_1', \vec{p}_1) = \frac{M}{V} \delta_{s_1' s_1} \delta_{\mathcal{S}_1' \mathcal{S}_1} (1 + 2\mathcal{S}_1) \quad (2.3)$$

for the time component, i.e., for the "charge density" responsible of the longitudinal nuclear response and

$$\begin{aligned} J_i(\vec{p}_1', \vec{p}_1) = & i \frac{\mu_0 M}{eV} \chi_{s_1'}^{\dagger} (\vec{\sigma} \times \vec{q})_i \chi_{s_1} \left\{ (1 + 2\mathcal{S}_1) (1 + \mu_p) + (1 - 2\mathcal{S}_1) \mu_n \right\} \delta_{s_1' s_1} + \\ & + \frac{\mu_0 M}{eV} (\vec{p}_1 + \vec{p}_1')_i \delta_{s_1' s_1} \delta_{\mathcal{S}_1' \mathcal{S}_1}, \quad i = 1, 2, 3 \end{aligned} \quad (2.4)$$

(with $\vec{q} = \vec{p}_1' - \vec{p}_1$) for the space components, responsible for the transverse one.

In (2.4) the first term in the RHS is the spin current, whereas the second is of electric character.

The transverse nuclear response is defined as

$$4\pi S_T(\vec{q}, \omega) = \left(\delta_{ij} - \frac{q_i q_j}{q^2} \right) W_{ij}(\vec{q}, \omega) \quad (2.5)$$

where, in the one particle - one hole (1p-1h) space,

$$W_{ij}(\vec{q}, \omega) = (2\pi)^3 V M c^2 A \sum_{\vec{p}, \vec{p}'} \delta^{(4)}(q + p - p') \theta(p' - k_F) \theta(k_F - p) \cdot \\ \cdot \langle F | \hat{J}_i^\dagger | \vec{p}', \vec{p} \rangle \langle \vec{p}', \vec{p} | \hat{J}_j | F \rangle \quad (2.6)$$

$|F\rangle$ being the particle-hole vacuum, $\hat{J}$ the second quantized expression of (2.4) and A the mass number of the target nucleus.

The insertion of (2.6) into (2.5) leads then to

$$4\pi S_T(\vec{q}, \omega) = \frac{-\hbar^2 k_F^2}{M} \frac{A^2}{\pi g} \text{Im} \Pi^{(0)}(Q, \nu) \left\{ Q^2 [(1 + \mu_p)^2 + \mu_n^2] + \left[1 - \left(\frac{\nu}{Q} - \frac{Q}{2} \right)^2 \right] \right\} \quad (2.7)$$

Q being in units of the Fermi wave number k_F and ν of twice the Fermi energy; also $g = 2k_F^3/3\pi^2$ is the nuclear matter density and $\Pi^{(0)}(Q, \nu)$ the polarization propagator for a free Fermi gas.

In (2.7) the first term in the RHS stems from the spin current, whereas the second, representing a small correction (cf., Fig. 1), is of electric nature.

As it stands (2.7) refers to a non-interacting Fermi gas of point nucleons. The particle-hole polarization propagator reads, accordingly,

$$\Pi^{(0)}(\vec{q}, \omega) = \Pi^N(\vec{q}, \omega) + \Pi^A(\vec{q}, \omega) \quad (2.8)$$

where Π^N is the standard Lindhard function and

$$\Pi^A(\vec{q}, \omega) = -\frac{4}{9} \left[\Phi_\Delta(\vec{q}, \omega) + \Phi_\Delta(-\vec{q}, \omega) \right] \quad (2.9)$$

with

$$\Phi_\Delta(\vec{q}, \omega) = \int \frac{d\vec{k}}{(2\pi)^3} \frac{\Theta(k_F - k)}{\epsilon_{\vec{q}+\vec{k}}^\Delta - \epsilon_{\vec{k}} - \hbar\omega - i\Gamma_\Delta/2} \quad (2.10)$$

where

$$\epsilon_{\vec{k}} = \frac{\hbar^2 \vec{k}^2}{2M}, \quad \epsilon_{\vec{q}+\vec{k}}^\Delta = \frac{\hbar^2 (\vec{q}+\vec{k})^2}{2M_\Delta} + \hbar\omega_\Delta \quad (2.11)$$

and $\hbar\omega_\Delta = 2.14 m_\pi c^2$ is the mass difference between the Δ and the nucleon in terms of the pionic mass.

If one allows the following frequency dependence for the width of the Δ

$$\Gamma_\Delta = \bar{\Gamma} \left\{ \frac{(\hbar\omega)^2 - (m_\pi c^2)^2}{(\hbar\omega_\Delta)^2 - (m_\pi c^2)^2} \right\}^{3/2} \Theta(\hbar\omega - m_\pi c^2) \quad (2.12)$$

with $\bar{\Gamma} = 120$ MeV, then

$$\text{Re } \Pi^A(\vec{q}, \omega) = -\frac{4}{9\pi^2} \frac{M_\Delta k_F}{\hbar^2 Q} \left\{ g(Q, \nu) + g(Q, -\nu) \right\} \quad (2.13)$$

where

$$\begin{aligned} g(Q, \nu) = & \frac{1}{Q} \left(\nu_\Delta - \nu + Q^2/2 \right) + \\ & + \frac{1}{4} \left\{ 1 + \frac{1}{Q^2} \left[\gamma_\Delta^2 - \left(\nu_\Delta - \nu + \frac{Q^2}{2} \right)^2 \right] \right\} \ln \left[\frac{\gamma_\Delta^2 + \left(\nu_\Delta - \nu + Q^2/2 + Q \right)^2}{\gamma_\Delta^2 + \left(\nu_\Delta - \nu + Q^2/2 - Q \right)^2} \right] - \\ & - \frac{\gamma_\Delta}{Q^2} \left(\nu_\Delta - \nu + \frac{Q^2}{2} \right) \left\{ \tan^{-1} \left[\frac{1}{\gamma_\Delta} \left(\nu_\Delta - \nu + \frac{Q^2}{2} + Q \right) \right] - \tan^{-1} \left[\frac{1}{\gamma_\Delta} \left(\nu_\Delta - \nu + \frac{Q^2}{2} - Q \right) \right] \right\} \end{aligned} \quad (2.14)$$

and

$$\text{Im } \Pi^A(\vec{q}, \omega) = -\frac{2}{9\pi^2} \frac{M_\Delta \kappa_F}{\hbar^2 Q} \left\{ f(Q, \nu) + f(Q, -\nu) \right\} \quad (2.15)$$

where

$$\begin{aligned} f(Q, \nu) = & \left[1 + \frac{1}{Q^2} \left\{ \gamma_\Delta^2 - \left(\nu_\Delta - \nu + \frac{Q^2}{2} \right)^2 \right\} \right] \left\{ \tan^{-1} \left[\frac{1}{\gamma_\Delta} \left(\nu_\Delta - \nu + \frac{Q^2}{2} + Q \right) \right] - \tan^{-1} \left[\frac{1}{\gamma_\Delta} \left(\nu_\Delta - \nu + \frac{Q^2}{2} - Q \right) \right] \right\} + \\ & + \frac{\gamma_\Delta}{Q^2} \left(\nu_\Delta - \nu + \frac{Q^2}{2} \right) \ln \left[\frac{\gamma_\Delta^2 + \left(\nu_\Delta - \nu + \frac{Q^2}{2} + Q \right)^2}{\gamma_\Delta^2 + \left(\nu_\Delta - \nu + \frac{Q^2}{2} - Q \right)^2} \right] - 2 \frac{\gamma_\Delta}{Q} \end{aligned} \quad (2.16a)$$

with

$$\gamma_\Delta = \frac{M_\Delta \Gamma_\Delta}{\hbar^2 \kappa_F^2}, \quad \nu_\Delta = \frac{M_\Delta \hbar \omega_\Delta}{\hbar^2 \kappa_F^2}. \quad (2.16b)$$

In order to emphasize the single particle aspect of the nuclear response, improving upon the simple Fermi gas treatment, one should at least insert self-energies in the nucleonic lines.

We have stressed in the past, instead, the collective behaviour of $S_T(q, \omega)$. To do so we have analyzed ⁽¹⁰⁾ the transverse response in ⁵⁶Fe with the polarization propagator in ring approximation

$$\Pi^{\text{ring}}(\vec{q}, \omega) = \frac{\Pi^{(0)}(\vec{q}, \omega)}{1 - V_{ph}^{\sigma=1, \tau=1}(\vec{q}, \omega) \Pi^{(0)}(\vec{q}, \omega)} \quad (2.17)$$

as far as the magnetic response is concerned (first term in the RHS of (2.7)); for the small electric one (totally neglected in ref.(10)) we have kept the simple Fermi gas approach.

We have employed the following particle-hole interaction

$$V_{ph}^{\sigma=1, \tau=1}(\vec{q}, \omega) = 4 \frac{f_{\pi}^2}{\mu_{\pi}^2} \left\{ \frac{q^2}{\omega^2/c^2 - q^2 - \mu_{\pi}^2} C_{\pi} \Gamma_{\pi}^2(q_{\lambda}^2) + g' \Gamma_{\pi}^2(q_{\lambda}^2) \right\} \quad (2.18)$$

and the πNN (ρNN) vertex form factor

$$\Gamma_{\pi, \rho}(q_{\lambda}^2) = \frac{\Lambda_{\pi, \rho}^2 - (m_{\pi, \rho} c^2)^2}{\Lambda_{\pi, \rho}^2 - (\hbar\omega)^2 + (\hbar c q)^2} \quad (2.19)$$

with $\Lambda_{\pi} = 1.3$ GeV and $\Lambda_{\rho} = 2$ GeV.

In (2.18) the πNN vertex form factor has been attributed to the Landau-Migdal parameter g' also and the strong coupling for the ρ meson has been chosen ($C_{\rho} = 2.18$). In addition, according to the Chew and Low model, for the strength of the $\pi-\Delta$ coupling we have set $f_{\pi}^* = 2f_{\pi}$ ($f_{\pi}^2/4\pi\hbar c = 0.08$). Finally the usual dipole electromagnetic form factor for the proton and the neutron

$$G_M^{p,n}(q_{\lambda}^2) = \frac{1}{[1 + q_{\lambda}^2/(839 \text{ MeV})^2]^2} \quad (2.20)$$

has been used at each γNN vertex.

Our analysis has proved itself not incompatible with the data in ^{56}Fe for a set of six momentum transfers ranging from 210 MeV/c to 410 MeV/c, as shown in Fig.1 for the case $q=370$ MeV/c. However our lp-lh approach falls below the experimental data already near the maximum of the response (though this deficiency can be cured by an adjustment of the Fermi wavenumber) and, more dramatically so, as one proceeds in the domain of deeper inelasticity. It is also worth noticing in Fig.1 the smallness of the electric contribution as well as the negligible influence of the Δ -width.

3. Meson exchange currents

The shortcomings of the $1p-1h$ theory mentioned at the end of the previous Section reflect, in part, our too rough treatment of the nuclear dynamics associated with the single particle motion in the medium. In particular the insertion of self-energies along the nucleonic lines (sometimes this is done by introducing a nucleon effective mass M^*) would extend the nuclear response to higher frequencies. An effect in the same direction could be achieved by replacing the θ functions in the polarization propagator (2.8) with a smoother and more realistic finite temperature Fermi distribution.

However the above mentioned improvements would not be enough to cure the disagreement between theory and experiment, which therefore unambiguously indicates the need of two particle-two hole ($2p-2h$) states to keep in touch with the physics.

One natural way to reach these excited states is through the mesonic degrees of freedom, properly accounted for with the two-body meson exchange currents (MEC), which make the simultaneous ejection of two nucleons out of the nucleus possible.

We limit ourselves to exploring the role of the longest range component of the MEC, which is carried by the pion. The corresponding Feynman diagrams are shown in Fig.2. It is likely that the MEC of shorter range, produced by the exchange of heavier mesons, are somewhat suppressed by the correlations among the nucleons.

Donnelly and Van Orden ⁽¹⁾ derive the expressions of the two-body currents associated with each of the diagrams in Fig.2 starting from an effective, chiral invariant Lagrangian with a pseudovector pion-nucleon coupling. They neglect the width of the Δ resonance, which is a good approximation in the energy range we explore, as we have pointed out in the previous section.

By performing the non-relativistic reduction and keeping only the leading terms in the $(1/M)$ expansion these authors get ⁽¹⁾

$$\vec{J}^{\pi}(\vec{p}_1' - \vec{p}_1, \vec{p}_2' - \vec{p}_2) = - \frac{4M^2 c^4}{V^2 \hbar c} \frac{f_{\pi}^2}{\mu_{\pi}^2} \chi_{s_1'}^+ \frac{\vec{\sigma} \cdot \vec{R}_1}{R_1^2 + \mu_{\pi}^2} \chi_{s_1} \chi_{s_2'}^+ \frac{\vec{\sigma} \cdot \vec{R}_2}{R_2^2 + \mu_{\pi}^2} \chi_{s_2} \times (\vec{R}_2 - \vec{R}_1) 4g_1 \mathcal{F}_E \quad (3.1)$$

for the pion-in-flight current,

$$\vec{J}^{\text{contact}}(\vec{p}_1' - \vec{p}_1, \vec{p}_2' - \vec{p}_2) = - \frac{4M^2 c^4}{V^2 \hbar c} \frac{f_{\pi}^2}{\mu_{\pi}^2} \left[\chi_{s_1'}^+ \vec{\sigma} \chi_{s_1} \chi_{s_2'}^+ \frac{\vec{\sigma} \cdot \vec{R}_2}{R_2^2 + \mu_{\pi}^2} \chi_{s_2} - \chi_{s_1'}^+ \frac{\vec{\sigma} \cdot \vec{R}_1}{R_1^2 + \mu_{\pi}^2} \chi_{s_1} \chi_{s_2'}^+ \vec{\sigma} \chi_{s_2} \right] 4g_1 \mathcal{F}_E \quad (3.2)$$

for the sea-gull or contact current and

$$\begin{aligned} \vec{J}^{\Delta}(\vec{p}_1' - \vec{p}_1, \vec{p}_2' - \vec{p}_2) = & \frac{4M}{3V^2} \frac{k^* \hbar f_{\pi} (\hbar c)^{3/2}}{\mu_{\pi}^2 (M_{\Delta}^2 - M^2)} \left\{ (2M_{\Delta} + 3M) \times 4 \mathcal{F}_D \times \right. \\ & \times \left[i(\vec{q} \times \vec{R}_2) \delta_{s_1' s_1} \chi_{s_2'}^+ \frac{\vec{\sigma} \cdot \vec{R}_2}{R_2^2 + \mu_{\pi}^2} \chi_{s_2} \delta_{s_2} + i(\vec{q} \times \vec{R}_1) \chi_{s_1'}^+ \frac{\vec{\sigma} \cdot \vec{R}_1}{R_1^2 + \mu_{\pi}^2} \chi_{s_1} \delta_{s_2' s_2} \delta_{s_1} \right] + \\ & + (2M_{\Delta} + M) \left[\chi_{s_1'}^+ \frac{\vec{\sigma} \cdot \vec{R}_1}{R_1^2 + \mu_{\pi}^2} \chi_{s_1} \chi_{s_2'}^+ \vec{q} \times (\vec{R}_1 \times \vec{\sigma}) \chi_{s_2} - \right. \\ & \left. \left. - \chi_{s_1'}^+ \vec{q} \times (\vec{R}_2 \times \vec{\sigma}) \chi_{s_1} \chi_{s_2'}^+ \frac{\vec{\sigma} \cdot \vec{R}_2}{R_2^2 + \mu_{\pi}^2} \chi_{s_2} \right] 4g_1 \mathcal{F}_E \right\} \quad (3.3) \end{aligned}$$

for the Δ current.

