N = 1 SUPERGRAVITY, THE WEAK SCALE AND
THE LOW ENERGY PARTICLE SPECTRUM

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E R R A T A

- In page 2, lines 16 and 17, one should have

$$c = 4B-A \quad ; \quad 2B = A-1$$

- In page 5, line 14, one should read

$$\omega = v_2/v_1$$

- In page 17, line 15, one should have

$$(\mu_1^2 - \mu_2^2)/M_Z^2 = \epsilon \ll 1$$

- In page 27, line 12, one should have

$$M \gtrsim 2m \gtrsim 2M_Z$$

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A B S T R A C T

We consider in detail the radiative $SU(2) \times U(1)$ breaking induced by broken $N = 1$ supergravity. We include in this study some parameters (b quark Yukawas, mixed Higgs mass terms) neglected in a previous analysis. We show that there are essentially two types of mechanisms leading to radiative $SU(2) \times U(1)$ breaking. The first of them occurs for a t quark mass $m_t \gtrsim 60$ GeV. The second one appears for arbitrary small values of m_t (without using a large A). These two mechanisms correspond to two different classes of local SUSY GUTs. We present results for the low energy supersymmetric particle spectrum. In some of the possible scenarios presented, squarks, sleptons and/or gauginos are accessible to present accelerator energies. In other scenarios, the scalar neutrinos may be the lightest R odd particles. We finally remark that the $SU(5)$ prediction for m_b/m_τ is substantially suppressed for a very heavy t quark.

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1. - INTRODUCTION

It has recently been shown ^{1),2)} that one can use $N = 1$ local supersymmetry coupled to grand unified theories (GUTs) in order to solve the gauge hierarchy problem. In these local SUSY GUTs, $N = 1$ supergravity is broken at an intermediate mass scale ($\sim 10^{10}$ GeV) so that the gravitino mass $m_{3/2}$ is of the order of the weak scale M_W . Below the Planck mass, one is left with a globally supersymmetric Lagrangian plus certain explicit soft terms ¹⁾⁻³⁾ (characterized by the scale $m_{3/2}$) which break supersymmetry. The breaking of the GUT symmetry may or may not be achieved by the same superfield which breaks $N = 1$ supergravity.

One of the interesting features of local SUSY GUTs (also present in radiative global models ⁴⁾) is that the $SU(2) \times U(1)$ symmetry may be broken as a direct consequence of the breaking of supersymmetry. This is because we have dimensionful terms in the Lagrangian of order $m_{3/2}$ with which we can play. One can try to break $SU(2) \times U(1)$ at the tree level ^{5),6)} using the explicit soft terms left by supergravity. However, as we will comment below, this turns out to be problematic ⁷⁾⁻⁹⁾. A more attractive possibility ^{1),3),10)} is trying to break the $SU(2) \times U(1)$ symmetry radiatively, starting from the boundary conditions dictated by the unification group + $N = 1$ supergravity at the GUT scale ³⁾. In a recent letter ¹¹⁾ we have shown how $SU(2) \times U(1)$ breaking can be radiatively generated for a heavy enough ($m_t \gtrsim 50$ GeV) top quark [see also Refs. 9) and 12)]. We present here a more detailed analysis of the general mechanisms for radiative $SU(2) \times U(1)$ breaking in GUTs coupled to $N = 1$ supergravity. This includes new results as well as general formulae which can be used for the analysis of the $SU(2) \times U(1)$ breaking mechanisms in different local SUSY GUTs. Interesting constraints between the masses of the SUSY particles appear including the possibility of detecting them at present accelerator energies.

Before starting with the mechanism of Weinberg-Salam breaking, let us briefly review some facts about the coupling of $N = 1$ supergravity to GUTs. If the chiral interactions of the GUT are governed by a superpotential $W(Z)$, the potential for the scalars Z_i takes the form ¹³⁾

$$V = e^{\frac{K}{M^2}} \left(\sum_i \left| \frac{\partial W}{\partial Z_i} + \frac{\partial K}{\partial Z_i^*} \frac{W}{M^2} \right|^2 - \frac{3}{M^2} |W|^2 \right) + \text{gauge terms} \quad (1)$$

where $K = K(Z_i, Z_i^*)$ is the Kahler potential which is an arbitrary real function of its arguments. It is customary to consider W as a sum of two terms corresponding to the observable sector (i.e., quarks, leptons, gauge non-singlet sector)

and to a hidden (singlet) sector which is used to break supersymmetry. This division is artificial and unnecessary; the same superfields which break the GUT symmetry can be used to break supersymmetry. In particular, the adjoint of $SU(5)$ can be used to break both the gauge symmetry and supersymmetry at the Planck mass ^{1),3),14),15)}. Let us notice also that W may also contain non-renormalizable terms since the whole theory is non-renormalizable ^{3),10),14)}.

What will be interesting for us in the following is the effective Lagrangian below the Planck mass and the GUT scale obtained from Eq. (1). It is well known that one obtains at low energies a supersymmetric Lagrangian plus some terms breaking explicitly but softly the global supersymmetry. The general form of the effective scalar potential is ¹⁶⁾:

$$V_{\text{eff}} = \sum_i \left| \frac{\partial \tilde{W}}{\partial \tilde{z}_i} \right|^2 + |m|^2 \sum_i |\tilde{z}_i|^2 + (A m \tilde{W}_3 + \text{h.c.}) + \\ + 2 (B m \tilde{W}_2 + \text{h.c.}) + (C m \tilde{W}_1 + \text{h.c.}) + \text{gauge terms} \quad (2)$$

where $\tilde{W}$ is an effective low energy superpotential and $\tilde{W}_3$, $\tilde{W}_2$ and $\tilde{W}_1$ are its trilinear, bilinear and linear parts. Here $|m|$ is of the order of the gravitino mass and A, B, C are complex numbers with $|A| \sim |B| \sim 1$ and $C = 2B - A$. (For a minimal Kahler potential ¹³⁾, $K = \sum_i |\tilde{z}_i|^2$, the further relation exists $B = A - 1$.) All the non-supersymmetric terms in (2) give rise only to logarithmic divergences ¹⁷⁾ and hence (in principle) do not spoil the solution for the hierarchy problem.

Concerning the rest of the effective Lagrangian, the Yukawas (and fermion masses) are induced by the effective superpotential $\tilde{W}$:

$$\mathcal{L}_{Y,M} = \sum_{i,j} \frac{\partial \tilde{W}}{\partial \tilde{z}_i \partial \tilde{z}_j} \psi_i \psi_j + \text{h.c.} \quad (3)$$

Concerning the gauge interactions, a term of the form

$$\left[f_{ab}(z) F^a F^b \right] \quad (4)$$

where F^a is the chiral Yang-Mills strength and $f_{ab}(Z)$ is an arbitrary analytic function of the chiral superfields, gives rise to the usual kinetic terms for $f_{ab} = \delta_{ab}$. If, e.g., $f_{ab} = (1+Y)\delta_{ab}$ with Y a singlet superfield with a non-vanishing F_Y component, one can also have tree-level masses for the gauginos ^{13),3)}. The "gauge terms" in Eq. (2) are also obtained ¹³⁾ from the kinetic term (4)

$$V_{\text{gauge}} = \frac{1}{2} \text{Re} \left(f_{ab}^{-1} D^a D^b \right) \quad (5)$$

where D^a is the usual gauge auxiliary field.

The non-supersymmetric terms in Eq. (2) can be written in a supersymmetric fashion by introducing a spurious superfield ^{17),18)}

$$\eta = (\theta\theta) m \quad (6)$$

Then one can rewrite the soft terms as :

$$\sum_i \left[z_i^* (\eta + \eta^*) z_i \right]_D + A \left[\eta \tilde{W}_3 \right]_F + 2B \left[\eta \tilde{W}_2 \right]_F + C \left[\eta \tilde{W}_1 \right]_F + \text{h.c.} \quad (7)$$

2. - THE LOW ENERGY SCALAR POTENTIAL AND THE RENORMALIZATION GROUP

The effective potential, Eq. (2), applies at the Planck mass. However, gauge and chiral interactions will alter the general form (2) below the Planck mass ³⁾. For example, the masses for scalars belonging to different SU(5) representations will in general be renormalized differently. The same will happen, e.g., with the trilinear scalar couplings in (2). Since the GUT scale is so close to the Planck scale ($M_X \approx 3 \times 10^{16}$ GeV, $M = M_P / \sqrt{8\pi} = 2.4 \times 10^{18}$ GeV) the effect will be numerically negligible and we will assume that the general form (2) is still good at the GUT scale. Moreover, it may well be that the GUT scale coincides in fact with the Planck scale M , as happens in certain models ^{1),3),14),15)}.