In the above formulas

$$\vec{R}_1 = \vec{p}_1' - \vec{p}_1, \quad \vec{R}_2 = \vec{p}_2' - \vec{p}_2 \quad (3.4a)$$

$$\mathcal{F}_E = (1 - \delta_{s_1 s_2})(1 - \delta_{s_1' s_1})(1 - \delta_{s_2' s_2}) \quad (3.4b)$$

$$\mathcal{F}_D = \delta_{s_1' s_1} \delta_{s_2' s_2} \quad (3.4c)$$

and $h^2 = 0.290$, $K^* = 5.0$ are the $\pi N\Delta$ and $\gamma N\Delta$ coupling constants entering into the Peccei Lagrangian.

Notice that in the non-relativistic domain, where the space components alone of the MEC survive, mesonic phenomena will affect only the transverse nuclear response $S_T(q, \omega)$.

We point out also that the MEC depend upon two and not four momenta separately.

Turning to the nuclear many-body problem, the two-body MEC can connect the vacuum to the vacuum (in the elastic scattering process we shall not consider here), to 1p-1h states or to 2p-2h states. The associated matrix elements are easily deduced in the frame of the second quantization formalism (compare ref.1). They turn out to be

$$\begin{aligned} \langle \vec{p} \vec{h} | \vec{J}^{MEC} | F \rangle &= \sum_{\vec{p}_1} \theta(k_F - p_1) \theta(p - k_F) \theta(k_F - h) \\ &\times \{ \vec{J}^{MEC}(\vec{p} - \vec{h}, \vec{p}_1 - \vec{p}_1) - \vec{J}^{MEC}(\vec{p} - \vec{p}_1, \vec{p}_1 - \vec{h}) \} \end{aligned} \quad (3.5)$$

and

$$\begin{aligned} \langle \vec{p} \vec{p}' \vec{h} \vec{h}' | \vec{J}^{MEC} | F \rangle &= \theta(p - k_F) \theta(p' - k_F) \theta(k_F - h) \theta(k_F - h') \\ &\times \{ \vec{J}^{MEC}(\vec{p} - \vec{h}, \vec{p}' - \vec{h}') - \vec{J}^{MEC}(\vec{p} - \vec{h}', \vec{p}' - \vec{h}) \} \end{aligned} \quad (3.6)$$

with obvious meaning of the symbols. In the above formulae the splitting in the direct and exchange terms is also clearly apparent; remarkably from the property of the MEC to vanish when either one of their arguments is zero, it follows that the direct p-h matrix element (3.5) is also vanishing.

The knowledge of the matrix elements (3.6) allows to calculate the contribution of 2p-2h excitations to the transverse nuclear response function. The latter is again given by the formulae (2.5) and (2.6) with 2p-2h excited states and dividing by 4 to account for the identity of particles and holes in the final states.

With a standard calculation one obtains

$$4\pi S_T^{MEC}(\vec{q}, \omega) = \frac{AV^5}{32(2\pi)^9(Mc^2)^3} \int d\vec{p} \int d\vec{p}' \int d\vec{k} \int d\vec{k}' \theta(p-k_F) \theta(p'-k_F) \times \\ \times \theta(k_F-k) \theta(k_F-k') \delta^{(3)}(\vec{q} + \vec{p} + \vec{p}' - \vec{k} - \vec{k}') \delta\left\{\hbar\omega - \frac{\hbar^2}{2M}(\vec{p}^2 + \vec{p}'^2 - \vec{k}^2 - \vec{k}'^2)\right\} \\ \times \left(\delta_{ij} - \frac{q_i q_j}{q^2}\right) \sum_{\substack{s_p s_p' \\ s_k s_k'}} \sum_{\substack{s_p s_p' \\ s_k s_k'}} J_i^{MEC\dagger}(\vec{p} - \vec{k}, \vec{p}' - \vec{k}') J_j^{MEC}(\vec{p} - \vec{k}, \vec{p}' - \vec{k}') \quad (3.7)$$

where J^i is the i-th space component of any of the MEC.

In (3.7) we have neglected the Pauli exchange contributions: they have been estimated to be small ⁽¹⁾, but this point should be further explored. We shall numerically evaluate (3.7) in Section 5.

Here we remind that the theory outlined in this section amounts to evaluate 49 perturbative contributions to $S_T(\vec{q}, \omega)$. They are proportional to the imaginary part of the diagrams arising from the folding together of the 7 diagrams of Fig.3. Some of them are displayed in Fig.4.

The graphs of Fig.3 are conveniently grouped in four classes: in the first the photon is directly absorbed by a pion in flight, in the second the photon produces a pion via a contact interaction (pair term of a pseudoscalar πN coupling), in the third class the photon couples to a Δ already present in the medium, whereas in the last a Δ -h pair is excited by the photon out of the ground state.

These diagrams play different roles in different kinematical domains, as will be explored in Section 5.

In concluding this section we notice that, although not explicitly exhibited in the previous formulae, form factors have been attached to each vertex of the diagrams of Fig.4. In particular we use (2.20) for the γ_{NN} and $\gamma_{N\Delta}$ electromagnetic vertices and (2.19) for the π_{NN} and $\pi_{N\Delta}$ ones.

However the contact and pion-in-flight diagrams require special attention; in fact it has been shown ⁽¹¹⁾ that the corresponding currents (3.1) and (3.2) do not violate the gauge invariance of the theory as long as the same form factors are utilized for both. On the other hand the pertinent electromagnetic vertices are of a different nature, being associated with the pion photoproduction in the contact term and with the direct $\gamma\pi\pi$ coupling in the pion-in-flight one, respectively.

In order to avoid the violation of gauge invariance which would occur by utilizing the appropriate form factors, we adopt the standard procedure of assuming for the $\gamma\pi\pi$ vertex form factor the same expression (2.20) as for the γ_{NN} one. Moreover we utilize the axial form factor

$$G_A(|\vec{q}-\vec{k}|) = \frac{1}{[1 + \hbar^2(\vec{q}-\vec{k})^2/m_A^2 c^2]^2} \quad (3.8)$$

in the $\gamma\pi NN$ vertex of the contact term ($\vec{q}$ and $\vec{k}$ being the wave numbers of the photon and of the pion respectively). The cut-off mass m_A recently suggested for (3.8) on the basis of the cloudy bag model ⁽¹²⁾ is not much different from the one assumed in (2.20), which has the same dipole form. So, to preserve as much as possible the gauge invariance of the MEC, we actually use $m_A c^2 = 839$ MeV.

4. The nucleon-nucleon correlations

The MEC were revisited in the previous section in a basis of uncorrelated nucleons. Actually the protons and the neutrons in the nucleus are correlated, among other things, via the exchange of the same pion we have previously looked at as the main carrier of the MEC.

To remedy this inconsistency we take the view to account for all the diagrams where a single pion is exchanged. Accordingly, in this section we associate a current to each of the four Feynman diagrams of Fig.5: they indeed represent the absorption of a photon (the case of pion absorption will be considered in the next section) by a pion-correlated pair of nucleons.

Applying the standard Feynman rules one gets, for the correlation current, the expression

$$\begin{aligned}
 J_{\mu}^{\text{corr}}(P_1', P_1, P_2', P_2) = & \\
 = - \frac{g_{\pi}^2}{\mu_{\pi}^2 V^2 \hbar c} & \left\{ \bar{U}(P_1', s_1') \chi_{s_1'}^{\dagger} \Gamma_{\mu}^{(1)} S_F^{(+)}(P_1' - q, M) \tau_i^{(1)} \gamma_{\nu} k_2^{\nu} \gamma_5 U(P_1, s_1) \chi_{s_1} + \right. \\
 & + \bar{U}(P_1', s_1') \chi_{s_1'}^{\dagger} \gamma_{\nu} k_2^{\nu} \gamma_5 \tau_i^{(1)} S_F^{(+)}(P_1' + q, M) \Gamma_{\mu}^{(1)} U(P_1, s_1) \chi_{s_1} \left. \right\}^* \quad (4.1) \\
 & \times \frac{1}{\mu_{\pi}^2 - k_2^2} \bar{U}(P_2', s_2') \gamma_{\lambda} k_2^{\lambda} \gamma_5 \chi_{s_2'}^{\dagger} \tau_i^{(2)} \chi_{s_2} U(P_2, s_2) + [1 \leftrightarrow 2]
 \end{aligned}$$

where

$$S_F^{(+)}(p, M) = \frac{\gamma_0 \sqrt{\hbar^2 \vec{p}^2 + M^2 c^2} - \hbar \vec{\gamma} \cdot \vec{p} + M c}{\frac{2}{\hbar} \sqrt{\hbar^2 \vec{p}^2 + M^2 c^2} \left(\frac{p_0}{c} - \sqrt{\hbar^2 \vec{p}^2 + M^2 c^2} \right)} \quad (4.2)$$

is the positive-frequency part of the Feynman propagator for the nucleon and Γ_μ is given by (2.1').

In the non-relativistic limit (4.1) yields for the space components

$$\begin{aligned}
 J_i^{corr}(\vec{p}_1, \vec{p}_1', \vec{p}_2, \vec{p}_2') = & \frac{4M^2 c^4}{V^2 \hbar c} \frac{g_\pi^2}{\mu_\pi^2} \left\{ \frac{1}{[2M\omega/\hbar - \vec{q} \cdot (\vec{p}_1 + \vec{p}_1' - \vec{k}_2)]} \times \right. \\
 & \times \left[\chi_{s_1'}^+ \{i(\vec{\sigma} \times \vec{q})_i (\vec{k}_2 \cdot \vec{\sigma})\} \chi_{s_1} \{ \mathcal{F}_E(\mu_s + 2g_2 \mu_v) + g_2 \mathcal{F}_D(\mu_v + 2g_1 \mu_s) \} + \right. \\
 & + (\vec{p}_1' + \vec{p}_1 - \vec{k}_2)_i \chi_{s_1'}^+ (\vec{k}_2 \cdot \vec{\sigma}) \chi_{s_1} \{ (1 + 2g_2) \mathcal{F}_E + g_2 (1 + 2g_1) \mathcal{F}_D \} \Big] + \\
 & + \frac{1}{[2M\omega/\hbar - \vec{q} \cdot (\vec{p}_1 + \vec{p}_1' + \vec{k}_2)]} \times \\
 & \times \left[\chi_{s_1'}^+ \{ -i(\vec{k}_2 \cdot \vec{\sigma}) (\vec{\sigma} \times \vec{q})_i \} \chi_{s_1} \{ \mathcal{F}_E(\mu_s - 2g_2 \mu_v) + g_2 \mathcal{F}_D(\mu_v + 2g_1 \mu_s) \} - \right. \\
 & - (\vec{p}_1' + \vec{p}_1 + \vec{k}_2)_i \chi_{s_1'}^+ (\vec{k}_2 \cdot \vec{\sigma}) \chi_{s_1} \{ (1 - 2g_2) \mathcal{F}_E + g_2 (1 + 2g_1) \mathcal{F}_D \} \Big] \Big\} \times \\
 & \times \frac{1}{k_2^2 + \mu_\pi^2} \chi_{s_2'}^+ (\vec{k}_2 \cdot \vec{\sigma}) \chi_{s_2} + \{1 \rightleftharpoons 2\} \quad (4.3)
 \end{aligned}$$

and for the time component

$$\begin{aligned}
 J_0^{corr}(\vec{p}_1, \vec{p}_1', \vec{p}_2, \vec{p}_2') = & \\
 = & \frac{16 M^3 c^6 g_\pi^2}{V^2 \hbar c \mu_\pi^2} \frac{1}{\{ [2M\omega/\hbar - \vec{q} \cdot (\vec{p}_1' + \vec{p}_1)]^2 - (\vec{q} \cdot \vec{k}_2)^2 \}} \frac{1}{(k_2^2 + \mu_\pi^2)} \times \\
 & \times \chi_{s_1'}^+ (\vec{k}_2 \cdot \vec{\sigma}) \chi_{s_1} \chi_{s_2'}^+ (\vec{k}_2 \cdot \vec{\sigma}) \chi_{s_2} \left\{ [2M\omega/\hbar - \vec{q} \cdot (\vec{p}_1' + \vec{p}_1)] 2g_2 \mathcal{F}_E - \right. \\
 & - (\vec{q} \cdot \vec{k}_2) [\mathcal{F}_E + g_2 (1 + 2g_1) \mathcal{F}_D] \Big\} + \{1 \rightleftharpoons 2\} \quad (4.4)
 \end{aligned}$$

which is of pure electric nature.

So, at variance with the MEC, in leading order of the $1/M$ expansion, the correlation diagrams contribute to the charge longitudinal nuclear response, which could entail a breaking of gauge invariance. In (4.3) $\mu_s = 1 + \mu_p + \mu_n = 0.88$ and $\mu_v = 1 + \mu_p - \mu_n = 4.70$.

An important observation should be made: when a dynamical pion is exchanged between the nucleons in the graphs of Fig.5, two kinds of contributions, differing in the time ordering, arise. One of them is associated with the renormalization of the wave function and the other is usually referred to as recoil term. It is well known⁽¹³⁾ that a strong cancellation occurs between them. These considerations, however, do not apply to our case, where the pion is static.

Notice that the currents (4.3) and (4.4), unlike the MEC, depend upon four momenta separately (a fact with heavy consequences for the numerical analysis) and display a magnetic (spin) and an electric component (the first and second terms, respectively, in each square bracket in the case of the spatial current).

Their contribution to the transverse nuclear response is again obtained through the vacuum, 2p-2h matrix elements (3.6) and then utilizing the formulae (2.5) and (2.6) with the same proviso discussed in the previous section. Dropping once more the Pauli exchange diagrams, one arrives to an expression analogous to (3.7) with the correlation current in place of the MEC.

The introduction of the correlation current amounts to account for 16 further perturbative contributions to $S_T(q, \omega)$ which arise from the folding together of the 4 diagrams of Fig.6: the six topologically distinct ones are shown in Fig. 7.

In dealing with the diagrams of Fig.7 we point out that we have kept the two nucleonic lines between the pionic and electromagnetic vertices strictly off the mass shell (this prescription is reflected in the Θ -functions which are present in formulae (5.7) to (5.13)). In so doing we neglect the contribution of self-energy insertions on nucleonic lines, which to this order are divergent.

The same considerations apply also to the 56 perturbative terms corresponding to the interference between the correlation current and the MEC (see Fig.8) which again arise from the folding together of the 7 diagrams of Fig.3 and the 4 ones of Fig.6. Thus our theory, which consistently treats the one pion exchange at the level of currents, includes 121 perturbative diagrams altogether.

An important point should be stressed namely that if the pion can be considered as the main carrier of the MEC, other mesons as well participate in building the correlations among nucleons, in particular the ϱ meson.

We do not incorporate it explicitly since, as was pointed out by Dickhoff⁽¹⁴⁾, it produces too much suppression of the tensor force. On the contrary we account for the contribution of heavier mesons empirically. For this purpose we separate out the central and tensor components of the pion exchange force, easily recognizable in the diagrams associated with the correlations.

We add then to the former the Landau-Migdal parameter g' which embodies the short range repulsion of the particle-hole interaction. For the tensor piece we choose the πNN vertex form factor in order to reproduce the effective tensor force of ref.14 which is based on a G-matrix calculation. This can be achieved by setting $\Lambda_\pi \cong 0.8 \div 0.9$ GeV in (2.19). We have used the same cut-off mass in the central interaction and in the MEC as well.

5. The two particle-two hole nuclear response for deep inelastic electron scattering

The general form of the 2p-2h transverse nuclear response to an excitation induced by an electron is given by (3.7) as far as the MEC are concerned. Following ref. 1) one can simplify such an expression by introducing the dimensionless variables

$$\begin{aligned}\vec{x}_1 &= \frac{1}{2k_F} (\vec{p} + \vec{k}) \quad , \quad \vec{\ell}_1 = \frac{1}{k_F} (\vec{p} - \vec{k}) \\ \vec{x}_2 &= \frac{1}{2k_F} (\vec{p}' + \vec{k}') \quad , \quad \vec{\ell}_2 = \frac{1}{k_F} (\vec{p}' - \vec{k}')\end{aligned}\quad (5.1)$$

thus getting

$$\begin{aligned}4\pi S_T^{\text{MEC}}(\vec{q}, \omega) &= \frac{AV^5 k_F^7 / c^6}{32(2\pi)^7 M^2 \hbar^2} \int \frac{d\vec{\ell}_1}{\ell_1^3} \int \frac{d\vec{\ell}_2}{\ell_2^3} A(\ell_1, \ell_2, \nu) \delta^{(3)}(\vec{Q} - \vec{\ell}_1 - \vec{\ell}_2) \times \\ &\times \sum_{\text{all } S} \sum_{\text{all } J} \left(\delta_{ij} - \frac{1}{Q^2} Q_i Q_j \right) J_i^\dagger(\vec{\ell}_1, \vec{\ell}_2) J_j(\vec{\ell}_1, \vec{\ell}_2)\end{aligned}\quad (5.2)$$

where the currents are any of the MEC and the function

$$\begin{aligned}A(\ell_1, \ell_2, \nu) &\equiv \frac{\ell_1^3 \ell_2^3}{(2\pi)^2} \int d\vec{x}_1 \int d\vec{x}_2 \delta(\nu - \vec{x}_1 \cdot \vec{\ell}_1 - \vec{x}_2 \cdot \vec{\ell}_2) \times \\ &\times \theta\left(|\vec{x}_1 + \frac{\vec{\ell}_1}{2}| - 1\right) \theta\left(|\vec{x}_2 + \frac{\vec{\ell}_2}{2}| - 1\right) \theta\left(1 - |\vec{x}_1 - \frac{\vec{\ell}_1}{2}|\right) \theta\left(1 - |\vec{x}_2 - \frac{\vec{\ell}_2}{2}|\right)\end{aligned}\quad (5.3)$$

is analytically calculated and extensively discussed in ref. 1).

The evaluation of (5.3) can be carried out up to the following two-dimensional integral

$$\begin{aligned}4\pi S_T^{\text{MEC}}(q, \omega) &= \frac{6A^2 M^2 c^2}{(2\pi)^4 \hbar^2 k_F^2} \left(\frac{g_n^2}{m_n^2 \hbar c} \right)^2 \frac{1}{Q} \int_0^\infty dx \int_0^\infty dy \frac{A(x, y, \nu)}{x^2 y^2} \times \\ &\times \theta(x - y + Q) \theta(x + y - Q) \theta(y - x + Q) \sum_{i=1}^6 \mathcal{D}_i(x, y)\end{aligned}\quad (5.4)$$

where $x = \ell_1$, $y = |\vec{\ell}_1 - \vec{Q}|$ and

$$\mathcal{D}_1(x, y) = \frac{4 x_T^2 y^2 x^2}{(x^2 + m_\pi^2)^2 (y^2 + m_\pi^2)^2} \quad (5.5a)$$

$$\mathcal{D}_2(x, y) = 2 \left[\frac{x^2}{(x^2 + m_\pi^2)^2} + \frac{y^2}{(y^2 + m_\pi^2)^2} + \frac{x_T^2}{(x^2 + m_\pi^2)(y^2 + m_\pi^2)} \right] \quad (5.5b)$$

$$\mathcal{D}_3(x, y) = - \frac{4 x_T^2}{(x^2 + m_\pi^2)(y^2 + m_\pi^2)} \left[\frac{x^2}{x^2 + m_\pi^2} + \frac{y^2}{y^2 + m_\pi^2} \right] \quad (5.5c)$$

$$\begin{aligned} \mathcal{D}_4(x, y) = a^2 Q^2 & \left[\frac{x^2(x^2 + x_L^2)}{(x^2 + m_\pi^2)^2} + \frac{y^2(y^2 + y_L^2)}{(y^2 + m_\pi^2)^2} + \frac{2 Q^2 x_T^2}{(x^2 + m_\pi^2)(y^2 + m_\pi^2)} \right] + \\ & + 2 b^2 Q^2 x_T^2 \left[\frac{x^2}{(x^2 + m_\pi^2)^2} + \frac{y^2}{(y^2 + m_\pi^2)^2} \right] \end{aligned} \quad (5.5d)$$

$$\mathcal{D}_5(x, y) = -4 a Q \left[\frac{y^2 y_L}{(y^2 + m_\pi^2)^2} - \frac{x^2 x_L}{(x^2 + m_\pi^2)^2} + \frac{Q x_T^2}{(x^2 + m_\pi^2)(y^2 + m_\pi^2)} \right] \quad (5.5e)$$

$$\mathcal{D}_6(x, y) = \frac{4 a Q^2 x_T^2}{(x^2 + m_\pi^2)(y^2 + m_\pi^2)} \left[\frac{x^2}{x^2 + m_\pi^2} + \frac{y^2}{y^2 + m_\pi^2} \right]. \quad (5.5f)$$

In the above

$$a = \frac{\kappa^* h (2M_\Delta + M) (\hbar c)^{5/2} K_F^2}{3 M f_\pi (M_\Delta^2 - M^2)}, \quad b = \frac{\kappa^* h (2M_\Delta + 3M) (\hbar c)^{5/2} K_F^2}{3 M f_\pi (M_\Delta^2 - M^2)}$$

and

$$m_\pi = \frac{\mu_\pi}{K_F}, \quad x_T^2 = x^2 - x_L^2, \quad x_L = \frac{y^2 - x^2 - Q^2}{2Q}, \quad y_L = \frac{y^2 - x^2 + Q^2}{2Q}.$$

Formula (5.5a) represents the pion-in-flight contribution, (5.5b) the contact one and (5.5c) the interference between them; (5.5d) is associated with the Δ degree of freedom and

the last two are the interference of the Δ with the contact and pion-in-flight currents respectively.

Concerning the relative importance of these terms we notice that the well known destructive interference between the contact and pion-in-flight holds as well in the 2p-2h frame. Moreover the Δ current contribution dominates at large q , owing to the linear q -dependence of its coupling.

Let us now discuss the correlation current. Its contribution to the transverse nuclear response can be split into six terms (diagrammatically displayed in Fig.7) according to the formula

$$4\pi S_T^{corr}(q, \omega) = \frac{A^2}{(2\pi)^4} \left(\frac{f_\pi^2}{4\pi\hbar c} \right)^2 \frac{1}{m_\pi^4} \frac{3M_c^2}{\pi\hbar^2 k_F^2} \sum_{i=1}^6 A_i(Q, \nu) \quad (5.6)$$

where

$$A_1(Q, \nu) = 6 \int d\vec{t} d\vec{\ell} \theta(t-1) \theta(1-|\vec{t}+\vec{\ell}-\vec{Q}|) \frac{\ell^4}{(\ell^2+m_\pi^2)^2} \frac{\theta(|\vec{t}-\vec{Q}|-1)}{[2\nu+Q^2-2\vec{Q}\cdot\vec{t}]^2} \\ \times \left\{ Q^2(\mu_s^2+\mu_v^2) + 4 \left[t^2 - \frac{1}{Q^2} (\vec{Q}\cdot\vec{t})^2 \right] \right\} I \left[\nu - \vec{t}\cdot(\vec{Q}-\vec{\ell}) + \frac{1}{2}(\vec{Q}-\vec{\ell})^2, \ell \right] \quad (5.7)$$

$$A_2(Q, \nu) = \int d\vec{t} d\vec{\ell} \theta(t-1) \theta(1-|\vec{t}+\vec{\ell}-\vec{Q}|) \frac{\ell^2}{(\ell^2+m_\pi^2)^2} \frac{\theta(1-|\vec{t}+\vec{\ell}|)}{[\nu-\vec{Q}\cdot(\vec{t}+\vec{\ell}-\vec{Q}/2)]} \\ \times \frac{\theta(|\vec{t}-\vec{Q}|-1)}{[\nu-\vec{Q}\cdot(\vec{t}-\vec{Q}/2)]} \left\{ (\vec{Q}\cdot\vec{\ell})^2 (3\mu_s^2-\mu_v^2) - 4\ell^2 \left[\vec{t}\cdot(\vec{t}+\vec{\ell}) - \frac{1}{Q^2} (\vec{Q}\cdot\vec{t}) \vec{Q}\cdot(\vec{t}+\vec{\ell}) \right] \right\} \\ \times I \left[\nu - \vec{t}\cdot(\vec{Q}-\vec{\ell}) + \frac{1}{2}(\vec{Q}-\vec{\ell})^2, \ell \right] \quad (5.8)$$

$$A_3(Q, \nu) = 6 \int d\vec{t} d\vec{\ell} \theta(1-t) \theta(|\vec{t}-\vec{\ell}+\vec{Q}|-1) \frac{\ell^4}{(\ell^2+m_\pi^2)^2} \frac{\theta(1-|\vec{t}+\vec{Q}|)}{[2\nu-Q^2-2\vec{Q}\cdot\vec{t}]^2} \times \left\{ Q^2(\mu_s^2+\mu_v^2) + 4 \left[t^2 - \frac{1}{Q^2} (\vec{Q}\cdot\vec{t})^2 \right] \right\} I \left[\nu - \vec{t}\cdot(\vec{Q}-\vec{\ell}) - \frac{1}{2}(\vec{Q}-\vec{\ell})^2, \ell \right] \quad (5.9)$$

and, for $j = 4, 5, 6$

$$A_j(Q, \nu) = \int d\vec{x}_1 d\vec{\ell}_1 d\vec{\ell}_2 \delta^{(3)}(\vec{Q}-\vec{\ell}_1-\vec{\ell}_2) \theta\left(|\vec{x}_1+\frac{\vec{\ell}_1}{2}|-1\right) \theta\left(1-|\vec{x}_1-\frac{\vec{\ell}_1}{2}|\right) \times \int d\vec{x}_2 \delta(\nu-\vec{x}_1\cdot\vec{\ell}_1-\vec{x}_2\cdot\vec{\ell}_2) \theta\left(|\vec{x}_2+\frac{\vec{\ell}_2}{2}|-1\right) \theta\left(1-|\vec{x}_2-\frac{\vec{\ell}_2}{2}|\right) \frac{1}{(\ell_1^2+m_\pi^2)(\ell_2^2+m_\pi^2)} \times a_j(\vec{x}_1, \vec{x}_2, \vec{\ell}_1, \vec{\ell}_2, \vec{Q}) \quad (5.10)$$