We will assume the minimal content for the low energy effective theory : three generations of quarks and leptons and two Higgs doublets H_1 and H_2 of opposite hypercharge. We will not consider light singlets since, in the context of GUTs, they jeopardize the gauge hierarchy (if coupled to H_1 and H_2) ⁸⁾. Under these conditions, the most general effective superpotential will have the form :

$$\tilde{W} = \sum_{\text{generations}} (h_u Q_L U_R H_2 + h_D Q_L D_R H_1 + h_L L_L E_R H_1) + \mu' H_1 H_2 \quad (8)$$

This includes the usual Yukawas (in a self-explanatory notation) plus a possible mass term for the Higgses. Equations (2) and (8) give us, for the complete scalar potential ^{*)} :

$$V = V_4 + V_3 + V_2 + V_{\text{gauge}} \quad (9a)$$

$$V_4 = |\sum (h_L L_L E_R + h_D Q_L D_R)|^2 + |\sum h_U Q_L U_R|^2 + \sum |h_U Q_L H_2|^2 + \sum |h_D Q_L H_1|^2 + \sum |h_U U_R H_2 + h_D D_R H_1|^2 + \sum |h_L E_R H_1|^2 + \sum |h_L L_L H_1|^2 \quad (9b)$$

$$V_{\text{gauge}} = \frac{1}{2} g'^2 \left(\sum \left(\frac{1}{6} |Q_L|^2 + \frac{1}{3} |D_R|^2 - \frac{2}{3} |U_R|^2 - \frac{1}{2} |L_L|^2 + |E_R|^2 + \frac{1}{2} |H_2|^2 - \frac{1}{2} |H_1|^2 \right) \right)^2 + \frac{1}{8} g^2 \left(\sum \vec{Q}_L^* \vec{\tau} Q_L + \vec{L}_L^* \vec{\tau} L_L + \vec{H}_1^* \vec{\tau} H_1 + \vec{H}_2^* \vec{\tau} H_2 \right)^2 + \quad (9c)$$

+ SU(3) D² term

$$V_3 = m \left(\sum A_U h_U Q_L U_R H_2 + A_D h_D Q_L D_R H_1 + A_L h_L L_L E_R H_1 \right) + h.c. + \mu' \left(\sum h_U H_1^* Q_L U_R + h_D H_2^* Q_L D_R + h_L H_2^* L_L E_R \right) + h.c. \quad (9d)$$

$$V_2 = \sum m_Q^2 |Q_L|^2 + m_U^2 |U_R|^2 + m_D^2 |D_R|^2 + m_L^2 |L_L|^2 + m_E^2 |E_R|^2 + \mu_1^2 |H_1|^2 + \mu_2^2 |H_2|^2 - \mu_3 (H_1 H_2 + h.c.) \quad (9e)$$

where the sums run over the three generations and we denote $\mu_3^2 = -2Bm\mu'$. Concerning the fermionic Lagrangian we will have, apart from the usual Yukawas, a mass = μ' for the Higgsinos and gaugino masses M_3 , M_2 and M_1 .

The trilinear couplings in V_3 may be dangerous if the A_U , A_D and A_L are large ⁷⁾. Charge- and colour-breaking absolute minima can appear if the following inequalities are not respected ^{7), 9)} :

^{*)} We use the same symbol for a superfield and its scalar component.

$$\mu_2^2 + m_Q^2 + m_U^2 > m^2 (2|A_U| - 3) \quad (10a)$$

$$\mu_1^2 + m_Q^2 + m_D^2 > m^2 (2|A_D| - 3) \quad (10b)$$

$$\mu_1^2 + m_L^2 + m_E^2 > m^2 (2|A_L| - 3) \quad (10c)$$

For equal scalar masses, this just gives the well-known condition $|A| < 3$. If the conditions (10) are respected and if the squark and slepton (masses)² are positive, no unwanted minima occur in Eq. (9). The relevant potential¹⁹⁾ for the breaking of the symmetry is then ^{*)} :

$$V(\text{Higgs}) = \frac{g^2 + g'^2}{8} (|H_1|^2 - |H_2|^2)^2 + \mu_1^2 |H_1|^2 + \mu_2^2 |H_2|^2 - \mu_3^2 (H_1 H_2 + \text{h.c.}) \quad (11)$$

This potential is minimized¹⁹⁾ for

$$v^2 = v_1^2 + v_2^2 = 2 \frac{(\mu_1^2 - \mu_2^2 - (\mu_1^2 + \mu_2^2) \cos 2\theta)}{(g^2 + g'^2) \cos 2\theta} \quad (12)$$

where $\langle H_1^0 \rangle = v_1$ and $\langle H_2^0 \rangle = v_2$. The angle θ is defined as :

$$\cos 2\theta \equiv (\omega^2 - 1)/(\omega^2 + 1) \quad ; \quad \sin 2\theta \equiv 2\omega/(\omega^2 + 1) \quad (13)$$

with $\omega \equiv v_1/v_2$. The minimization conditions also give

$$\sin 2\theta = 2\mu_3^2/(\mu_1^2 + \mu_2^2) \quad (14)$$

The W gauge bosons would then acquire a mass $M_W^2 = g^2 v^2/2$. In order that the potential be bounded below, one must impose the condition¹⁹⁾ :

$$2|\mu_3^2| < \mu_1^2 + \mu_2^2 \quad (15)$$

which must be obeyed not only at the M_W scale but all over the energy range $M_W - M_X$ so that the scalars do not take the unwanted direction $v_1 = v_2 \rightarrow \infty$. One must also be sure that there is not a lower symmetric minimum at the scale M_W . This requires :

^{*)} We will take μ_3^2 positive.

$$|\mu_3|^4 > \mu_1^2 \mu_2^2 \quad (16)$$

All the mass parameters and couplings in Eqs. (11)-(16) should be evaluated at the scale $\sim M_W$ where we want the $SU(2) \times U(1)$ symmetry to be broken. Thus we have to calculate how the masses and couplings in Eqs. (9) and (11) are renormalized when going from the GUT scale to the weak scale³⁾. At the GUT scale, certain boundary conditions implied by the general equations (2) and (4) should apply. These are³⁾:

$$A_U = A_D = A_L = A \quad (17a)$$

$$m_Q^2 = m_U^2 = m_D^2 = m_L^2 = m_E^2 = \mu_1^2 - \mu'^2 = \mu_2^2 - \mu'^2 = m^2 \quad (17b)$$

$$M_1 = M_2 = M_3 = M \quad (17c)$$

where M_i are the gaugino masses corresponding to $SU(3) \times SU(2) \times U(1)$. In the case of simple $SU(5)$ or $SO(10)$ GUTs one also adds

$$h_D = h_L \quad (18)$$

These boundary conditions are non-trivial and impose relationships among the masses of the particles at low energies. If we neglect the Yukawa couplings of the first two generations, mixings and CP phases, we have in principle seven free parameters:

$$m, M, A, \mu', \mu_3^2, h_t, h_b \quad (19)$$

However, there are two restrictions to be imposed. First, $SU(2) \times U(1)$ must be broken at the appropriate scale:

$$\nu^2 = \nu_1^2 + \nu_2^2 = 2 \frac{M_W^2}{g^2} \quad (20)$$

Second, the bottom-quark mass experimental value should be imposed:

$$m_b = h_b(2m_b)\nu_1 = h_b \nu \cos\theta \simeq 5 \text{ GeV} \quad (21)$$

This leaves us with a total of five free parameters. In a previous letter ¹¹⁾ we presented results neglecting the parameters h_b , μ_3^2 and μ' . Here we give a more detailed analysis including non-negligible h_b and μ_3^2 . In order to simplify the (already cumbersome) analysis somewhat, we will take, however, $\mu'^2 \ll m^2$. We will comment later on the effect of the inclusion of a non-vanishing μ' . It may seem contradictory to neglect μ' but not $\mu_3^2 = |B||m\mu'|^2$. Notice, however, that those two quantities are independent and, e.g., taking $|B| \sim 2$ and $\mu' \sim m/4$ one obtains $\mu_3^2 \approx m^2$ but $\mu'^2 \approx m^2/16$ [implying $\sin 2\theta(M_X) \approx 16/17$], hence one can have reasonably large $\sin 2\theta$ with a small μ'^2 . We remark again that the effect of a non-negligible μ'^2 is easy to understand and nothing especially interesting occurs for a non-vanishing value. We will show at the end of Section 6 what is the effect of a non-vanishing μ' in the cases of interest.

An interesting question is the origin of the μ' (or μ_3^2) terms in the potential in specific local SUSY GUTs. One may have a $\mu'H_1H_2$ term at the tree level by fine tuning a small μ' in a given local SUSY GUT ²⁰⁾. Alternatively, μ' and μ_3^2 terms can appear radiatively ^{21),9),14)}. For example, in a missing partner mechanism ²²⁾, the diagram of Figs. 1a and 1b [with $\eta = m(\theta\theta)$] will give rise to μ' and μ_3^2 terms. Another possibility is the use of a superheavy chiral singlet χ coupled to H_1 and H_2 with a superpotential :

$$W_\chi = \lambda \chi^3 + M_\chi^2 \chi^2 + \lambda' \chi H_1 H_2 \quad (22)$$

The diagrams of Figs. 1c and 1d will produce μ' and μ_3^2 terms. This singlet is not dangerous since it decouples at low energies.

If the μ_3^2 term is produced radiatively, one would expect it to be small compared to m^2 , leading to very small $\sin 2\theta(M_X) \approx \mu_3^2/m^2$. This depends, however, on the details of the diagram and the size of the coupling constants. Another way to obtain a sizeable $\mu_3^2 \sim m^2$ involves the use of a "heavy sliding singlet". This heavy sliding singlet ²³⁾ would give a tree-level mass $\sim m$ to the Higgses without fine tuning ^{*)}. Anyway it seems that the required term μ_3^2 does appear in general in local SUSY GUTs.

Concerning the renormalization group equations ^{19),3)}, the relevant equations for the g_i , h_U , h_D , h_L couplings, the trilinear couplings A_U , A_D , A_L and all the masses of the different particles are given in Appendix I. We will comment in the following two sections about their solution.

*) We thank G.G. Ross for pointing out to us this possibility.

3. - THE YUKAWA EVOLUTION EQUATIONS AND THE m_b/m_τ RATIO

An important rôle in the present analysis is played by the evolution equations of the Yukawa couplings^{19),24)}. We will neglect the couplings of the first two generations. The equations for Y_t , Y_b and Y_τ (where $Y = h^2/(4\pi)^2$) are shown in (A.I.3). They form a set of three coupled non-linear differential equations which we have solved numerically. However, in the limit in which $h_b, h_\tau \ll h_t$ one can solve the equations analytically. One obtains the simple result (recall $Y \equiv h^2/(4\pi)^2$) :

$$Y_t(t) = Y_t(0) E_1(t) / (1 + 6 Y_t(0) F_1(t)) \quad (23a)$$

$$Y_b(t) = Y_b(0) E_2(t) / (1 + 6 Y_t(0) F_1(t))^{1/6} \quad (23b)$$

$$Y_\tau(t) = Y_\tau(0) E_3(t) \quad (23c)$$

$$F_1(t) \equiv \int_0^t E_1(t') dt' \quad ; \quad t = 2 \ln(M_x/Q) \quad (24)$$

where the E_i 's are :

$$E_1(t) = \left(1 + b_3 \frac{\alpha_3(0)}{4\pi} t\right)^{16/3b_3} \left(1 + b_2 \frac{\alpha_2(0)}{4\pi} t\right)^{3/b_2} \left(1 + b_1 \frac{\alpha_1(0)}{4\pi} t\right)^{13/9b_1} \quad (25a)$$

$$E_2(t) = \left(1 + b_1 \frac{\alpha_1(0)}{4\pi} t\right)^{-2/3b_1} E_1(t) \quad (25b)$$

$$E_3(t) = \left(1 + b_2 \frac{\alpha_2(0)}{4\pi} t\right)^{3/b_2} \left(1 + b_1 \frac{\alpha_1(0)}{4\pi} t\right)^{3/b_1} \quad (25c)$$

b_i being the coefficients of the corresponding β functions (see Appendix I). Let us concentrate for the moment on the result for the top quark (23a). Notice that for a small $Y_t(0)$ one re-obtains the well-known gauge anomalous dimensions result. However, for $Y_t(0) \rightarrow \infty$ (or $Y_t(0) \sim 1$) one obtains the result

$$Y_t(t) \simeq E_1(t) / 6 F_1(t) \quad (26)$$

independent of the original value of $Y_t(0)$, i.e., there is a fixed point at (26). At the scale M_W ($t \approx 67$) one obtains $E_1 \approx 13$ and $F_1 \approx 290$ which gives an upper bound for the top-quark ^{19),11),9)} mass ^{*)} :

$$m_t = h_t \nu_2 \leq h_t \nu \lesssim 190 \text{ GeV} \quad (27)$$

This is an interesting bound which is just a consequence of supersymmetry and perturbative unification. A useful numerical formula for m_t as a function of $h_t(0)$ is obtained from (23a) :

$$m_t = 3.6 h_t(0) \nu_2 / \sqrt{1 + 11 \times h_t^2(0)} \quad (28)$$

Looking at formulae (23b,c) one observes that the mass of the bottom quark decreases as m_t increases whereas m_τ does not depend directly on m_t . Using these formulae, one obtains that the usual SU(5) prediction for m_b/m_τ gets modified when a heavy top-quark exists in the following way :

$$\frac{m_b(t)}{m_\tau(t)} = \left(\frac{\alpha_3(t)}{\alpha_{\text{sum}}} \right)^{8/9} \left(\frac{\alpha_1(t)}{\alpha_1(0)} \right)^{10/99} \left(1 + 6 Y_t(0) F_1(t) \right)^{-1/12} \quad (29)$$

$t \lesssim 67$

where $F_1(t)$ is given in Eq. (24). Equation (29) indicates that a sufficiently large $Y_t(0)$ (heavy top-quark) can lead to an arbitrarily small m_b/m_τ ratio. This is shown numerically in Fig. 2. The same applies if, when the top-quark is replaced by the U quark of a new heavier fourth generation, the same reduction of the ratio m_b/m_τ appears ^{**)} . Hence arguments which limit the number of possible generations to three on the grounds of the SU(5) prediction for m_b/m_τ should be revised. Let us notice, however, that the suppression factor in the right-hand side of Eq. (29) is limited by perturbativity ($Y_t(0) \lesssim 1$) to be greater than ~ 0.50 . Notice also that if we know the values of m_b/m_τ and $\alpha_3, \alpha_2, \alpha_1$ with enough precision, we should be able to extract from Fig. 2 a prediction for the mass of the top-quark (of course, two-loop corrections should be included). Unfortunately, this is not the case and the uncertainty in the QCD parameter Λ is too large to draw any definite conclusion.

*) All over the paper we have taken $\alpha_{\text{sum}} = 1/24$, $M_X = 2.8 \times 10^{16} \text{ GeV}$ and $\sin 2\theta_W(M_W) = 0.23$.

**) The fact that a large m_t leads to a reduction of the m_b/m_τ ratio was already remarked in Ref. 19).

In the numerical calculations shown in the rest of the paper, we have solved numerically the equations of the Yukawas for a large number of initial conditions $Y_t(0)$ and $Y_b(0) = Y_\tau(0)$. For $Y_b \ll Y_t$ the approximation in Eqs. (23) and (29) is extremely good. Notice that the condition in Eq. (21) implies that the quantities $\sin 2\theta(M_W)$ and $h_b(M_W), h_\tau(M_W)$ are related to each other :

$$h_{b,\tau}(M_W) = g \, m_{b,\tau}(M_W) \frac{\sqrt{1+\omega^2}}{\sqrt{2} M_W} \quad (30)$$

where $\omega = \tan \theta = v_2/v_1$. If we impose a condition $h_b \leq 1$, this implies a (not very stringent) upper bound for $\tan \theta$, $\tan \theta \leq 1.6 \times 10^4$ ($\sin 2\theta \gtrsim 1.2 \times 10^{-3}$). A more stringent bound is obtained in each case by imposing $h_b(M_W) \leq h_\tau(M_W)$, in order to get $m_t > m_b$. This implies also, in general, $\omega = v_2/v_1 > 1$ which, when inserted in Eq. (30), gives lower bounds on $h_{b,\tau}$:

$$\begin{aligned} h_b(M_W) &\gtrsim 1/40 \\ h_\tau(M_W) &\gtrsim 1/95 \end{aligned} \quad (31)$$

From what we have explained above, it follows that one can use as free parameters in the evolution equations for the Yukawas the quantities $Y_t(0)$ (or $Y_t(M_W)$) and $\sin 2\theta(M_W)$ (or ω) instead of $Y_t(0)$ and $Y_b(0)$. We proceed now to the discussion of the solutions for the rest of the renormalization group equations.

4. - THE EVOLUTION OF THE MASS PARAMETERS

The renormalization group equations shown in Appendix I are difficult to solve so that eventually one will have to integrate them numerically. Many of the equations, however, are easy to integrate analytically, at least in certain limits, and we will try to reduce in this section the numerical computation to a minimum.

The evolution of the gauge couplings and gaugino masses is trivial :

$$\alpha_i(t) = \alpha_i(0) / (1 + \beta_i t) \quad ; \quad M_i(t) = M_i / (1 + \beta_i t) \quad (32)$$

where $\beta_i \equiv \tilde{\alpha}_i(0)b_i$, $\tilde{\alpha}_i(0) = \alpha_i(0)/(4\pi)$ and b_i are the coefficients of the Callan-Symanzik β function. For the minimal low energy content

$$b_3 = -3 \quad ; \quad b_2 = 1 \quad ; \quad b_1 = 11$$

$$\alpha_3(0) = \alpha_2(0) = \frac{5}{3} \alpha_1(0) = \alpha_{\text{sum}} \simeq 1/24. \quad (33)$$

The solution of the equation for μ_3^2 is :

$$\mu_3^2(t) = \mu_3^2(0) (1+\beta_2 t)^{3/2b_2} (1+\beta_1 t)^{1/2b_1} \exp\left(\frac{-1}{2} \int_0^t (3Y_t + 3Y_b + Y_\tau) dt'\right) \quad (34)$$

where we have neglected the effect of the Yukawas of the first two generations. This equation shows that, for not too large Yukawas, μ_3^2 grows at low energies. This will turn out to be important as we will show below. In the interesting limit $h_b, h_\tau \ll h_t$ one can do the integral in Eq. (34) and obtain

$$\mu_3^2(t) = \mu_3^2(0) (1+\beta_2 t)^{3/2b_2} (1+\beta_1 t)^{1/2b_1} (1 + 6 Y_t(0) F_1(t))^{-1/4} \quad (35)$$

where $F_1(t)$ is given in Eq. (24). For $t = \ln(M_X^2/M_W^2) \simeq 67$ the $SU(2) \times U(1)$ contribution in (35) gives a factor $\simeq 1.4$. The Y_t contribution flattens the evolution of μ_3^2 .