with

$$a_4(\vec{x}_1, \vec{x}_2, \vec{\ell}_1, \vec{\ell}_2, \vec{Q}) = \frac{\theta(|\vec{x}_2+\vec{\ell}_2/2-\vec{Q}|-1)}{[2\nu-\vec{Q}\cdot(2\vec{x}_2+\vec{\ell}_2-\vec{Q})]} \cdot \frac{\theta(|\vec{x}_1+\vec{\ell}_1/2-\vec{Q}|-1)}{[2\nu-\vec{Q}\cdot(2\vec{x}_1+\vec{\ell}_1-\vec{Q})]} \times \left\{ (2\mu_v^2-3\mu_s^2) [\vec{Q}\times(\vec{\ell}_1\times\vec{\ell}_2)]^2 + 2(3\mu_s-2\mu_v)(\vec{\ell}_1\cdot\vec{\ell}_2) [\vec{Q}\times(\vec{\ell}_1\times\vec{\ell}_2)]\cdot(\vec{x}_1-\vec{x}_2+\frac{\vec{\ell}_1-\vec{\ell}_2}{2}) + 4(\vec{\ell}_1\cdot\vec{\ell}_2)^2 \left[(\vec{x}_1+\frac{\vec{\ell}_1}{2})\cdot(\vec{x}_2+\frac{\vec{\ell}_2}{2}) - \frac{1}{Q^2} \{ \vec{Q}\cdot(\vec{x}_1+\frac{\vec{\ell}_1}{2}) \} \{ \vec{Q}\cdot(\vec{x}_2+\frac{\vec{\ell}_2}{2}) \} \right] \right\} \quad (5.11)$$

$$a_5(\vec{x}_1, \vec{x}_2, \vec{\ell}_1, \vec{\ell}_2, \vec{Q}) = 2 \frac{\theta(|\vec{x}_1+\vec{\ell}_1/2-\vec{Q}|-1) \theta(1-|\vec{x}_2-\vec{\ell}_2/2+\vec{Q}|)}{[2\nu-\vec{Q}\cdot(2\vec{x}_1+\vec{\ell}_1-\vec{Q})][2\nu-\vec{Q}\cdot(2\vec{x}_2-\vec{\ell}_2+\vec{Q})]} \times \left\{ -(3\mu_s^2+2\mu_v^2) [\vec{Q}\times(\vec{\ell}_1\times\vec{\ell}_2)]^2 + 2(3\mu_s+2\mu_v) (\vec{x}_1+\vec{x}_2+\frac{\vec{\ell}_1-\vec{\ell}_2}{2})\cdot[\vec{Q}\times(\vec{\ell}_1\times\vec{\ell}_2)](\vec{\ell}_1\cdot\vec{\ell}_2) - 20(\vec{\ell}_1\cdot\vec{\ell}_2)^2 \left[(\vec{x}_1+\frac{\vec{\ell}_1}{2})\cdot(\vec{x}_2-\frac{\vec{\ell}_2}{2}) - \frac{1}{Q^2} \{ \vec{Q}\cdot(\vec{x}_1+\frac{\vec{\ell}_1}{2}) \} \{ \vec{Q}\cdot(\vec{x}_2-\frac{\vec{\ell}_2}{2}) \} \right] \right\} \quad (5.12)$$

$$\begin{aligned}
 a_6(\vec{x}_1, \vec{x}_2, \vec{\ell}_1, \vec{\ell}_2, \vec{Q}) = & \frac{\theta(1-|\vec{x}_1-\vec{\ell}_1/2+\vec{Q}|)\theta(1-|\vec{x}_2-\vec{\ell}_2/2+\vec{Q}|)}{[2\nu-\vec{Q}\cdot(2\vec{x}_1-\vec{\ell}_1+\vec{Q})][2\nu-\vec{Q}\cdot(2\vec{x}_2-\vec{\ell}_2+\vec{Q})]} \times \\
 & \times \left\{ (2\mu_V^2-3\mu_S^2)[\vec{Q}\times(\vec{\ell}_1\times\vec{\ell}_2)]^2 - 2(3\mu_S-2\mu_V)\left(\vec{x}_1-\vec{x}_2-\frac{\vec{\ell}_1-\vec{\ell}_2}{2}\right)\cdot[\vec{Q}\times(\vec{\ell}_1\times\vec{\ell}_2)](\vec{\ell}_1\cdot\vec{\ell}_2) + \right. \\
 & \left. + 4(\vec{\ell}_1\cdot\vec{\ell}_2)^2\left[\left(\vec{x}_1-\frac{\vec{\ell}_1}{2}\right)\cdot\left(\vec{x}_2-\frac{\vec{\ell}_2}{2}\right) - \frac{1}{Q^2}\{\vec{Q}\cdot(\vec{x}_1-\vec{\ell}_1/2)\}\{\vec{Q}\cdot(\vec{x}_2-\vec{\ell}_2/2)\}\right] \right\} \quad (5.13)
 \end{aligned}$$

In formulae (5.7) to (5.9)

$$\begin{aligned}
 I(\alpha, \ell) \equiv & \int d\vec{x}_2 \delta(\alpha-\vec{x}_2\cdot\vec{\ell}) \theta\left(|\vec{x}_2+\frac{\vec{\ell}}{2}|-1\right) \theta\left(1-|\vec{x}_2-\frac{\vec{\ell}}{2}|\right) \\
 = & \frac{2\pi}{\ell^3} \left\{ \frac{1}{2} \left[\left(\alpha - \frac{\ell^2}{2} \right)^2 - \ell^2 \right] \left[\theta\left(\alpha - \frac{\ell^2}{2} - \ell \right) - \theta\left(\alpha - \frac{\ell^2}{2} + \ell \right) \theta(\ell-2) \right] + \right. \\
 & \left. + \ell^2 \alpha \theta(\alpha) \theta(2-\ell) - \frac{1}{2} \left[\left(\alpha + \frac{\ell^2}{2} \right)^2 - \ell^2 \right] \theta\left(\alpha + \frac{\ell^2}{2} - \ell \right) \theta(2-\ell) \right\} \quad (5.14)
 \end{aligned}$$

In the terms A_1, A_2 and A_3 the two photon lines are attached to the same bubble of the corresponding graph (see Fig. 7). These terms are given here as six dimensional integrals, but, by a suitable change of variables, one more integration can be carried out analytically: numerical methods should then be used.

In the terms A_4, A_5 and A_6 the two photons are attached to different bubbles. For these terms an accurate numerical evaluation is more difficult: thus we have resorted to the approximation of setting the two hole momenta both equal to zero which seems to be adequate when both q and ω are large.

Concerning the size of the various terms A_1 is in general dominant although A_3 tends to be as large at small q or small

ω . Instead A_4 , A_5 and A_6 are generally quite negligible and in particular their magnetic component vanishes as q goes to zero.

We discuss now the interference of the correlation diagram with the MEC.

Let us start from the Δ current. One gets, dropping as usual the Pauli exchange contribution, the following expression

$$S_T^{\Delta\text{-corr}}(q, \omega) = \frac{A^2 f_n^3 k^* h M^2}{(\hbar c)^{3/2} 2^6 \pi^8 m_\pi^4 (M_\Delta^2 - M^2)} [A(Q, \nu) - B(Q, \nu)] \quad (5.15)$$

with

$$\begin{aligned} A(Q, \nu) = & \int d\vec{t} \theta(t-1) \int d\vec{t}_1 \theta(1-|\vec{t}-\vec{t}_1|) \int d\vec{t}_2 \delta(\vec{Q}-\vec{t}_1-\vec{t}_2) I\left(\nu + \frac{t_1^2}{2} - \vec{t} \cdot \vec{t}_1, t_2\right) \cdot \\ & \cdot \frac{\theta(|\vec{t}-\vec{Q}|-1)}{[2\nu + Q^2 - 2\vec{Q} \cdot \vec{t}]} \frac{1}{(\ell_2^2 + m_\pi^2)} \left\{ \mu_\nu \frac{\ell_2^2}{\ell_2^2 + m_\pi^2} \left[2\left(\frac{M_\Delta}{M} + 1\right) Q^2 \ell_2^2 - (\vec{Q} \cdot \vec{t}_2)^2 \right] + \right. \\ & \left. + \frac{Q^2}{\ell_1^2 + m_\pi^2} \left(2\frac{M_\Delta}{M} + 1 \right) \left[\frac{1}{2} \mu_\nu \{ Q^2 \ell_2^2 - (\vec{Q} \cdot \vec{t}_2)^2 \} + (\vec{t}_1 \cdot \vec{t}_2) \left\{ (\vec{t} \cdot \vec{t}_2) - \frac{1}{Q^2} (\vec{t} \cdot \vec{Q})(\vec{t}_2 \cdot \vec{Q}) \right\} \right] \right\} \end{aligned} \quad (5.16)$$

$$\begin{aligned} B(Q, \nu) = & \int d\vec{s} \theta(1-s) \int d\vec{t}_1 \theta(|\vec{t}_1 + \vec{s}|-1) \int d\vec{t}_2 \delta(\vec{Q}-\vec{t}_1-\vec{t}_2) I\left(\nu - \frac{t_1^2}{2} - \vec{s} \cdot \vec{t}_1, t_2\right) \cdot \\ & \cdot \frac{\theta(1-|\vec{s}+\vec{Q}|)}{[2\nu - Q^2 - 2\vec{Q} \cdot \vec{s}]} \frac{1}{(\ell_2^2 + m_\pi^2)} \left\{ \mu_\nu \frac{\ell_2^2}{\ell_2^2 + m_\pi^2} \left[2\left(\frac{M_\Delta}{M} + 1\right) Q^2 \ell_2^2 - (\vec{Q} \cdot \vec{t}_2)^2 \right] + \right. \\ & \left. + \frac{Q^2}{\ell_1^2 + m_\pi^2} \left(2\frac{M_\Delta}{M} + 1 \right) \left[\frac{1}{2} \mu_\nu \{ Q^2 \ell_2^2 - (\vec{Q} \cdot \vec{t}_2)^2 \} - (\vec{t}_1 \cdot \vec{t}_2) \left\{ (\vec{s} \cdot \vec{t}_2) - \frac{1}{Q^2} (\vec{s} \cdot \vec{Q})(\vec{t}_2 \cdot \vec{Q}) \right\} \right] \right\} \end{aligned} \quad (5.17)$$

In the previous formulae $I(\alpha, \ell_2)$ is given by (5.14) and four of the nine integrations can be performed, thus leaving five-dimensional integrals to be treated with numerical methods. It should also be noticed that we have found here more convenient to keep the variable $\vec{t} = \vec{p}/k_F$ in $A(Q, \nu)$ and

the variable $\vec{s} = \vec{k}/k_F$ in $B(Q, \nu)$.

It is interesting to introduce at this stage the 2p-2h propagator of the spin-isospin polarization. For this purpose we disentangle the two photon lines in all the diagrams associated with the correlation and the Δ currents (the contact and pion-in-flight terms are peculiar of photon absorption). Keeping only the magnetic part of S_T^{corr} , S_T^Δ and $S_T^{\Delta\text{-corr}}$, which is dominant, one gets

$$\text{Im } \Pi^{2p-2h}(q, \omega) = \frac{8\pi^2 M g}{A^2 \hbar^2 q^2} \frac{(-1)}{G_M^2(q_\lambda^2)(\mu_S^2 + \mu_V^2)} \left\{ S_T^{\text{corr}}(q, \omega) + S_T^\Delta(q, \omega) + S_T^{\Delta\text{-corr}}(q, \omega) \right\} \quad (5.18)$$

Via a dispersion relation, its real part could be obtained as well. We will utilize (5.18) in Sect.6 to discuss the relationship between photon and pion absorption.

Concerning the interference of the contact and pion-in-flight currents with the correlation one, the expressions of their contribution to the transverse nuclear response are the following

$$S_T^{\text{cont-corr}}(q, \omega) = - \frac{3A^2 M^2}{8\pi^6 \hbar^2 \kappa_F^2} \left(\frac{g_\pi^2}{4\pi \hbar c} \right)^2 \frac{1}{m_\pi^4} \left[C(Q, \nu) - D(Q, \nu) \right] \quad (5.19)$$

with

$$\begin{aligned} C(Q, \nu) = & \int d\vec{t} \theta(t-1) \int d\vec{\ell}_1 \theta(1-|\vec{t}-\vec{\ell}_1|) \int d\vec{\ell}_2 \delta(\vec{Q}-\vec{\ell}_1-\vec{\ell}_2) \frac{1}{\ell_2^2 + m_\pi^2} \times \\ & \times I\left(\nu + \frac{\ell_1^2}{2} - \vec{t} \cdot \vec{\ell}_1, \ell_2\right) \frac{\theta(|\vec{t}-\vec{Q}|-1)}{2\nu + Q^2 - 2\vec{Q} \cdot \vec{t}} \left\{ \mu_V \left[\frac{2\ell_2^2(\vec{Q} \cdot \vec{\ell}_2)}{\ell_2^2 + m_\pi^2} + \frac{\ell_2^2 Q^2 - (\vec{Q} \cdot \vec{\ell}_2)^2}{\ell_1^2 + m_\pi^2} \right] - \right. \\ & \left. - \left[\frac{2\ell_2^2}{\ell_2^2 + m_\pi^2} - \frac{2\vec{\ell}_1 \cdot \vec{\ell}_2}{\ell_1^2 + m_\pi^2} \right] \left[\vec{t} \cdot \vec{\ell}_2 - \frac{1}{Q^2} (\vec{t} \cdot \vec{Q})(\vec{\ell}_2 \cdot \vec{Q}) \right] \right\} \end{aligned} \quad (5.20)$$

$$\begin{aligned}
 D(Q, \nu) = & \int d\vec{s} \, \theta(1-s) \int d\vec{\ell}_1 \, \theta(|\vec{s} + \vec{\ell}_1| - 1) \int d\vec{\ell}_2 \, \delta(\vec{Q} - \vec{\ell}_1 - \vec{\ell}_2) \frac{1}{\ell_2^2 + m_\pi^2} \times \\
 & \times I\left(\nu - \frac{\ell_1^2}{2} - \vec{s} \cdot \vec{\ell}_1, \ell_2\right) \frac{\theta(1 - |\vec{s} + \vec{Q}|)}{2\nu - Q^2 - 2\vec{s} \cdot \vec{Q}} \left\{ \mu_\nu \left[\frac{2\ell_2^2(\vec{Q} \cdot \vec{\ell}_2)}{\ell_2^2 + m_\pi^2} + \frac{\ell_2^2 Q^2 - (\vec{Q} \cdot \vec{\ell}_2)^2}{\ell_1^2 + m_\pi^2} \right] + \right. \\
 & \left. + \left[\frac{2\ell_2^2}{\ell_2^2 + m_\pi^2} - \frac{2(\vec{\ell}_1 \cdot \vec{\ell}_2)}{\ell_1^2 + m_\pi^2} \right] \left[\vec{s} \cdot \vec{\ell}_2 - \frac{1}{Q^2} (\vec{s} \cdot \vec{Q})(\vec{\ell}_2 \cdot \vec{Q}) \right] \right\} \quad (5.21)
 \end{aligned}$$