Let us turn now to the masses of the scalars. The equations for the scalars of the first two generations can be easily solved since their Yukawas are negligible. One finds for the mass of any scalar :

$$m_\phi^2(1^{\text{st}}, 2^{\text{nd}} \text{ gen.}) = m^2 + 2M^2 \left(\sum_{i=1}^3 C_i \tilde{\alpha}_i(0) f_i(t) \right) \quad (36)$$

where the quantities $f_i(t)$, which will be used very often all over the paper, are defined

$$f_i(t) = \frac{1}{\beta_i} \left(1 - \frac{1}{(1+\beta_i t)^2} \right) \quad (37)$$

C_i is the quadratic Casimir corresponding to each scalar [$C = (N^2-1)/2N$ for $SU(N)$, $C = Y^2$ for $U(1)$]. Once the $SU(2) \times U(1)$ symmetry is broken, there will appear additional contributions to all the scalar masses coming from the $\frac{1}{2}D^2$

term in the scalar potential. We will talk about these at the end of the paper. Notice from Eq. (36) that the masses of the scalars remain $\approx$ gravitino mass for vanishing gaugino masses. A large gaugino mass drives on the contrary large scalar masses.

The situation for the scalars of the third generation and the Higgs doublets is more complicated since in general the couplings h_t, h_b cannot be neglected. These are seven independent mass parameters : $m_Q^2, m_U^2, m_D^2, m_L^2, m_E^2, \mu_1^2, \mu_2^2$ in the notation of Section 2. Fortunately, there are four linear combinations of masses whose Yukawa contributions to their anomalous dimensions vanish. Thus their evolution equations can also be analytically solved. These linear combinations are :

$$m_4^2 \equiv m_D^2 + m_U^2 - 2 m_Q^2 \quad (38a)$$

$$m_5^2 \equiv \frac{2}{3} \mu_1^2 - m_U^2 \quad (38b)$$

$$m_6^2 \equiv \frac{3}{2} m_D^2 + m_L^2 - \mu_1^2 \quad (38c)$$

$$m_7^2 \equiv m_L^2 - \frac{1}{2} m_E^2 \quad (38d)$$

The solutions to their evolution equations are :

$$m_4^2 = M^2 (-3 \tilde{\alpha}_2(0) f_2(t) + \tilde{\alpha}_1(0) f_1(t)) \quad (39a)$$

$$m_5^2 = -\frac{m^2}{3} + M^2 \left(-\frac{8}{3} \tilde{\alpha}_3(0) f_3(t) + \tilde{\alpha}_2(0) f_2(t) - \frac{5}{9} \tilde{\alpha}_1(0) f_1(t) \right) \quad (39b)$$

$$m_6^2 = \frac{3}{2} m^2 + M^2 \left(4 \tilde{\alpha}_3(0) f_3(t) + \frac{1}{3} \tilde{\alpha}_1(0) f_1(t) \right) \quad (39c)$$

$$m_7^2 = \frac{1}{2} m^2 + M^2 \left(\frac{3}{2} \tilde{\alpha}_2(0) f_2(t) - \frac{1}{2} \tilde{\alpha}_1(0) f_1(t) \right) \quad (39d)$$

There are, however, three mass parameters with non-vanishing Yukawa contributions to their anomalous dimensions. These are precisely m_U^2, m_D^2 and m_E^2 . In order to use matrix notation, from now on we shall call

$$m_1^2 \equiv m_U^2 \quad ; \quad m_2^2 \equiv m_D^2 \quad ; \quad m_3^2 \equiv m_E^2 \quad (40a)$$

$$\gamma_1 \equiv \gamma_t \quad ; \quad \gamma_2 \equiv \gamma_b \quad ; \quad \gamma_3 \equiv \gamma_r \quad (40b)$$

$$A_1 \equiv A_t \quad ; \quad A_2 \equiv A_b \quad ; \quad A_3 \equiv A_r \quad (40c)$$

With this notation, the evolution equation for $\vec{m}^2 \equiv (m_1^2, m_2^2, m_3^2)$ can be written

$$\frac{d\vec{m}^2}{dt} = \vec{G}(t) - (M(t)\vec{m}^2(t)) \quad (40')$$

where $M(t)$ is a 3×3 matrix

$$M(t) = \begin{pmatrix} 6\gamma_1 & \gamma_1 & 0 \\ \gamma_2 & 6\gamma_2 & \gamma_2 \\ 0 & 3\gamma_3 & 4\gamma_3 \end{pmatrix} \quad (41)$$

and the three functions $G_i(t)$ are given in Appendix II. They are simple functions of t , $Y_i(t)$ and $A_i(t)$. The general solution to Eq. (40) has the form

$$\vec{m}^2(t) = \exp\left(-\int_0^t M(t') dt'\right) \left\{ \vec{m}^2(0) + \int_0^t dt' \exp\left(\int_0^{t'} M(t'') dt''\right) \vec{G}(t') \right\} \quad (42)$$

One usually solves Eq. (42) numerically since it implies the diagonalization of matrix $M(t)$ and the integration of functions of $Y_i(t)$ and $A_i(t)$. One obtains something of the form :

$$m_i^2(t) = M^2 \tilde{a}_i + M A M \tilde{b}_i + m^2 \tilde{c}_{1i} + A^2 m^2 \tilde{c}_{2i} \quad (43)$$

where the general (complicated) expression of the coefficients $\tilde{a}_j$, $\tilde{b}_j$, $\tilde{c}_{1j}$ and $\tilde{c}_{2j}$ in terms of the $Y_i(t)$ are given in Appendix II. Notice that once we have obtained the m_i^2 ($i = 1, 2, 3$), the other four masses are related to the m_i^2 ($i = 1-7$) as follows

$$m_Q^2 = -\frac{m_4^2}{2} + \frac{1}{2} m_2^2 + \frac{1}{2} m_1^2 \quad (44a)$$

$$m_L^2 = m_7^2 + \frac{1}{2} m_3^2 \quad (44b)$$

$$\mu_1^2 = \frac{3}{2} m_2^2 + \frac{1}{2} m_3^2 + m_7^2 - m_6^2 \quad (44c)$$

$$\mu_2^2 = \frac{3}{2} m_1^2 + \frac{3}{2} m_5^2 \quad (44d)$$

To obtain the solution of Eq. (42) one needs to know the evolution of the $A_i(t)$. The renormalization group equations for these can be written :

$$\frac{d\vec{A}(t)}{dt} = \vec{G}_A(t) - (N(t)\vec{A}(t)) \quad (45)$$

where $N(t)$ is a 3×3 matrix [not the same as $m(t)$] :

$$N(t) = \begin{pmatrix} 6\gamma_1 & \gamma_2 & 0 \\ \gamma_1 & 6\gamma_2 & \gamma_3 \\ 0 & 3\gamma_2 & 4\gamma_3 \end{pmatrix} \quad (46)$$

and the three $G_{A_i}(t)$ ($i = 1, 2, 3$) are in this case very simple known functions (proportional to M/m) which we give in Appendix II. The solution of Eq. (45) has again the form :

$$\vec{A}(t) = \exp\left(-\int_0^t dt' N(t')\right) \left\{ \vec{A}(0) + \int_0^t dt' \left(\exp\left(\int_0^{t'} dt'' N(t'')\right) \vec{G}_A(t') \right) \right\} \quad (47)$$

In the case of the first two generations, the Yukawa couplings may be neglected and one gets ^{*)} :

$$A_U(1^{st}, 2^{nd} \text{ gen.}) = A + \tilde{\alpha}_{sum} \frac{M}{m} \left(\frac{16}{3} h_3(t) + 3 h_2(t) + \frac{13}{15} h_1(t) \right) \quad (48a)$$

$$A_D(1^{st}, 2^{nd} \text{ gen.}) = A + \tilde{\alpha}_{sum} \frac{M}{m} \left(\frac{16}{3} h_3(t) + 3 h_2(t) + \frac{7}{15} h_1(t) \right) \quad (48b)$$

$$A_L(1^{st}, 2^{nd} \text{ gen.}) = A + \tilde{\alpha}_{sum} \frac{M}{m} \left(3 h_2(t) + \frac{9}{5} h_1(t) \right) \quad (48c)$$

where the functions $h_i(t)$ are defined

$$h_i(t) = t / (1 + \beta_i t) \quad (49)$$

^{*)} Here we neglect the third generation Yukawa contribution to the H_1 and H_2 field's wave function renormalization.

5. - THE BREAKING OF THE SU(2)×U(1) SYMMETRY

Up to now, we have shown how to obtain the evolution equations for the mass parameters, μ_3^2 and m_1^2 . We must impose now that the SU(2)×U(1) symmetry is broken at the appropriate scale. This amounts to imposing $M_W^2 = g^2 v^2/2$ with v^2 given by Eq. (12). Simple algebra converts the condition of SU(2)×U(1) breaking into the following equation

$$(\mu_1^2 - \omega^2 \mu_2^2) / (\omega^2 - 1) = \frac{M_Z^2}{2} \quad (50)$$

where $\omega = \tan\theta = v_2/v_1$ and M_Z is the Z^0 boson mass. All the quantities in (50) (ω , μ_1^2 and μ_2^2) should be evaluated at the M_W scale. Notice that in the literature up to now only the limit $\omega \rightarrow \infty$, which gives $\mu_2^2 = -M_Z^2/2$ has been treated in detail. We consider here the general case with arbitrary $\omega > 1$. Using Eqs. (44c-d), the condition (50) can be written

$$\sum_i m_i^2 \phi_i(\omega) + \left(\frac{3}{2} m_5^2 \omega^2 + m_6^2 - m_7^2 \right) / (\omega^2 - 1) = -\frac{M_Z^2}{2} \quad (51a)$$

where ϕ_j is the vector

$$\phi_j = \left(3\omega^2/2(\omega^2-1), -3\omega^2/2(\omega^2-1), -1/2(\omega^2-1) \right) \quad (51b)$$

which tends to $(\frac{3}{2}, 0, 0)$ as $\omega \rightarrow \infty$. Equations (51) can be further transformed in order to show the explicit dependence on M^2 and m^2 as follows :

$$\sum_i \phi_i m_i^2 + \frac{m^2}{2} \left(\frac{2-\omega^2}{\omega^2-1} \right) + M^2 \frac{3}{2} K(t) = -\frac{M_Z^2}{2} \quad (52)$$

where $K(t) = (m_5^2 + m^2/3)/M^2$ does not depend on m^2 or M^2 . Inserting in (52) the general form of the m_i 's given by (43), one finally obtains the SU(2)×U(1) breaking condition as a second order equation on the variable M

$$aM^2 + bM + c = 0 \quad (53)$$

with coefficients :

$$a \equiv \sum_i \phi_i \tilde{a}_i + \frac{3}{2} K \quad (54a)$$

$$b \equiv A m \sum_i \tilde{b}_i \phi_i \quad (54b)$$

$$c \equiv m^2 \sum_i \phi_i (\tilde{c}_{1i} + A^2 \tilde{c}_{2i}) + \frac{m^2}{2} \left(\frac{2 - \omega^2}{\omega^2 - 1} \right) + \frac{M_Z^2}{2} \quad (54c)$$

where the coefficients $\tilde{a}_j$, $\tilde{b}_j$, $\tilde{c}_{1j}$ and $\tilde{c}_{2j}$ were introduced in Eq. (43). Equations (53) and (54) explicitly show the dependence of the $SU(2) \times U(1)$ breaking condition on the parameters A , m , M and $\sin 2\theta(M_W)$. The dependence on the Yukawa couplings is complicated and it is hidden in the coefficients $\tilde{a}_j$, $\tilde{b}_j$, $\tilde{c}_{1j}$ and $\tilde{c}_{2j}$. We have explored in detail numerically the solutions of Eq. (53) for a large range of values in the parameter space $(A, m, M, \sin 2\theta, Y_t)$ and the results are presented in the following section. Before discussing these results, let us comment on the general form of the equations and the mechanisms of symmetry breaking. A strong constraint on the parameter space is given by the condition $b^2 \geq 4ac$. Thus, for example, in the limit $m \rightarrow 0$, $M_Z^2 = -M_W^2/2a$ and the quantity a has to be negative and this constrains the values of Y_t and $\sin^2\theta(M_W)$. In particular, if we want to have a $M^2 \gg m^2$ we need an almost vanishing a . This condition $a \approx 0$ in turn only allows for a restricted range of values of Y_t (and hence, top-quark masses). This will show up in the numerical results shown in the next section.

Equations (53) and (54) are extremely simplified in the limit $M \rightarrow 0$. This just implies $C = 0$. Further simplification is obtained if we assume $h_b, h_\tau \ll h_t$ and $\omega \rightarrow \infty$. This limit was already considered in Ref. 11). In this case, Eqs. (43), (44) and (50) yield the simple expressions :

$$\mu_1^2 = m^2 \quad (55a)$$

$$\mu_2^2 = \frac{3}{2} m^2 \left(\frac{1 + Y_t(0) F_1(6 - 2A^2)}{(1 + 6 Y_t(0) F_1)^2} - \frac{1}{3} \right) \quad (55b)$$

and the $SU(2) \times U(1)$ breaking condition is just $\mu_2^2 = -M_Z^2/2$. Thus in this limit $Y_t(0)$ has to be large enough in order to get a negative μ_2^2 . Alternatively, one may use a large A but this could violate the bounds in Eqs. (10) ^{*)}. Since the bracket in (55) has to be negative, we can translate this condition into a condition for the top-quark mass (for the case $M = 0$) :

*) One may have, however, large A 's if one assumes that we live in an unstable local minimum which will eventually decay into charge and colour breaking minima. See Ref. 25).

$$m_t \geq m_t^{\max} \sqrt{G(A)/(1+G(A))} \quad (56a)$$

$$G(A) \equiv \sqrt{\left(\frac{A^2-1}{2}\right)^2 + 2} + \frac{(1-A^2)}{2} \quad (56b)$$

where $m_t^{\max} \approx 190$ GeV is the upper bound for the t quark that we mentioned in Section 3. It is obvious from Eqs. (56) that one can always break $SU(2) \times U(1)$ with an arbitrarily small m_t by using large A 's since the lower bound behaves when $A \rightarrow \infty$ as

$$m_t \geq m_t^{\max} \sqrt{2}/A \quad (M=0, A \rightarrow \infty) \quad (57)$$

This feature is not specific to this approximation ($M=0, h_b, h_t \ll h_t$). It happens also in the general case and it is due to the sign of the A^2 term in the evolution equations of μ_2^2 .

From the above example, Eq. (55), and from the results of Refs. 9),11),12), it is clear that one can break $SU(2) \times U(1)$ radiatively by having a heavy top-quark which drives a negative mass for μ_2^2 (Fig. 4a). But this is not the only way by which one can obtain the required symmetry breaking. This is very easily seen from Eq. (50). Assume that μ_1^2 and μ_2^2 are very similar, $(\mu_1^2 - \mu_2^2)/M_Z^2 = \epsilon \ll 1$. This is what happens if all the Yukawas are very small since the renormalization group equations for μ_1^2 and μ_2^2 only differ in Yukawa terms. Then the $SU(2) \times U(1)$ breaking condition, Eq. (50), takes the form

$$\epsilon \left(\frac{\omega^2}{\omega^2 - 1} \right) = \frac{1}{2} + \frac{\mu_1^2}{M_Z^2} \quad (58)$$

Although ϵ is assumed to be small, that can be compensated by having an ω sufficiently close to 1. This is the second mechanism for $SU(2) \times U(1)$ breaking advertized in the abstract ^{*)}. Notice that the smaller the top-quark mass (Y_t) the smaller ϵ is and, hence, the smaller $(\omega^2 - 1)$ has to be. This seems to

*) We understand that this mechanism has also been considered by Jones and Ross ²⁶⁾. A similar mechanism in the context of the "supersymmetric Coleman-Weinberg" mechanism has been considered by Kounnas, Lahanas, Nanopoulos and Quiros ²⁷⁾.

imply that, in order for this second mechanism of symmetry breaking to work, one has to fine tune the parameters at M_X in such a way that $\sin 2\theta(M_W) \approx 1$. Interestingly enough this happens not to be the case. The parameter $\sin 2\theta \equiv 2\mu_3^2 / (\mu_1^2 + \mu_2^2)$ evolves with energy in such a way that values of $\sin 2\theta(M_X)$ in the range 0.60-1.0 evolve towards a $\sin 2\theta(M_W) \approx 1.0$ at low energies. This is due to the fact that, as we remarked in Section 4, μ_3^2 increases at low energies. We will show how this phenomenon occurs numerically in the following section but it can also be seen analytically in the approximation $h_b, h_t \rightarrow 0$, which is in fact the appropriate limit for this second mechanism. In that limit one has :

$$\frac{d(\mu_1^2 + \mu_2^2)}{dt} = 6 \tilde{\alpha}_2 M_2^2 + 2 \tilde{\alpha}_1 M_1^2 \quad (59a)$$

$$\frac{d\mu_3^2}{dt} = \left(\frac{3}{2} \tilde{\alpha}_2 + \frac{1}{2} \tilde{\alpha}_1 \right) \mu_3^2 \quad (59b)$$

One can integrate for $\sin 2\theta = 2\mu_3^2 / (\mu_1^2 + \mu_2^2)$ to obtain

$$\sin 2\theta(t) = \sin 2\theta(0) (1 + \beta_2 t)^{3/2 b_1} (1 + \beta_1 t)^{1/2 b_1} \left(1 + \frac{M^2}{2m^2} (3\tilde{\alpha}_2(0) f_2(t) + \tilde{\alpha}_1(0) f_1(t)) \right)^{-1} \quad (60)$$

where the $f_i(t)$ are defined in Eq. (37). At the $SU(2) \times U(1)$ breaking scale one has $t \approx 67$ and one gets :

$$\sin 2\theta(M_X) = \sin 2\theta(M_W) \times 0.71 (1 + 0.53 M^2/m^2) \quad (61)$$

Notice that in this limit one can obtain $\sin 2\theta(M_W) \approx 1$ with initial values for $\sin 2\theta(M_X)$ in the range 0.7 to 1.0, i.e., no fine tuning is required. With slightly non-vanishing Y_t one would obtain a slightly large range. We will show how Eqs. (59)-(61) are modified in the case of a non-negligible μ' at the end of the following section.

The two mechanisms of symmetry breaking are qualitatively exemplified in Figs. 3a and 3b. In the first mechanism, μ_2^2 becomes negative due to a strong enough top-quark Yukawa ⁴⁾. μ_1^2 is relatively flat since its evolution is flattened by the Yukawa coupling Y_t [see, e.g., Eq. (35)]. In the second mechanism (Fig. 4b), both μ_1^2 and μ_2^2 remain relatively flat, since the Yukawa couplings are small. $\mu_1^2 > \mu_2^2$ though, since Y_t is bigger than Y_b . μ_3^2 evolves at low

energies towards $(\mu_1^2 + \mu_2^2)/2$, leading to $\sin 2\theta(M_W) \approx 1$. Of course, there are also symmetry breaking situations in which a "linear combination" of the above two mechanisms is at work, as we will see in the following section.