and

$$S_T^{\pi\text{-corr}}(q, \omega) = \frac{3A^2 M^2}{4\pi^6 \hbar^2 k_F^2} \left(\frac{f_\pi^2}{4\pi \hbar c} \right)^2 \frac{1}{m_\pi^4} [E(Q, \nu) - F(Q, \nu)] \quad (5.22)$$

with

$$\begin{aligned}
 E(Q, \nu) = & \int d\vec{t} \, \theta(t-1) \int d\vec{\ell}_1 \, \theta(1 - |\vec{t} - \vec{\ell}_1|) \int d\vec{\ell}_2 \, \delta(\vec{Q} - \vec{\ell}_1 - \vec{\ell}_2) \\
 & \times I\left(\nu + \frac{\ell_1^2}{2} - \vec{t} \cdot \vec{\ell}_1, \ell_2\right) \frac{\ell_2^2}{(\ell_1^2 + m_\pi^2)(\ell_2^2 + m_\pi^2)^2} \frac{\theta(1 - |\vec{t} - \vec{Q}| - 1)}{2\nu + Q^2 - 2\vec{Q} \cdot \vec{t}} \times \\
 & \times \left\{ \mu_\nu [Q^2 \ell_2^2 - (\vec{Q} \cdot \vec{\ell}_2)^2] + 2(\vec{\ell}_1 \cdot \vec{\ell}_2) \left[\vec{t} \cdot \vec{\ell}_2 - \frac{1}{Q^2} (\vec{\ell}_2 \cdot \vec{Q})(\vec{t} \cdot \vec{Q}) \right] \right\} \quad (5.23)
 \end{aligned}$$

$$\begin{aligned}
 F(Q, \nu) = & \int d\vec{s} \, \theta(1-s) \int d\vec{\ell}_1 \, \theta(|\vec{s} + \vec{\ell}_1| - 1) \int d\vec{\ell}_2 \, \delta(\vec{Q} - \vec{\ell}_1 - \vec{\ell}_2) \\
 & \times I\left(\nu - \frac{\ell_1^2}{2} - \vec{s} \cdot \vec{\ell}_1, \ell_2\right) \frac{\ell_2^2}{(\ell_1^2 + m_\pi^2)(\ell_2^2 + m_\pi^2)^2} \frac{\theta(1 - |\vec{s} + \vec{Q}|)}{2\nu - Q^2 - 2\vec{Q} \cdot \vec{s}} \times \\
 & \times \left\{ \mu_\nu [Q^2 \ell_2^2 - (\vec{Q} \cdot \vec{\ell}_2)^2] - 2(\vec{\ell}_1 \cdot \vec{\ell}_2) \left[\vec{s} \cdot \vec{\ell}_2 - \frac{1}{Q^2} (\vec{\ell}_2 \cdot \vec{Q})(\vec{s} \cdot \vec{Q}) \right] \right\} \quad (5.24)
 \end{aligned}$$

Again in (5.20) to (5.24) $I(\alpha, \ell_2)$ is given by (5.14) and the numerical problem can be reduced to a five-dimensional integration.

Before comparing with the experimental data we split, as anticipated in Section 4, all the contributions to the transverse nuclear response associated with the correlation current into the central and tensor components relative to the pionic line. The corresponding formulas will be given in the Appendix.

We set $\Lambda_\pi = 0.9$ GeV and $g' = 0.5$: the latter brings our central interaction close to the one calculated via a G-matrix ⁽¹⁵⁾. In Figs. 9 to 12 the transverse nuclear response measured in deep inelastic electron scattering on ⁵⁶Fe is displayed for several momentum transfers together with our theoretical predictions. It is clearly seen that the 2p-2h excitations remarkably improve the agreement in the quasi-elastic peak region, particularly around the maximum of the nuclear response.

At the same time they yield a significant contribution in the energy range between the quasi-elastic and the Δ peaks, which more than double the one stemming from the MEC alone, as it appears by inspecting the above figures. Notice that the central correlations are far from negligible with respect to the tensor ones in the kinematical region we explore. This can be inferred from Table I where a detailed analysis of the various contributions to the nuclear response is given.

For energies above 150 MeV we are still somewhat below the experimental data; indeed in this region the pion electroproduction cross section should also be included. Moreover we should not forget that the present calculation is affected by $\sim 20\%$ uncertainty which can be attributed to the neglect

of Pauli exchange contributions in the 2p-2h response, as well as of the mean field effect on the nucleonic lines.

In conclusion we believe that our results, which give a reasonable fit to the data, shed light on the crucial role of nucleon-nucleon correlations in deep inelastic electron scattering.

6. The absorption of pions at threshold and of real photons

A) Pion absorption

The pion field in nuclei is strongly coupled to the nucleonic degrees of freedom. This can be expressed in the 1p-1h framework by the Dyson equation for the pion propagator in the medium

$$D(q, \omega) = D_0(q, \omega) + D_0(q, \omega) q^2 4 \frac{f_\pi^2}{\mu_\pi^2} \tilde{\Pi}(q, \omega) D(q, \omega) \quad (6.1)$$

where D_0 is the free pion propagator and $\tilde{\Pi}$ the p-h polarization propagator irreducible with respect to one pion exchange and summed to all orders. Neglecting Pauli exchange graphs it reads

$$\tilde{\Pi}(q, \omega) = \frac{\Pi^0(q, \omega)}{1 - 4(f_\pi^2/\mu_\pi^2) q^2 \Pi^0(q, \omega)} \quad (6.2)$$

and the solution of (6.1) is

$$D^{-1}(q, \omega) = \omega^2/c^2 - q^2 \mu_\pi^2 - q^2 4 \frac{f_\pi^2}{\mu_\pi^2} \tilde{\Pi}(q, \omega) \quad (6.3)$$

In the region $\hbar\omega \simeq m_\pi c^2$ and q small the pion self energy (or optical potential) $-q^2 4(f_\pi^2/\mu_\pi^2) \tilde{\Pi}(q, \omega)$ as given by (6.2) is purely real and dominated by the Δ contribution. However the 1p-1h description is insufficient, since it is well known that the pion-nucleus optical potential also contains an absorptive piece. The latter arises from the coupling of the pion to 2p-2h excitations and, as far as p-wave absorption is concerned, it has been sometimes parametrized as

$$2\hbar\omega \operatorname{Im} \langle \vec{q} | U_{\text{opt}} | \vec{q} \rangle = q^2 4\pi \varrho^2 \operatorname{Im} C_0 \quad (6.4)$$

where ϱ is the nuclear density.

This optical potential is well determined from the π -mesic atom data which provide an unambiguous determination of its imaginary part, namely

$$\operatorname{Im} C_0 \cong (0.05 \div 0.06) \mu_\pi^{-6}. \quad (6.5)$$

However the 2p-2h excitations contribute also to the real part of U_{opt} and the corresponding parameter $\operatorname{Re} C_0$ is far less known than (6.5). Indeed the real part of C_0 adds to the 1p-1h expression (6.2) for the optical potential, which in turn is very sensitive to the actual value of g' .

A different type of parametrization incorporates the effect of the 2p-2h excitations into the bare polarization propagator Π^0 . The full pion-nucleus optical potential is accordingly expressed as

$$2\hbar\omega \langle \vec{q} | U_{\text{opt}} | \vec{q} \rangle = \frac{-q^2 \alpha(q, \omega)}{1 - g' \alpha(q, \omega)} \quad (6.6)$$

where

$$\alpha(q, \omega) = 4 \frac{f_\pi^2}{\mu_\pi^2} \Pi^0(q, \omega) - 4\pi \varrho^2 \mathcal{E}_0 \quad (6.7)$$

and its imaginary part reads

$$2\hbar\omega \operatorname{Im} \langle \vec{q} | U_{\text{opt}} | \vec{q} \rangle \cong \frac{q^2 4\pi \varrho^2 \operatorname{Im} \mathcal{E}_0}{\{1 - g' [4(f_\pi^2/\mu_\pi^2) \operatorname{Re} \Pi^0 - 4\pi \varrho^2 \operatorname{Re} \mathcal{E}_0]\}^2} \quad (6.8)$$

Therefore the value of $\operatorname{Im} \mathcal{E}_0$ depends critically upon both g' and $\operatorname{Re} \mathcal{E}_0$. For instance with $g' = 1/3$ and $\operatorname{Re} \mathcal{E}_0 = -0.1 \mu_\pi^{-6}$

one would get $\text{Im } \mathcal{C}_0 = 0.09 \mu_\pi^{-6}$ whereas $g' = 0$ would correspond to the value (6.5). However g' and $\text{Re } \mathcal{C}_0$ are not independent: in fact they are linked by the data fixing the real part of the optical potential.

Tauscher⁽¹⁶⁾ has performed a very careful analysis of the π -mesic atom data with the special aim of determining separately the different parameters g' (called $\xi/3$ in his work), $\text{Im } \mathcal{C}_0$ and $\text{Re } \mathcal{C}_0$. His best fit yields $g' = 0.4 \pm 0.17$, $\text{Im } \mathcal{C}_0 = 0.11 \mu_\pi^{-6}$ and $\text{Re } \mathcal{C}_0 = 0.2 \mu_\pi^{-6}$. The extreme value $g' = 0.57$ is consistent with

$$\text{Im } \mathcal{C}_0 = 0.18 \mu_\pi^{-6}; \quad \text{Re } \mathcal{C}_0 \cong 0.4 \mu_\pi^{-6}. \quad (6.9)$$

The large difference between (6.5) and (6.9) reflects processes of vertex renormalization for the pion absorbed by a pair of correlated nucleons (LLEE effect⁽¹⁷⁾). Microscopically this corresponds to diagrams of the type shown in Fig.13, where only the Landau-Migdal force is retained in the interaction lines between the incoming pion and the 2p-2h sector of the graphs (pion exchange represents a distortion effect which should not appear in the π -nucleus optical potential).

By comparing (6.8) with (6.4) one obtains the relationship between the two parametrizations:

$$\text{Im } \mathcal{C}_0 = \frac{\text{Im } \mathcal{C}_0}{\left\{ 1 - g' \left[4 \left(\frac{p_\pi^2}{\mu_\pi^2} \right) \text{Re } \Pi^0 - 4 \pi g^2 \text{Re } \mathcal{C}_0 \right] \right\}^2}. \quad (6.10)$$

In the frame of the microscopic approach described in the previous section we have evaluated the absorptive p-wave pion-nucleus optical potential through the imaginary part of

the 2p-2h polarization propagator, $\text{Im} \Pi^{2p-2h}$ of eq. (5.18). Since this quantity does not include vertex renormalizations, it has to be related to the "bare" quantity $\mathcal{E}_0$ according to the formula

$$\text{Im} \mathcal{E}_0 = - \frac{1}{\pi} \frac{f_n^2}{\mu_n^2} \frac{1}{g^2} \text{Im} \Pi^{2p-2h}(q=0, \hbar\omega=m_n c^2). \quad (6.11)$$

We recall that for the p-wave π -absorption only the correlation diagrams of Fig.8 and the Δ diagrams together with their interference come into play. Moreover the electric contribution must be dropped, as we are considering a magnetic process.

With $\Lambda_\pi = 0.8$ GeV and $g' = 0.5$ we obtain

$$\text{Im} \mathcal{E}_0 = 0.23 \mu_n^{-6} \quad (6.12)$$

which is far too large with respect to the dressed experimental value (6.5), but quite compatible with (6.9). Thus this outcome asks for a strong LLEE effect in the p-wave optical potential, even allowing for sizable uncertainties and limitations in the present calculation. Indeed the only substantial factor we can think of to lower (6.12) down to (6.5) is the denominator of (6.10).

Before reaching this conclusion we have explored all possible other effects which could influence our calculation by reducing the estimate (6.12).

To start with we notice that in (6.12) the central contribution amounts to $0.03 \mu_n^{-6}$ only and is further reduced when the Pauli exchange diagrams are properly included. Moreover it depends upon g' and becomes vanishingly small for $g' = 1/3$,

as a result of the cancellation between the short range repulsion embodied in g' and the central attraction provided by the pion. However this uncertainty does not affect our estimate by more than 13%, even by totally neglecting the central contribution.

A further effect which could bring our value of $\text{Im } \mathcal{C}_0$ down lies in the binding energies (obviously neglected in our Fermi gas approach) of the two nucleons absorbing the pion. Although aware of the risk of double counting ^(4, 13), to get an orientation, we have phenomenologically included a binding of ~ 20 MeV in the appropriate energy denominators. This reduces (6.12) to ⁽⁺⁾

$$\text{Im } \mathcal{C}_0 \cong 0.18 \mu_n^{-6} \quad (6.13)$$

still far from the value of $\text{Im } C_0$.

We do not believe that the Pauli exchange diagrams will essentially alter the situation, nor will the distorsion of the emitted nucleons in their way out of the nucleus, owing to the transparency of the nuclear matter ⁽¹⁸⁾.

Therefore we are led to ascribe the difference between (6.13) and (6.5) to a strong LEE quenching, as it is expressed by equation (6.10). On the other hand the factor of ~ 3 thereby needed seems to be compatible with the experimental fits of Tauscher ⁽¹⁶⁾ providing that the 2p-2h polarization propagator has a large, negative real part. Indeed our value $\text{Im } \mathcal{C}_0 = 0.18 \mu_n^{-6}$ would require, in his analysis, $g' \cong 0.6$ and $\text{Re } \mathcal{C}_0 \cong 0.36 \mu_n^{-6}$.

(+) Notice that binding effects do not influence in a significant way the 2p-2h nuclear response in the deep inelastic electron scattering.

This point will be made more quantitative in a forthcoming paper, where we intend to calculate microscopically the $\text{Re}\Pi^{2p-2h}(q, \omega)$, in order to perform a fully consistent RPA theory based on a (1p-1h, 2p-2h) irreducible kernel.