6. - DISCUSSION OF THE NUMERICAL RESULTS

We have investigated numerically the range of the parameters M , m , A , $\sin 2\theta$ and m_t which fulfil the conditions of $SU(2) \times U(1)$ breaking, Eqs. (53) and (54). We have not neglected h_b and h_t which are not free parameters and are related to $\sin 2\theta$ through Eqs. (21) and (18). In particular, one cannot neglect h_b when $\sin 2\theta(M_W) \rightarrow 0$ ($v_1 \ll v_2$) since the constraint Eq. (21) requires $h_b \rightarrow \infty$ in this limit. However, in general, one has $h_b < h_t$ in order to get $m_t > m_b$ ($v_2 \ll v_1$ does also require $h_t > h_b$). One can thus have in principle h_b 's as large as h_t and that is why it is necessary to include non-vanishing h_b 's (and hence h_t 's) when $\sin 2\theta$ is small. In order to do that, we have integrated numerically the renormalization group equations (Appendix II-3) of the Yukawa couplings for a large range of initial conditions $h_t(0)$ and $h_b(0)$.

$SU(2) \times U(1)$ symmetry breaking occurs for wide ranges of the parameters. In order not to bore the reader we will not present too many numerical results but will show the most significant plots and comment about what happens for other ranges of the parameters. We will pay special attention to the dependence of the other parameters and the symmetry breaking on $\sin 2\theta$, since this has not been analyzed in detail in the literature. What we show in Figs. 4-7 is how the parameters M , m , A , $\sin 2\theta$ and m_t have to be related in order to obtain $SU(2) \times U(1)$ breaking at the appropriate scale.

Figure 4 represents a typical plot showing the dependence of the gaugino mass at M_X (M) on the value of $\sin 2\theta(M_W)$ for various values of the top Yukawa coupling $h_t(0)$. Each line corresponds to a value of $h_t(0)$ (from 0.7 to 0.06) and the small numbers on the lines indicate to what value of m_t it corresponds. Figure 4 shows the case with $A = 1$ and $m = 100$ GeV. Up to now, only the limit in the left-hand side had been explored ^{9),11),12)} but with a negligible h_b . The plot shows that $SU(2) \times U(1)$ is more easily broken for $\sin 2\theta(M_W) \approx 0.95-1.0$ (i.e., smaller values of gaugino masses and top-quarks are required). As $\sin 2\theta(M_W)$ decreases, $SU(2) \times U(1)$ breaking requires larger and larger gaugino masses and light t quarks (e.g., $m_t \lesssim 50$ GeV) do not seem possible. As one approaches

$\sin 2\theta(M_W) \sim 0.1$ things improve a little bit and gaugino masses decrease somewhat. This is due to the fact we discussed above that, for small $\sin 2\theta(M_W)$ one cannot neglect h_b . A non-negligible h_b collaborates in obtaining small μ_2^2 and facilitates the breaking of $SU(2) \times U(1)$. In this plot what is going on is that in a very small region $[\sin 2\theta(M_W) \approx 0.95-1.00]$ it is the second mechanism discussed in the previous section which is operative. In the rest of the values of $\sin 2\theta(M_W)$ it is the first mechanism (negative μ_2^2) which is at work. Thus one can have light t quarks but only for $\sin 2\theta(M_W) \approx 1.0$. In fact we have somewhat separated the $h_t(0) = 0.06$ line ($m_t \sim 30$ GeV) from the vertical $\sin 2\theta = 1.0$ line to facilitate the drawing of the figure but they should essentially be one on the top of each other. It seems then that in order to have, e.g., a t quark with mass ~ 30 GeV one has to "fine tune" the value of $\sin 2\theta(M_W) \approx 0.99$. However, the situation changes completely if we draw the same plot versus the value of $\sin 2\theta$ as measured at the GUT scale, $\sin 2\theta(M_X)$. This is in fact the quantity which has a real physical meaning and which should be given eventually by the correct (if it exists) local SUSY GUT. Then the plot is "deformed" to that in Fig. 5. It is especially the right-hand part of the plot which changes most. This is due to the fact that, as we discussed in the previous section, μ_3^2 has quite a flat scale-dependence for large h_t but changes substantially for small h_t 's. The dashed border on the right-hand side corresponds to the analytic limit $h_b, h_t \rightarrow 0$ given by Eq. (61). Curves to the right-hand side of that limit would violate the bound $\mu_1^2 + \mu_2^2 \geq 2\mu_3^2$ ($\sin 2\theta \leq 1$). Figure 5 shows that there is no need for a "fine tuning" at the GUT level. There is a wider range of values $\sin 2\theta(M_X) \approx 0.65-1.0$ leading to $SU(2) \times U(1)$ breaking with a light t quark (even massless!). This anyway implies that a local SUSY GUT with a light t quark should have $\mu_3^2 \sim m_{3/2}^2$.

There are several other points of interest in Fig. 5. The vertical line $\sin 2\theta(M_X) = 1.0$ does also (approximately) correspond to $\sin 2\theta(M_W) \approx 1.0$. This means that for points in that line (and above the dashed limit) $\sin 2\theta$ remains constant and ≈ 1.0 from M_X to M_W , leading to a flat potential with $v_1 = v_2$. Figure 5 shows that only scenarios with $m_t \lesssim 50$ GeV can have that structure. Another interesting point is that (contrary to what happened in previous analyses) one can have vanishing gaugino masses with arbitrary small t quark masses as long as $\sin 2\theta(M_X)$ approaches the value 0.71.

It is also interesting to notice that Eq. (61) is quite a good approximation for light enough (e.g., $\lesssim 30$ GeV) t quarks, as can be observed from Fig. 5.

An obvious conclusion from Figs. 4 and 5 is the well-known fact that one needs heavier t quarks in order to get light gauginos (for a given $\sin 2\theta$). Another point is the dependence of these results on m and A (Figs. 4 and 5 correspond to $A = 1$ and $m = 100$ GeV). As is well known, a large A facilitates the symmetry breaking, i.e., one needs smaller gaugino masses. As an example, we show in Fig. 6 results for $A = 2$ and $m = 100$. What happens essentially is that Fig. 5 gets "pushed" downwards but the qualitative form of the plot remains (just lighter t quarks and gauginos are required). The analytic limit in the right-hand side, Eq. (61), does not depend on A . We do not present results for $A > 3$ since then the bounds in Eq. (10) would be violated ⁷⁾.

Concerning the m dependence, the plots depend on m primarily through M^2/m^2 so that, e.g., for $m = 200$ GeV we could multiply the M scale in Figs. 4, 5 and 6 by a factor two and that would resemble very much the actual result for $m = 200$ GeV. Of course, this is only a good approximation for $M, m \gg M_Z$ but still the general shape of the plots remains the same as that of Figs. 4, 5 and 6.

The content of Fig. 4 can be displayed in a different (and interesting) way by plotting the gaugino mass at M_X (M) versus the top-quark mass for different values of the parameter $\sin 2\theta(M_X)$. This is shown for $A = 1$ and $m = 100$ GeV in Fig. 7. One observes again that one can break $SU(2) \times U(1)$ with an arbitrary small m_t by having $\sin 2\theta(M_X) \geq 0.6$. Also one can have vanishing gaugino masses for any value of the t quark mass. Notice that for $M \geq 2m \geq 2M_W$ the allowed values of the top-quark mass are restricted to the range

$$50 \text{ GeV} \leq m_t \leq 65 \text{ GeV} \quad (M \geq 2m) \quad (62)$$

This corresponds to what we discussed in Section 5 with respect to the $M \gg m$ limit of Eq. (53). That limit is obtained for $a \rightarrow 0$. a is a function of Y_t and $\sin 2\theta$ and the restricted range in Eq. (62) corresponds to the approximate solutions of $a \rightarrow 0$. A similar result is obtained for other values of A .

The small $\sin 2\theta(M_X)$ curves (right-hand side) in Fig. 7 should correspond to the small μ_3^2 results presented in Ref. 11) and, in particular, to Figs. 1 and 2 of that reference. Notice, however, that in the present calculation we have not neglected the h_b coupling and that is why the upper sectors of the curves $\sin 2\theta(M_X) = 0, 1; 0.2$ bend a little bit.

In all the presented results, the bounds in Eqs. (15) and (16) are respected. The same happens with the limits on the trilinear couplings, Eqs. (10a-c). In the latter case, the fulfilment of the condition usually implies values for A slightly smaller than 3. Usually (especially for not too large Yukawa couplings) the condition (10c) is stronger than the other two.

Let us finally comment on the effect of the inclusion in the calculation of a non-negligible μ' (see Section 2). There will still be the two simple $SU(2) \times U(1)$ breaking mechanisms discussed in the previous section. In the case of the first mechanism (large m_t 's drive a negative μ_2^2) a large μ'^2 would make the symmetry breaking slightly more difficult (larger gaugino masses and m_t 's) since μ_2^2 would start at M_X from a larger positive value $\mu_2^2(M_X) = m^2 + \mu'^2$.

In the case of the second mechanism one in general needs larger values of $\sin 2\theta(M_X)$ in order to obtain the desired $\sin 2\theta(M_W) \approx 1.0$. However, one still has a finite range of values for $\sin 2\theta(M_X)$ leading to $\sin 2\theta(M_W) \approx 1.0$ as long as $\mu' \lesssim m$. This can easily be seen from the renormalization group equations in the limit of negligible Yukawa couplings $h_t, h_b, h_z \rightarrow 0$ which, as we mentioned in the previous section, is the appropriate limit to simulate this second mechanism. In the case of a non-negligible μ' , Eqs. (59) are changed to

$$\frac{d(\mu_1^2 + \mu_2^2)}{dt} = 6\tilde{\alpha}_2 M_2^2 + 2\tilde{\alpha}_1 M_1^2 + (6\tilde{\alpha}_2 + 2\tilde{\alpha}_1) \mu'^2 \quad (63a)$$

$$\frac{d\mu_3^2}{dt} = \left(\frac{3}{2}\tilde{\alpha}_2 + \frac{1}{2}\tilde{\alpha}_1\right) \mu_3^2 + 3\tilde{\alpha}_2 \mu' M_2 + \tilde{\alpha}_1 \mu' M_1 \quad (63b)$$

$$\frac{d\mu'^2}{dt} = (3\tilde{\alpha}_2 + \tilde{\alpha}_1) \mu'^2 \quad (63c)$$

which are integrated to yield

$$\mu_1^2 + \mu_2^2 = 2m^2 + M^2(3\tilde{\alpha}_2(0)f_2(t) + \tilde{\alpha}_1(0)f_1(t)) + \int_0^t (6\tilde{\alpha}_2 + 2\tilde{\alpha}_1) \mu'^2(t') dt' \quad (64a)$$

$$\mu_3^2(t) = \mu_3^2(0) (1 + \beta_2 t)^{3/2b_2} (1 + \beta_1 t)^{1/2b_1} + \mu'(t) M (3\tilde{\alpha}_2(0)h_2(t) + \tilde{\alpha}_1(0)h_1(t)) \quad (64b)$$

$$\mu'^2(t) = \mu'^2(0) (1 + \beta_2 t)^{3/b_2} (1 + \beta_1 t)^{1/b_1} \quad (64c)$$

Using these equations one obtains the formula equivalent to that in Eq. (61) for the case of a non-negligible μ' :

$$\sin 2\theta(M_W) = \frac{1.4 \times \sin 2\theta(M_X) + 0.83 (\tilde{\mu}' \tilde{M} / (1 + \tilde{\mu}'^2))}{1 + 0.82 (\tilde{\mu}'^2 / (1 + \tilde{\mu}'^2)) + 0.53 (\tilde{M}^2 / (1 + \tilde{\mu}'^2))} \quad (65)$$

where $\tilde{\mu}' \equiv \mu'/m$, $\tilde{M} = M/m$. This formula with $\sin 2\theta(M_W) = 1$ would give rise to the boundary line equivalent to those which appear in Figs. 5 and 6 but for the $\mu' \neq 0$ case. Using Eq. (65), one easily convinces oneself that a finite range for $\sin 2\theta(M_X)$ (0.73-1.0 for $\mu'/m = \frac{1}{4}$, 0.77-1.0 for $\mu'/m = \frac{1}{2}$, 0.88-1.0 for $\mu'/m = 1$) leads to $\sin 2\theta(M_W) \approx 1.0$. These ranges are slightly enlarged when one considers the physical case with $m_t \gtrsim 20$ GeV. We conclude that the second possibility for symmetry breaking exists and is still "natural" when we incorporate a $\mu' \neq 0$ in the equations. The numerical results will not be much changed as long as μ' is not too large.

Before coming to the conclusions let us briefly discuss the typical particle spectrum appearing in this type of scenario with radiative $SU(2) \times U(1)$ breaking induced from supergravity.

7. - LOW ENERGY SPECTRUM

There are four different sources for masses of the squarks and sleptons in this type of theory. First, there is a "supersymmetric" mass which is equal to the mass of the corresponding supersymmetric fermionic partner. Second, there is a "gravitino induced" mass which is the one we have been discussing in detail in the previous sections. Third, there is a contribution to the scalar masses coming from the $D^2/2$ term in Eq. (9) which appears once the Higgses get their vacuum expectation value. All these three types of masses are of the form $(\phi_L^* \phi_L)$ or $(\phi_R^* \phi_R)$. Finally, there is a fourth contribution to scalar masses ($\Delta I_W = \frac{1}{2}$ masses) coming from the trilinear scalar couplings which are of the form $h m_A v (\phi_L^* \phi_R + \text{h.c.})$. The magnitude of the first of these four contributions is obvious and, for the sake of completeness, we give the general expressions for the second and third contributions to the scalar masses of the different squarks and sleptons in Appendix III. Concerning the fourth type of mass ($\Delta I_W = \frac{1}{2}$ masses), these are obtained from the equations for V_3 in (9) and Eqs. (47) and (48) by substituting the appropriate vacuum expectation values. Eventually, one would have to diagonalize a mass matrix for each quark and lepton flavour of the form

$$\begin{pmatrix} \phi_R^* & \phi_1^* \\ \phi_R & \phi_L \end{pmatrix} \begin{pmatrix} m_{I=0}^R & m_{I=1/2} \\ m_{I=1/2} & m_{I=0}^L \end{pmatrix} \quad (66)$$

but in general one can neglect the off-diagonal terms for the first two generations or even for the third one if the gravitino mass is large.

Apart from the squarks and sleptons there are Higgs scalars in the low energy theory. Of the eight real fields contained in H_1 and H_2 , three are swallowed by the weak gauge bosons. There remains a complex scalar with mass [see Ref. 19)]

$$m_{H^\pm}^2 = \mu_1^2 + \mu_2^2 + M_w^2 \quad (67a)$$

and three neutral scalars with masses

$$m^2(H_c) = \mu_1^2 + \mu_2^2 \quad (67b)$$

$$m^2(H_{a,b}) = \frac{1}{2} \left(m^2(H_c) + M_2^2 \pm \sqrt{(m^2(H_c) + M_2^2)^2 - 4m^2(H_c)M_2^2 \cos^2 2\theta} \right) \quad (67c)$$

where all the parameters have to be evaluated at the M_w scale.

The fermion spectrum contains the usual quarks and leptons and the gauginos. The gluinos only have Majorana masses as given by Eq. (32). The Winos and the Binos have both Majorana masses and $\Delta I_W = \frac{1}{2}$ masses by combining with the charged and neutral Higgsinos. The charged gaugino mass matrix takes the form (21), (28), (29) :

$$(\tilde{W}^+, \tilde{H}_2^+) \begin{pmatrix} M_2 & g\nu_1 \\ g\nu_2 & \mu' \end{pmatrix} \begin{pmatrix} \tilde{W}^- \\ \tilde{H}_1^- \end{pmatrix} \quad (68)$$

whose eigenvalues are (gv_1, gv_2) in the limit $M_2 \rightarrow 0$, $\mu' \rightarrow 0$ and $(M_2, g^2 v_1 v_2 / M_2)$ in the limit $M_2 \rightarrow \infty$, $\mu' \rightarrow 0$. There are four neutral supersymmetric fermions $\tilde{W}_3$, $\tilde{B}$, $\tilde{H}_1^0$ and $\tilde{H}_2^0$. Their mass matrix has been extensively studied in the literature (21), (28), (9), (29).

Let us comment now on the specific spectra which appear in this type of theory. Each range of the parameters allowed by $SU(2) \times U(1)$ breaking leads to a different low energy spectrum so that one cannot make predictions about, e.g., what the masses of the squarks and sleptons are till new experimental information restricts the number of parameters. One can show, however, some typical spectra appearing for each of the general schemes for $SU(2) \times U(1)$ breaking considered in the previous sections. Some specific examples are shown in the Table where masses at the scale M_W are shown for squarks ($\tilde{q}$), sleptons ($\tilde{\ell}$), gluino ($\tilde{g}$), Wino ($\tilde{W}$) and Bino ($\tilde{B}$). In the case of $\tilde{W}$ and $\tilde{B}$, it is the Majorana mass which is shown. Also shown are the masses of the Higgs scalars. In the case of $\tilde{q}$ and $\tilde{\ell}$, only the "gravitino induced" and " $D^2/2$ induced" scalar masses are included, i.e., in the case of the third generation, one has to add a contribution of the order of the fermionic partner mass.

Of the examples shown in the Table, cases a), b) and c) proceed essentially through mechanism 2 since $\sin 2\theta(M_W) \approx 1$. These cases lead to a t quark ~ 30 GeV but one can get smaller (or higher) values easily. The spectrum a) has relatively light $\tilde{\ell}$'s and $\tilde{q}$'s and one can get lower values by putting a slightly smaller gravitino mass (m). Thus, contrary to what happened in most globally supersymmetric radiative models, the supersymmetric particle spectrum may be accessible to present accelerators (including the $p\bar{p}$ collider). Spectra b) and c) show how a small t quark mass does not necessarily mean a light spectrum. In case b) one can have both relatively light gluinos and not too heavy $\tilde{q}$'s and $\tilde{\ell}$'s. In case c), on the contrary, both heavy gauginos and $\tilde{q}$'s, $\tilde{\ell}$'s are obtained. If, as advocated in some models, M should be small, it turns out that for a t quark ~ 30 GeV, one always gets spectra very similar to b), i.e., $\tilde{q}$'s and $\tilde{\ell}$'s lighter than ~ 100 GeV.