However even before performing an explicit calculation of $\text{Re}\Pi^{2p-2h}$ we can state, relying on the results of the 1p-1h sector, that the LLEE quenching is not a smooth function of energy and momentum. Indeed in the kinematical region of π absorption at threshold, which is well above the (N, N^{-1}) continuum, Π^0 is dominated by the Δ -hole propagator since both the real and imaginary parts of Π^N vanish.

On the contrary the energy-momentum region between the quasi-elastic and Δ -peaks, explored by deep inelastic electron scattering, lies just on the boundary of the (N, N^{-1}) continuum. Here $\text{Re}\Pi^N$ and $\text{Re}\Pi^\Delta$ almost cancel each other thus unaffected the bare 2p-2h nuclear response even when inserted in an RPA expression analogous to (6.10). Of course a precise discussion of these points should also include the real part of the 2p-2h polarization propagator.

Since we have seen that the bare 2p-2h polarization propagator well accounts for the experiment in deep inelastic electron scattering, but not for the absorptive piece of the pion-nucleus optical potential at threshold, we conclude that the two processes can be reconciled in a unique theoretical frame only if a strong LLEE quenching affects the latter quantity reducing it by about a factor of 3.

This view is supported on one hand by the analysis of the data performed by Tauscher and on the other by the extensive theoretical work of Oset and Weise ⁽¹⁹⁾, who evaluated

both real and imaginary parts of $\mathcal{E}_0$ in a microscopic approach. Their value for $\text{Im } \mathcal{E}_0$ ($\sim 0.09 \mu_\pi^{-6}$) was not too large, since they considered only the effects of isobar-hole excitations (whereas we have found that the contribution of the correlation diagrams and of their interference with the Δ is substantial). However they also found a large $\text{Re } \mathcal{E}_0$ which favours the existence of a LLEE effect.

As a short-cut to the microscopic approach we like to propose an approximate formula which spares the cumbersome calculations involved in the graphs entering into $\text{Im } \Pi^{2p-2h}$. For this purpose note that (6.11) can be extrapolated up to appreciable values of q .

Indeed it turns out that $\text{Im } \Pi^{2p-2h}$, for fixed energy, is practically independent upon the external momentum q as shown in Table II. As a consequence (6.11) can be directly extended to the region explored by deep inelastic electron scattering, where it accounts fairly well for the data if $\text{Im } \mathcal{E}_0$ is given by (6.9) (of course the pion-in-flight and the contact terms, albeit small, should be added separately to the analysis). Note that the extrapolation is also valid in an energy band of ~ 100 MeV centered around $\hbar\omega = m_\pi c^2$ since $\text{Im } \Pi^{2p-2h}$ is a smooth function of the energy.

In this way processes induced by different probes are actually linked together, in spite of the distinct nature of the couplings: transverse for the photon ($\vec{\sigma} \times \vec{q}$) and longitudinal for the pion ($\vec{\sigma} \cdot \vec{q}$).

It should be realized that the link does not apply to the cross sections of the actual processes, but exists only at a

local level, as was suggested by T.E.O. Ericson and Bernabeu⁽⁸⁾ for the axial current of the nucleon. In this conceptual frame the photon momentum has to be replaced by the local pion momentum in the medium $\vec{\nabla}\varphi$, which is naturally embodied in the p-wave pion-nucleus optical potential

$$\text{Im} (2\hbar\omega U_{opt}) = 4\pi \text{Im} e_0 (\vec{\nabla}\varphi)^2 \quad (6.14)$$

Finally we stress that the strong LLEE quenching we believe to exist in the p-wave pion absorption also applies to the Gamow-Teller strength near pion threshold. This quenching is associated with the Δ degree of freedom and likely also with the 2p-2h excitations.

It becomes thus important to explore through the actual calculation of the 2p-2h polarization propagator whether these two contributions still cooperate at low frequencies, and in which proportion, to quench the Gamow-Teller strength^(20,21).

B) Photon absorption

In terms of the transverse nuclear response function the cross section for photon absorption on nuclei reads, according to Drell and Walecka⁽²²⁾,

$$\sigma_y^{tot}(\omega) = (2\pi)^3 \frac{e^2}{Mc\omega} \frac{1}{A} S_T(\omega=cq) \quad (6.15)$$

In an infinite homogeneous system the (N, N^{-1}) domain is inaccessible to real photon absorption, as it occurs for pions. This is not so for a finite system, for photon energies up to $20 \div 30$ MeV, due to the discretization of the single particle levels.

As a consequence the crucial role in photoabsorption is again played by the $2p-2h$ excitations. Up to energies of about 150 MeV the currents of relevance for this process are, beyond the correlation one, the pion-in-flight and the contact : the possibility of linking real photons and pions absorption is thus ruled out.

We have performed the explicit evaluation of (6.15) including in S_T all the MEC and the magnetic component of the correlation current and then we have compared our σ_Y^{tot} with the parametrization of Bergere et al.⁽²³⁾

$$\sigma_Y^{M.Q.D.}(\omega) = \frac{ZN}{A} L e^{-\Delta/\hbar\omega} \frac{62.4}{(\hbar\omega)^{3/2}} \text{ mb} \quad (6.16)$$

for the quasideuteron component of the total experimental γ -absorption cross section^(7,24). In (6.16) $L = 10$ and $\Delta \cong 80$ MeV.

The results are shown in Fig.14, where two theoretical curves are displayed. The first one (a) does not incorporate any vertex renormalization, while the second (b) embodies them in the form of a LEE quenching of the order of ~ 3 for the correlation and Δ terms. It appears that the latter follows fairly well the "experimental" predictions (6.16).

In the same figure the cross section given by the old Levinger's quasideuteron model⁽²⁵⁾, i.e., the deuteronic one scaled by a constant factor, is also displayed. Its low energy behaviour is totally inadequate, whereas our nuclear matter description has the proper trend. This reflects the influence of the Pauli blocking on the two ejected nucleons, which becomes progressively stronger as the energy is lowered below $2\epsilon_F$ and is phenomenologically accounted for by the exponential factor in formula (6.16). This feature, suggested by Levinger⁽²⁶⁾ as an improvement to his model, is naturally embodied in our formalism.

In spite of the good agreement of our approach with the "experiment", we cannot look at our theory as satisfactory. Indeed the electric contribution to the correlation term has been, here, completely ignored.

Should we have incorporated it naïvely in our calculation without worrying about gauge invariance, we would have totally spoiled the agreement, the cross section becoming far too large.

Therefore any conclusion on the real photon absorption on nuclei should wait for a deeper understanding of the nature of the electric components in the correlation term, which however, according to our results, seems to be small.

Should this be the case, then the microscopic foundation of the quasideuteron model would rely mostly on the pion-in-flight and contact terms.

7. Conclusions

The purpose of this paper is to investigate the role played by the 2p-2h excitations in the spin-isospin nuclear response to various probes.

It is well known that a variety of phenomena in nuclear physics is affected by the 2p-2h states, whose origin is ultimately linked to the mesons exchanged among nucleons.

In this connection the role of the mesons in nuclei is dual: on the one hand they give rise to two-body currents, to be added to the conventional nucleonic one, which most naturally lead to the excitation of 2p-2h states; on the other, they correlate pairs of nucleons which will consequently act as a unique entity in the nuclear response to an external field.

In the present work we aimed at dealing consistently with these two roles of the pion by explicitly considering all the perturbative Feynman diagrams where it is once exchanged, also allowing for the Δ degree of freedom.

However while the pion may indeed be considered as the main carrier of the meson exchange currents, it cannot be looked upon as the major responsible of the nucleon-nucleon correlations in the nucleus. We have remedied this inconsistency through an "ad hoc" modification of the πNN vertex in the tensor component of the force carried by the pion, which brings it close to a realistic tensor effective interaction, and by adding to the central component the conventional Landau-Migdal parameter g' .

To test our theory we have analyzed the transverse cross-section from the inclusive deep inelastic electron scattering: it turns out that the 2p-2h excitations remarkably improve the agreement between theory and experiment, yielding a substantial contribution to the cross section between the quasi-elastic and the Δ peaks. This result is in line with the findings of Donnelly and Van Orden, but the existence of dynamical correlations among nucleons more than doubles the response as given by the MEC alone, bringing it closer to the experimental data over a wide range of momenta.

In spite of certain shortcomings of our approach (for example we have neglected the self-energy insertions on nucleonic lines, the antisymmetrization, the dynamical propagation of the internal pions, etc.), we feel having attained a satisfactory understanding of the inclusive deep inelastic electron scattering.

In order to check the validity of our description of the 2p-2h nuclear response with respect to other probes we have next investigated the absorptive p-wave pion-nucleus optical potential.

For this purpose we have evaluated the imaginary part of the 2p-2h polarization propagator for $q = 0$ and $\hbar\omega = m_\pi c^2$ utilizing the Feynman diagrams which describe the nucleon-nucleon correlation and the Δ resonance.

By comparing the results of our calculation with the analysis of the pionic atom data there emerges the necessity to introduce a strong LLEE quenching factor. Indeed, before getting absorbed, the pion polarizes the medium through the action of the short range Landau-Migdal force g' . This vertex

renormalization could be conveniently dealt with in an RPA scheme embodying both the 2p-2h and the 1p-1h polarization propagators.

On the basis of our microscopic analysis we have next addressed the question whether a link between the magnetic-photon and pion absorptions can be established in spite of the different nature of the couplings involved in the two processes and of the disparity in the momenta carried by the two probes at a fixed energy.

Our conclusion is that, in the absence of vertex renormalizations, the connection does indeed exist and it simply amounts to replace the photon momentum by the local pion momentum. In such a situation the photon absorption cross section at a given energy would be directly proportional to the p-wave absorptive π -nucleus optical potential for the same energy (apart from trivial form factor effects).

The existence of vertex renormalizations alters, however, the above simple picture since they crucially depend on q at a fixed energy. For instance they strongly reduce the absorption of pions while leaving practically unchanged the response to deep inelastic electron scattering.

Finally we have explored whether our theory can also account for photon absorption in nuclei in the energy range above the giant resonances. We have found that it successfully reproduces the trend of the cross section also at low energy, where the effect of the Pauli blocking on the two emitted nucleons is particularly felt.

As far as the magnitude of the cross section is concerned, we cannot claim at present to account for it. Indeed our evaluation of $\sigma_Y^{tot}(\omega)$ is quite sensitive to the electric component of the correlation current and a reliable evaluation of this term requires a better control of gauge invariance, which we have not yet achieved.

In summarizing, the study carried out in this paper shows that the 2p-2h states not only play a relevant role in the $\sigma = \tau = 1$ nuclear response as "bare" excitations but also emphasize, at low momenta, important collective features of nuclear structure.

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Appendix

We derive here explicitly the separation between central and tensor components in the 2p-2h response associated with the nucleon-nucleon correlations.

For this purpose let us rewrite (4.3) as follows

$$\begin{aligned}
 J_i^{\text{corr}}(\vec{p}_1, \vec{p}_1', \vec{p}_2, \vec{p}_2') &= \frac{4M^2 c^4}{V^2 \hbar c} \frac{g_n^2}{\mu_n^2} \frac{1}{(k_2^2 + \mu_n^2)} \left\{ D_+^{-1} \left[i(\vec{\sigma}_1 \times \vec{q})_i (\vec{k}_2 \cdot \vec{\sigma}_1) \{ \mathcal{F}_E(\mu_s + 2g_2 \mu_v) + \right. \right. \\
 &+ g_2 \mathcal{F}_D(\mu_v + 2g_1 \mu_s) \} + (p_1' + p_1 - k_2)_i (\vec{k}_2 \cdot \vec{\sigma}_1) \{ (1 + 2g_2) \mathcal{F}_E + g_2 (1 + 2g_1) \mathcal{F}_D \} \left. \right] (\vec{k}_2 \cdot \vec{\sigma}_2) + \\
 &+ D_-^{-1} \left[-i(\vec{k}_2 \cdot \vec{\sigma}_1) (\vec{\sigma}_1 \times \vec{q})_i \{ \mathcal{F}_E(\mu_s - 2g_2 \mu_v) + g_2 \mathcal{F}_D(\mu_v + 2g_1 \mu_s) \} - (p_1' + p_1 + k_2)_i (\vec{k}_2 \cdot \vec{\sigma}_1) \cdot \right. \\
 &\left. \left. * \{ (1 - 2g_2) \mathcal{F}_E + g_2 (1 + 2g_1) \mathcal{F}_D \} \right] (\vec{k}_2 \cdot \vec{\sigma}_2) \right\} + \{ 1 \rightleftharpoons 2 \}
 \end{aligned} \tag{A.1}$$

where

$$D_{\pm} = \frac{2M\omega}{\hbar} - \vec{q} \cdot (\vec{p}_1 + \vec{p}_1') \pm \vec{q} \cdot \vec{k}_2 \tag{A.2}$$

and the indices on the spin operators imply that they act between the spin states of the corresponding nucleon, e.g.:

$$(\vec{\sigma}_1 \times \vec{q})_i (\vec{k}_2 \cdot \vec{\sigma}_1) \equiv \chi_{s_1'}^+ (\vec{\sigma}_1 \times \vec{q})_i (\vec{k}_2 \cdot \vec{\sigma}) \chi_{s_1}. \tag{A.3}$$

We can now separate in (A.1) the central and tensor components of the product $(\vec{k}_2 \cdot \vec{\sigma}_1)(\vec{k}_2 \cdot \vec{\sigma}_2)$ according to

$$\begin{aligned}
 (\vec{k}_2 \cdot \vec{\sigma}_1)(\vec{k}_2 \cdot \vec{\sigma}_2) &= \frac{1}{3} k_2^2 \left\{ \vec{\sigma}_1 \cdot \vec{\sigma}_2 + [3(\vec{\sigma}_1 \cdot \hat{k}_2)(\vec{\sigma}_2 \cdot \hat{k}_2) - \vec{\sigma}_1 \cdot \vec{\sigma}_2] \right\} \\
 &= \frac{1}{3} k_2^2 \left\{ \vec{\sigma}_1 \cdot \vec{\sigma}_2 + S_{12}(\vec{k}_2) \right\}
 \end{aligned} \tag{A.4}$$

The current (A.1) will then be split as follows

$$J_i^{\text{corr}} = J_{i(c)}^{\text{corr}} + J_{i(\tau)}^{\text{corr}} \quad (\text{A.5})$$

where

$$\begin{aligned} J_{i(c)}^{\text{corr}}(\vec{p}_1, \vec{p}_1', \vec{p}_2, \vec{p}_2') &= \frac{4M^2 c^4}{V^2 \hbar c} \cdot \frac{f_\pi^2}{\mu_\pi^2} \cdot \frac{1}{3} \cdot \frac{k_2^2}{(k_2^2 + \mu_\pi^2)} \times \\ &\times \left\{ D_+^{-1} \left[i(\vec{\sigma}_1 \times \vec{q})_i (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \left\{ \mathcal{F}_E(\mu_s + 2g_2 \mu_v) + g_2 \mathcal{F}_D(\mu_v + 2g_1 \mu_s) \right\} + (p_1' + p_1 - k_2)_i (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \right. \right. \\ &\times \left. \left\{ \mathcal{F}_E(1 + 2g_2) + g_2 \mathcal{F}_D(1 + 2g_1) \right\} \right] + D_-^{-1} \left[-i(\vec{\sigma}_1 \cdot \vec{\sigma}_2) (\vec{\sigma}_1 \times \vec{q})_i \left\{ \mathcal{F}_E(\mu_s - 2g_2 \mu_v) + \right. \right. \\ &\left. \left. + g_2 \mathcal{F}_D(\mu_v + 2g_1 \mu_s) \right\} - (p_1' + p_1 + k_2)_i (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \left\{ \mathcal{F}_E(1 - 2g_2) + \mathcal{F}_D g_2(1 + 2g_1) \right\} \right] \right\} \quad (\text{A.6}) \\ &+ \{1 \leftrightarrow 2\} \end{aligned}$$

and the tensor piece is obtained by the difference between (A.1) and (A.6).

When the current (A.5) is utilized in formula (3.7) one gets, in principle, three different contributions to the transverse response, which arise from the products $J_{i(c)}^{\text{corr}\dagger} J_{j(c)}^{\text{corr}}$, $J_{i(\tau)}^{\text{corr}\dagger} J_{j(\tau)}^{\text{corr}}$ and $J_{i(c)}^{\text{corr}\dagger} J_{j(\tau)}^{\text{corr}}$ (or $J_{i(\tau)}^{\text{corr}\dagger} J_{j(c)}^{\text{corr}}$), respectively. The first two have a purely central (or tensor) character, the last one reflects the interference between the two terms of (A.5).

It turns out that the dominant terms of S_T^{corr} , namely A_1 and A_3 defined in (5.7) and (5.9), are simply split into the central and tensor components which amount to 1/3 and 2/3 of the total, respectively. Therefore (5.7), for example, will be rewritten as

$$A_1(Q, \nu) = A_1^c(Q, \nu) + A_1^T(Q, \nu) \quad (\text{A.7})$$

with

$$A_1^c(Q, \nu; g) = 2 \int d\vec{t} \theta(t-1) \int d\vec{t} \theta(1-|\vec{t}+\vec{t}-\vec{Q}|) \left\{ 3g' - \frac{\ell^2}{\ell^2+m_\pi^2} \right\}^2 \Gamma_\pi^4(\ell^2) \times$$

$$\times \frac{\theta(|\vec{t}-\vec{Q}|-1)}{[2\nu+Q^2-2\vec{Q}\cdot\vec{t}]^2} I \left[\nu - \vec{t}\cdot(\vec{Q}-\vec{t}) + \frac{1}{2}(\vec{Q}-\vec{t})^2, \ell \right] \left\{ Q^2(\mu_s^2+\mu_v^2) + 4 \left[t^2 - \frac{1}{Q^2}(\vec{Q}\cdot\vec{t})^2 \right] \right\}$$

(A.8)

and

$$A_1^T(Q, \nu) = 2 A_1^c(Q, \nu; g'=0). \quad (A.9)$$

In the above formulae we have explicitly included the π NN form factors; moreover the Landau-Migdal parameter g' appears in the central piece (A.8).

For the other terms of S_T^{corr} the situation is more complicated since central and tensor components can also interfere. For the sake of illustration, let us consider A_2 , defined in Eq.(5.8). Its purely central part reads

$$A_2^c(Q, \nu) = \frac{1}{9} \int d\vec{t} \theta(t-1) \int d\vec{t} \theta(1-|\vec{t}+\vec{t}-\vec{Q}|) \left\{ 3g' - \frac{\ell^2}{\ell^2+m_\pi^2} \right\}^2 \Gamma_\pi^4(\ell^2) \times$$

$$\times \frac{\theta(1-|\vec{t}+\vec{t}|)}{[\nu-\vec{Q}\cdot(\vec{t}+\vec{t}-\vec{Q}/2)]} \frac{\theta(|\vec{t}-\vec{Q}|-1)}{[\nu-\vec{Q}\cdot(\vec{t}-\vec{Q}/2)]} I \left[\nu - \vec{t}\cdot(\vec{Q}-\vec{t}) + \frac{1}{2}(\vec{Q}-\vec{t})^2, \ell \right] \times$$

$$\times \left\{ Q^2(3\mu_s^2-\mu_v^2) - 12 \left[\vec{t}\cdot(\vec{t}+\vec{t}) - \frac{1}{Q^2}(\vec{Q}\cdot\vec{t})(\vec{t}+\vec{t})\cdot\vec{Q} \right] \right\}$$

(A.10)

whereas the purely tensor component is

$$\begin{aligned}
 A_2^T(Q, \nu) = & \int d\vec{E} \theta(t-1) \int d\vec{e} \theta(1-|\vec{E}+\vec{e}-\vec{Q}|) \frac{e^2}{(e^2+m_\pi^2)^2} \Gamma_\pi^4(e^2) \times \\
 & \frac{\theta(1-|\vec{E}+\vec{e}|)}{[\nu-\vec{Q}\cdot(\vec{E}+\vec{e}-\vec{Q}/2)]} \frac{\theta(|\vec{E}-\vec{Q}|-1)}{[\nu-\vec{Q}\cdot(\vec{E}-\vec{Q}/2)]} \left\{ (3\mu_s^2 - \mu_v^2) \left[\frac{1}{3} (\vec{Q}\cdot\vec{e})^2 + \frac{1}{9} e^2 Q^2 \right] - \right. \\
 & \left. - \frac{8}{3} e^2 \left[\vec{E}\cdot(\vec{E}+\vec{e}) - \frac{1}{Q^2} (\vec{Q}\cdot\vec{E}) \vec{Q}\cdot(\vec{E}+\vec{e}) \right] \right\} I \left[\nu - \vec{E}\cdot(\vec{Q}-\vec{e}) + \frac{1}{2} (\vec{Q}-\vec{e})^2, e \right] \quad (A.11)
 \end{aligned}$$

and it is easily checked that their sum does not coincide with the expression (5.8) for A_2 (but for the electric part) showing that the interference piece is missing.

The separation performed in Eq.(A.5) also affects the contributions to the transverse nuclear response arising from the interference between the correlation current and the MEC (we have not split the latter into central and tensor components since they contain structures which are not simply reducible to one pion exchange).

Therefore Eqs.(5.16) and (5.17) are replaced by

$$A(Q, \nu) = A^c(Q, \nu) + A^T(Q, \nu) \quad (A.12)$$

and

$$B(Q, \nu) = B^c(Q, \nu) + B^T(Q, \nu) \quad (A.13)$$

with

$$\begin{aligned}
 A^c(Q, \nu) = & \frac{\mu_\nu}{3} \int d\vec{t} \theta(t-1) \int d\vec{\ell}_1 \theta(1-|\vec{t}-\vec{\ell}_1|) \int d\vec{\ell}_2 \delta(\vec{Q}-\vec{\ell}_1-\vec{\ell}_2) \times \\
 & \times I\left(\nu + \frac{\ell_1^2}{2} - \vec{t} \cdot \vec{\ell}_1, \ell_2\right) \frac{\theta(|\vec{t}-\vec{Q}|-1)}{2\nu + Q^2 - 2\vec{Q} \cdot \vec{t}} \left\{ \frac{\ell_2^2}{\ell_2^2 + m_\pi^2} - 3g' \right\} \Gamma_\pi^2(\ell_2^2) \times \\
 & \times \left\{ \frac{\Gamma_\pi^2(\ell_2^2)}{\ell_2^2 + m_\pi^2} \left[2\left(\frac{M_\Delta}{M} + 1\right) Q^2 \ell_2^2 - (\vec{Q} \cdot \vec{\ell}_2)^2 \right] + \frac{1}{2} \frac{\Gamma_\pi^2(\ell_1^2)}{\ell_1^2 + m_\pi^2} \left(2\frac{M_\Delta}{M} + 1 \right) \right. \\
 & \left. \times [Q^2 \ell_1^2 + (\vec{Q} \cdot \vec{\ell}_1)^2] \right\} \quad (A.14)
 \end{aligned}$$

$$\begin{aligned}
 A^T(Q, \nu) = & \int d\vec{t} \theta(t-1) \int d\vec{\ell}_1 \theta(1-|\vec{t}-\vec{\ell}_1|) \int d\vec{\ell}_2 \delta(\vec{Q}-\vec{\ell}_1-\vec{\ell}_2) I\left(\nu + \frac{\ell_1^2}{2} - \vec{t} \cdot \vec{\ell}_1, \ell_2\right) \times \\
 & \times \frac{\theta(|\vec{t}-\vec{Q}|-1)}{2\nu + Q^2 - 2\vec{Q} \cdot \vec{t}} \cdot \frac{\Gamma_\pi^2(\ell_2^2)}{(\ell_2^2 + m_\pi^2)} \left\{ \frac{2}{3} \mu_\nu \frac{\ell_2^2 \Gamma_\pi^2(\ell_2^2)}{\ell_2^2 + m_\pi^2} \left[2\left(\frac{M_\Delta}{M} + 1\right) Q^2 \ell_2^2 - (\vec{Q} \cdot \vec{\ell}_2)^2 \right] + \right. \\
 & + \frac{\Gamma_\pi^2(\ell_1^2)}{\ell_1^2 + m_\pi^2} \left[\mu_\nu Q^2 \{ \ell_2^2 Q^2 - (\vec{Q} \cdot \vec{\ell}_2)^2 \} - \frac{1}{3} \mu_\nu \ell_2^2 \{ \ell_1^2 Q^2 + (\vec{Q} \cdot \vec{\ell}_1)^2 \} + \right. \\
 & \left. \left. + 2(\vec{\ell}_1 \cdot \vec{\ell}_2) \{ Q^2(\vec{t} \cdot \vec{\ell}_2) - (\vec{t} \cdot \vec{Q})(\vec{\ell}_2 \cdot \vec{Q}) \} \right] \frac{1}{2} \left(2\frac{M_\Delta}{M} + 1 \right) \right\} \quad (A.15)
 \end{aligned}$$

$$\begin{aligned}
 B^c(Q, \nu) = & \frac{\mu_\nu}{3} \int d\vec{s} \theta(1-s) \int d\vec{\ell}_1 \theta(1-|\vec{\ell}_1+\vec{s}|-1) \int d\vec{\ell}_2 \delta(\vec{Q}-\vec{\ell}_1-\vec{\ell}_2) I\left(\nu - \frac{\ell_1^2}{2} - \vec{s} \cdot \vec{\ell}_1, \ell_2\right) \times \\
 & \times \frac{\theta(1-|\vec{s}+\vec{Q}|-1)}{2\nu - Q^2 - 2\vec{Q} \cdot \vec{s}} \left\{ \frac{\ell_2^2}{\ell_2^2 + m_\pi^2} - 3g' \right\} \Gamma_\pi^2(\ell_2^2) \left\{ \frac{\Gamma_\pi^2(\ell_2^2)}{\ell_2^2 + m_\pi^2} \left[2\left(\frac{M_\Delta}{M} + 1\right) Q^2 \ell_2^2 - \right. \right. \\
 & \left. \left. - (\vec{Q} \cdot \vec{\ell}_2)^2 \right] + \frac{\Gamma_\pi^2(\ell_1^2)}{\ell_1^2 + m_\pi^2} \frac{1}{2} \left(2\frac{M_\Delta}{M} + 1 \right) [Q^2 \ell_1^2 + (\vec{Q} \cdot \vec{\ell}_1)^2] \right\} \quad (A.16)
 \end{aligned}$$

$$\begin{aligned}
 B^T(Q, \nu) = & \int d\vec{s} \theta(1-s) \int d\vec{\ell}_1 \theta(|\vec{\ell}_1 + \vec{s}| - 1) \int d\vec{\ell}_2 \delta(\vec{Q} - \vec{\ell}_1 - \vec{\ell}_2) I\left(\nu - \frac{\ell_1^2}{2} - \vec{s} \cdot \vec{\ell}_1, \ell_2\right) \times \\
 & \times \frac{\theta(1 - |\vec{s} + \vec{Q}|)}{2\nu - Q^2 - 2\vec{Q} \cdot \vec{s}} \frac{\Gamma_n^2(\ell_2^2)}{(\ell_2^2 + m_n^2)} \left\{ \frac{2}{3} \mu_\nu \frac{\ell_2^2 \Gamma_n^2(\ell_2^2)}{\ell_2^2 + m_n^2} \left[2\left(\frac{M_A}{M} + 1\right) Q^2 \ell_2^2 - (\vec{Q} \cdot \vec{\ell}_2)^2 \right] + \right. \\
 & + \frac{1}{2} \left(2\frac{M_A}{M} + 1 \right) \frac{\Gamma_n^2(\ell_1^2)}{\ell_1^2 + m_n^2} \left[\mu_\nu Q^2 \{ \ell_2^2 Q^2 - (\vec{Q} \cdot \vec{\ell}_2)^2 \} - \frac{1}{3} \mu_\nu \ell_2^2 \{ \ell_1^2 Q^2 + (\vec{Q} \cdot \vec{\ell}_1)^2 \} - \right. \\
 & \left. \left. - 2(\vec{\ell}_1 \cdot \vec{\ell}_2) \{ Q^2(\vec{s} \cdot \vec{\ell}_2) - (\vec{s} \cdot \vec{Q})(\vec{\ell}_2 \cdot \vec{Q}) \} \right] \right\} \quad (A.17)
 \end{aligned}$$

In the interference between contact and correlation currents we write now, instead of (5.20) and (5.21),

$$C(Q, \nu) = C^C(Q, \nu) + C^T(Q, \nu) \quad (A.18)$$

and

$$D(Q, \nu) = D^C(Q, \nu) + D^T(Q, \nu) \quad (A.19)$$

with

$$\begin{aligned}
 C^C(Q, \nu) = & \frac{1}{3} \int d\vec{t} \theta(t-1) \int d\vec{\ell}_1 \theta(1 - |\vec{t} - \vec{\ell}_1|) \int d\vec{\ell}_2 \delta(\vec{Q} - \vec{\ell}_1 - \vec{\ell}_2) I\left(\nu + \frac{\ell_1^2}{2} - \vec{t} \cdot \vec{\ell}_1, \ell_2\right) \times \\
 & \times \Gamma_n^2(\ell_2^2) \left\{ \frac{\ell_2^2}{\ell_2^2 + m_n^2} - 3g' \right\} \frac{\theta(|\vec{t} - \vec{Q}| - 1)}{2\nu + Q^2 - 2\vec{t} \cdot \vec{Q}} \left\{ 2\mu_\nu \left[(\vec{Q} \cdot \vec{\ell}_2) \frac{\Gamma_n(\ell_2^2) G_A(\ell_1)}{\ell_2^2 + m_n^2} + \right. \right. \\
 & + (\vec{Q} \cdot \vec{\ell}_1) \frac{\Gamma_n(\ell_1^2) G_A(\ell_2)}{\ell_1^2 + m_n^2} \left. \right] - 2 \left[\vec{t} \cdot \vec{\ell}_2 - \frac{1}{Q^2} (\vec{t} \cdot \vec{Q})(\vec{\ell}_2 \cdot \vec{Q}) \right] \left[\frac{\Gamma_n(\ell_2^2) G_A(\ell_1)}{\ell_2^2 + m_n^2} + \right. \\
 & \left. \left. + \frac{\Gamma_n(\ell_1^2) G_A(\ell_2)}{\ell_1^2 + m_n^2} \right] \right\} \quad (A.20)
 \end{aligned}$$

$$\begin{aligned}
 C^T(Q, \nu) = & \int d\vec{t} \theta(t-1) \int d\vec{p}_1 \theta(1-|\vec{t}-\vec{p}_1|) \int d\vec{p}_2 \delta(\vec{Q}-\vec{p}_1-\vec{p}_2) I\left(\nu + \frac{p_1^2}{2} - \vec{p}_1 \cdot \vec{t}, p_2\right) \times \\
 & \times \frac{\Gamma_n^2(p_2^2)}{p_2^2 + m_n^2} \frac{\theta(|\vec{t}-\vec{Q}|-1)}{2\nu + Q^2 - 2\vec{t} \cdot \vec{Q}} \left\{ \mu_\nu \left[\frac{4}{3} p_2^2 (\vec{Q} \cdot \vec{p}_2) \frac{\Gamma_n(p_2^2) G_A(p_1)}{p_2^2 + m_n^2} + \right. \right. \\
 & + \left. \left\{ \frac{4}{3} p_2^2 Q^2 + \frac{2}{3} p_2^2 (\vec{Q} \cdot \vec{p}_2) - (\vec{Q} \cdot \vec{p}_2)^2 \right\} \frac{\Gamma_n(p_1^2) G_A(p_2)}{p_1^2 + m_n^2} \right] - \\
 & - \left[\vec{t} \cdot \vec{p}_2 - \frac{1}{Q^2} (\vec{t} \cdot \vec{Q})(\vec{p}_2 \cdot \vec{Q}) \right] \left[\frac{4}{3} p_2^2 \frac{\Gamma_n(p_2^2) G_A(p_1)}{p_2^2 + m_n^2} + \left(\frac{4}{3} p_2^2 - 2 \vec{p}_2 \cdot \vec{Q} \right) \frac{\Gamma_n(p_1^2) G_A(p_2)}{p_1^2 + m_n^2} \right] \left. \right\}
 \end{aligned} \tag{A.21}$$

$$\begin{aligned}
 D^c(Q, \nu) = & \frac{1}{3} \int d\vec{s} \theta(1-s) \int d\vec{p}_1 \theta(1-|\vec{s}+\vec{p}_1|) \int d\vec{p}_2 \delta(\vec{Q}-\vec{p}_1-\vec{p}_2) I\left(\nu - \frac{p_1^2}{2} - \vec{s} \cdot \vec{p}_1, p_2\right) \times \\
 & \times \frac{\theta(1-|\vec{s}+\vec{Q}|)}{2\nu - Q^2 - 2\vec{s} \cdot \vec{Q}} \Gamma_n^2(p_2^2) \left\{ \frac{p_2^2}{p_2^2 + m_n^2} - 3g' \right\} \left\{ 2\mu_\nu \left[(\vec{Q} \cdot \vec{p}_2) \frac{\Gamma_n(p_2^2) G_A(p_1)}{p_2^2 + m_n^2} + \right. \right. \\
 & + \left. (\vec{Q} \cdot \vec{p}_1) \frac{\Gamma_n(p_1^2) G_A(p_2)}{p_1^2 + m_n^2} \right] + 2 \left[\vec{s} \cdot \vec{p}_2 - \frac{1}{Q^2} (\vec{s} \cdot \vec{Q})(\vec{p}_2 \cdot \vec{Q}) \right] \left[\frac{\Gamma_n(p_2^2) G_A(p_1)}{p_2^2 + m_n^2} + \right. \\
 & \left. \left. + \frac{\Gamma_n(p_1^2) G_A(p_2)}{p_1^2 + m_n^2} \right] \right\}
 \end{aligned} \tag{A.22}$$

$$\begin{aligned}
 D^T(Q, \nu) = & \int d\vec{s} \theta(1-s) \int d\vec{p}_1 \theta(1-|\vec{s}+\vec{p}_1|) \int d\vec{p}_2 \delta(\vec{Q}-\vec{p}_1-\vec{p}_2) I\left(\nu - \frac{p_1^2}{2} - \vec{s} \cdot \vec{p}_1, p_2\right) \times \\
 & \times \frac{\theta(1-|\vec{s}+\vec{Q}|)}{2\nu - Q^2 - 2\vec{s} \cdot \vec{Q}} \frac{\Gamma_n^2(p_2^2)}{(p_2^2 + m_n^2)} \left\{ \mu_\nu \left[\frac{4}{3} p_2^2 (\vec{Q} \cdot \vec{p}_2) \frac{\Gamma_n(p_2^2) G_A(p_1)}{p_2^2 + m_n^2} + \left\{ \frac{4}{3} p_2^2 Q^2 - (\vec{Q} \cdot \vec{p}_2)^2 + \right. \right. \right. \\
 & + \left. \frac{2}{3} p_2^2 (\vec{Q} \cdot \vec{p}_2) \right\} \frac{\Gamma_n(p_1^2) G_A(p_2)}{p_1^2 + m_n^2} \left. \right] + \left[\vec{s} \cdot \vec{p}_2 - \frac{1}{Q^2} (\vec{s} \cdot \vec{Q})(\vec{p}_2 \cdot \vec{Q}) \right] \times \\
 & \times \left[\frac{4}{3} p_2^2 \frac{\Gamma_n(p_2^2) G_A(p_1)}{p_2^2 + m_n^2} + \left(\frac{4}{3} p_2^2 - 2 \vec{p}_2 \cdot \vec{Q} \right) \frac{\Gamma_n(p_1^2) G_A(p_2)}{p_1^2 + m_n^2} \right] \left. \right\}
 \end{aligned} \tag{A.23}$$

The function $I(\omega, \ell)$ occurring in the previous formulae is given in Eq.(5.14).

Finally the central and tensor parts of the interference between pion-in-flight and correlation currents simply amount to 1/3 and 2/3, respectively, of the total expression (5.22). The Landau-Migdal interaction has to be added to the central piece by the usual replacement

$$-\frac{1}{3} \frac{\ell^2}{\ell^2 + m_\pi^2} \Rightarrow g' - \frac{1}{3} \frac{\ell^2}{\ell^2 + m_\pi^2} . \quad (\text{A.24})$$

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Table captions

Table I - Partial contributions to the 2p-2h transverse response for $q = 410$ MeV/c. The following values have been used: $k_F = 1.2$ fm⁻¹, $\Lambda_\pi = 900$ MeV and $g' = 0.5$. In both (a) and (b) the energy (in MeV) is in the first column.

(a): in columns 2 (5) and 3 (6) are quoted the central and tensor contributions to A_1 (A_3), respectively; column 4 displays A_2 and in the last column the total contribution to S_T due to the correlation diagram is reported. The coefficient β is, according to (5.6),

$$\beta = \frac{A^2}{(2\pi)^6 m_\pi^4} \left(\frac{f_\pi^2}{4\pi\hbar c} \right)^2 \frac{3M^2 c^2}{\hbar^2 k_F^2}$$

(b): in column 2 is quoted the contribution to S_T coming from pion-in-flight and contact currents, in column 3 the one due to the Δ and in column 4 the global transverse response arising from the MEC. Columns 5 to 7 display the interferences of the correlation current with pion-in-flight, contact and Δ currents, respectively.

Table II - The imaginary part of the 2p-2h polarization propagator at fixed energy ($\hbar\omega = 140$ MeV) for different q values. In the first column is given q , in the second one the contributions to S_T included in Eq.(5.18), in column 3 the same divided by q^2 and by the electromagnetic form factor and finally in column 4 the value of $\text{Im}\Pi^{2p-2h}$.

Table Ia

$\hbar\omega$	βA_1^C	βA_1^T	βA_2	βA_3^C	βA_3^T	s_T^{corr}
50	11.3	20.3	-0.20	1.8	3.5	36.9
100	12.5	31.8	-0.17	0.7	2.0	58.3
150	23.6	47.8	-0.19	0.3	1.2	84.1
200	20.9	53.1	-0.12	0.2	0.7	83.4
250	17.3	51.1	-0.12	0.1	0.5	74.7
300	10.9	39.9	-0.10	0.07	0.4	55.3

Table Ib

$\hbar\omega$	π +Cont	Δ	s_T^{MEC}	π -Corr	Cont-Corr	Δ -Corr
50	7.3	3.3	6.7	2.0	-0.8	3.2
100	17.2	11.7	20.2	5.8	-5.2	11.9
150	16.7	19.1	28.8	8.0	-6.5	20.3
200	13.4	24.6	34.4	9.3	-4.5	29.5
250	10.1	28.9	38.4	8.9	-1.3	37.0
300	7.5	34.2	42.6	7.9	-0.3	43.7

Table II

$q(\text{fm}^{-1})$	S_T	$S_T/q^2 G_M^2(q^2)$	$\text{Im } \Pi^{2p-2h}$
0.01	$.85 \times 10^{-2}$	76.2	-2.36×10^{-4}
1.27	105.9	82.3	-2.55×10^{-4}
1.87	136.0	70.6	-2.19×10^{-4}
2.08	127.4	62.5	-1.94×10^{-4}

Figure Captions

Fig. 1 - The separated transverse magnetic response in ^{56}Fe at $q = 370 \text{ MeV}/c$ as a function of $\hbar\omega$. The experimental points (crosses) are taken from ref.6. The continuous line is the RPA response obtained with the interaction (2.18) where $g' = 0.7$; the dashed line is the electric contribution in zero order and the dot-dashed line is the RPA response with the finite Δ -width (2.12) in the Δ -hole polarization propagator. The Fermi wave-number is $k_F = 1.20 \text{ fm}^{-1}$.

Fig. 2 - The meson exchange currents diagrams. In the upper graphs the pion-in-flight and the contact terms are shown; in the lower ones the pionic current is coupled to a Δ intermediate state.

Fig. 3 - Diagrammatic representation of the matrix elements of the MEC between the ground state of a free Fermi gas and the 2p-2h states $|\vec{p}\vec{k}^{-1}, \vec{p}'\vec{k}'^{-1}\rangle$; the physical interpretation of the various graphs is given in the text.

Fig. 4 - Some of the 49 diagrams contributing to S_T^{MEC} .

Fig. 5 - Diagrams for the coupling of a photon to a pair of correlated nucleons (correlation "current").

Fig. 6 - Matrix elements of the correlation "current" between the ground state of a free Fermi gas and the 2p-2h states $|\vec{p}\vec{k}^{-1}, \vec{p}'\vec{k}'^{-1}\rangle$.

Fig. 7 - The six topologically distinct diagrams representing S_T^{corr} . Diagram (a) corresponds to A_1 , (b) to A_3 , (c) to A_2 , (d) to A_4 , (e) to A_6 and (f) to A_5 [see formulae (5.7) to (5.13) in the text].

Fig. 8 - Some examples of the diagrams corresponding to the interference between the correlation current and the MEC.

Fig. 9 - The separated transverse magnetic response in ^{56}Fe at $q = 210 \text{ MeV/c}$ as a function of $\hbar\omega$. The experimental points (bars) are taken from ref.6. The dot-dashed line is the 1p-1h RPA response; the continuous line is the global 2p-2h response, the dashed line its contribution from the MEC alone and the open circles represent the total 1p-1h, 2p-2h transverse response when both channels contribute. The Fermi wave number is $k_F = 1.20 \text{ fm}^{-1}$.

Fig.10 - The same as in Fig.9 at $q = 250 \text{ MeV/c}$.

Fig.11 - The same as in Fig.9 at $q = 370 \text{ MeV/c}$.

Fig.12 - The same as in Fig.9 at $q = 410 \text{ MeV/c}$.

Fig.13 - Diagrams representing the vertex renormalization for the absorption of a pion in the nuclear medium.

Fig.14 - Quasideuteron component of the total photon absorption cross section in ^{208}Pb . The dashed line represents the experimental fit (6.16), the dot-dashed line corresponds to our 2p-2h "bare" calculation and the continuous line to the renormalized one. The cross section given by the old Levinger's model is also shown (double-dot - dashed line).

^{56}Fe
 $q=370 \text{ MeV}/c$

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig 6

(a)

(b)

(c)

(d)

(e)

(f)

Fig. 7

Fig. 8

Fig. 9

Fig. 10

^{56}Fe
 $q=370 \text{ MeV}/c$

Fig. 11

Fig. 12

FIG. 13

Fig. 14