In the case of intermediate t quark masses (~ 60 - 70 GeV) we saw in the last section that in general large gaugino masses are required. This feeds into the $\tilde{q}$ and $\tilde{\ell}$ masses, giving rise to heavy spectra like the one shown in case d) of the Table. In this case as in examples e), f) and g), it is mechanism 1 (a negative μ_2^2) which is operative for the breaking of $SU(2) \times U(1)$. For a t quark with a larger mass like $m_t = 111$ GeV, as in examples e) and f), one can obtain slightly lighter gaugino masses and hence some relatively light charged $\tilde{\ell}$, as in case f). This example f) also presents quite light scalar neutrinos,

$\tilde{V} \sim 16$ GeV. This is due to the negative contribution of the $D^2/2$ term $(-M_Z^2/2 \times \cos 2\theta)$ to the $\tilde{V}$ mass. The U squarks also have negative $D^2/2$ contributions to their masses, which implies in general $m_{\tilde{U}_L}^2 < m_{\tilde{D}_L}^2$, $m_{\tilde{U}_R}^2 < m_{\tilde{D}_R}^2$. for example, the spectrum of the $\tilde{q}$'s and $\tilde{l}$'s of the first two generations is in case f) of the Table (in GeV) :

$$\begin{aligned} m(\tilde{D}_L) &= 155 & m(\tilde{U}_L) &= 136 \\ m(\tilde{D}_R) &= 142 & m(\tilde{U}_R) &= 137 & (GeV) \\ m(\tilde{E}_L) &= 75 & m(\tilde{\nu}_L) &= 16 \end{aligned} \quad (69)$$

Thus in the case of a non-negligible $\cos 2\theta$ ($v_2 \gg v_1$) one may have relatively light sneutrinos which may even be the lightest R odd particle. Finally, example g) of the Table shows a spectrum corresponding to a very heavy t quark, $m_t = 160$ GeV which requires small gaugino masses. For very large t quark masses, it is difficult to obtain lighter $\tilde{q}$ and $\tilde{l}$ masses since if one uses smaller m's, in general μ_2^2 gets "too much negative" and the condition in Eq. (15) is violated. Thus, on numerical grounds, it seems much easier to accommodate a light $\tilde{q}$ and $\tilde{l}$ spectrum in schemes with light t quark masses, i.e., schemes with $\sin 2\theta(M_W) \approx 1$.

8. - CONCLUSIONS

We have presented in the previous sections an analysis of the process of radiative $SU(2) \times U(1)$ breaking induced by the explicit terms coming from broken $N = 1$ supergravity. General formulae are given which must be satisfied for obtaining the symmetry breaking at the appropriate scale. We have also obtained analytic expressions which apply in some interesting limits. This includes some useful formulae for the evolution of the Yukawa coupling constants. The ratio m_b/m_τ is substantially reduced if a very heavy t quark exists, so that bounds on the number of generations based on the prediction for m_b/m_τ should be revised.

We have investigated numerically the restrictions on the parameters m , M , A , m_t and $\sin 2\theta(M_X)$ imposed by the condition of $SU(2) \times U(1)$ breaking. Non-negligible values for μ_3^2 , h_b and h_τ (which cannot be ignored for small $\sin 2\theta$) have been included in the analysis. For simplicity, the mass parameter μ' was neglected in the numerical calculations but we study the effect of a non-negligible μ' in the cases of interest. Two different symmetry breaking schemes corresponding

to different types of local SUSY GUTs emerge from this analysis. In the first of them, t quark masses $m_t \gtrsim 60$ GeV are required and one has $\mu_3^2 \ll m^2$ ($\sin 2\theta(M_X) \ll 1$) at the GUT scale. This could typically occur in a local SUSY GUT in which the μ_3^2 term is induced radiatively (e.g., models with a "missing partner" mechanism). The particle spectrum for $\tilde{q}$'s and $\tilde{l}$'s is usually $\gtrsim 100$ GeV (except for possible light sneutrinos). In the second mechanism, one can have arbitrarily light t quark masses and $\mu_3^2 \sim m^2$ at the GUT scale ($\sin 2\theta(M_W) \approx 1.0$). This would correspond to a local SUSY GUT with a tree-level contribution to μ_3^2 . One may have (but not necessarily) quite a light $\tilde{q}$ or $\tilde{l}$ spectrum, accessible to present accelerators.

In any of the two schemes, one may have vanishing gluino masses for any value of m_t . On the contrary, for $M \gtrsim 2m$ the t quark should have a mass $50 \lesssim m_t \lesssim 65$ GeV.

Certainly the present scenario for $SU(2) \times U(1)$ breaking desperately needs experimental help in order to reduce the number of free parameters. Notice, however, that there are 45 $\tilde{q}$'s and $\tilde{l}$'s, plus gauginos and Higgses and there are only five free parameters once one imposes $SU(2) \times U(1)$ breaking. Thus, in principle, this scheme has a strong predictive power. Furthermore, it provides us with a beautiful mechanism for symmetry breaking. We do not need any more to put by hand a negative μ^2 in a tree-level potential to produce symmetry breaking. Supersymmetry provides us with running scalar masses which induce the breaking naturally. This is certainly a very nice point in favour of supersymmetry.

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	a	b	c	d	e	f	g
m_t	29	29	30	63	111	111	160
m	25	50	100	50	200	50	150
M	17	5	71	141	108	49	6
A	1	1	1	1	0.5	1	1
$\sin 2\theta(M_X)$	0.787	0.679	0.779	0.76	0.195	0.038	0.187
$\sin 2\theta(M_W)$	0.999	0.999	0.996	0.42	0.43	0.43	0.45
$\tilde{q}$	50-52	51-52	215-220	380-396	300-363	120-150	143-160
$\tilde{l}$	26-28	49-50	105-114	84-122	207-219	67-75 $\tilde{\nu}_L \sim 16$	138-156
$\tilde{g}$	50	15	214	423	324	147	19
$\tilde{W}$	14	4	58	115	88	40	5
$\tilde{B}$	7	2	29	57	44	20	3
$H^\pm$	86	104	170	136	230	81	163
H_a	96	112	173	118	219	89	147
H_b	5	7	23	80	83	23	81
H_c	37	69	151	112	217	24	143

- TABLE -

Seven examples of low energy spectra. The masses are in Gevs.

APPENDIX I

We collect here the relevant renormalization group equations^(24),3),9) for couplings and mass parameters.

One has, for the gauge couplings

$$\frac{d g_i^2}{d t} = \frac{-b_i}{(4\pi)^2} g_i^4 \quad (I.1)$$

and for gaugino masses

$$\frac{d M_i}{d t} = \frac{-b_i}{(4\pi)^2} g_i^2 M_i \quad (I.2)$$

where $t = \ln(M_X^2/Q^2)$ and $b_3 = -3$, $b_2 = 1$ and $b_1 = 11$ for a minimal low energy particle content (three families and two Higgs doublets).

For the Yukawa couplings of the third generation one has

$$\frac{d Y_t}{d t} = Y_t \left(\frac{16}{3} \tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \frac{13}{9} \tilde{\alpha}_1 - 6 Y_t - Y_b \right) \quad (I.3a)$$

$$\frac{d Y_b}{d t} = Y_b \left(\frac{16}{3} \tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \frac{7}{9} \tilde{\alpha}_1 - Y_t - 6 Y_b - Y_\tau \right) \quad (I.3b)$$

$$\frac{d Y_\tau}{d t} = Y_\tau \left(3\tilde{\alpha}_2 + 3\tilde{\alpha}_1 - 3 Y_b - 4 Y_\tau \right) \quad (I.3c)$$

where we have denoted

$$Y \equiv h^2 / (4\pi)^2 \quad ; \quad \tilde{\alpha}_i \equiv \alpha_i / (4\pi) \quad (I.3d)$$

The equation for the μ_3^2 parameter is

$$\frac{d \mu_3^2}{d t} = \left(\frac{3}{2} \tilde{\alpha}_2 + \frac{1}{2} \tilde{\alpha}_1 - \frac{3}{2} Y_t - \frac{3}{2} Y_b - \frac{1}{2} Y_\tau \right) \mu_3^2 \quad (I.4)$$

where we have assumed here a negligible μ' .

The equations for the scalar masses in the notation of Section 2 are the following

$$\frac{dm_Q^2}{dt} = \left(\frac{16}{3} \tilde{\alpha}_3 M_3^2 + 3 \tilde{\alpha}_2 M_2^2 + \frac{1}{9} \tilde{\alpha}_1 M_1^2 \right) - Y_U (m_Q^2 + m_U^2 + \mu_2^2 + A_U^2 m^2) - Y_D (m_Q^2 + m_D^2 + \mu_1^2 + A_D^2 m^2) \quad (I.5a)$$

$$\frac{dm_U^2}{dt} = \left(\frac{16}{3} \tilde{\alpha}_3 M_3^2 + \frac{16}{9} \tilde{\alpha}_1 M_1^2 \right) - 2 Y_U (m_Q^2 + m_U^2 + \mu_2^2 + A_U^2 m^2) \quad (I.5b)$$

$$\frac{dm_D^2}{dt} = \left(\frac{16}{3} \tilde{\alpha}_3 M_3^2 + \frac{4}{9} \tilde{\alpha}_1 M_1^2 \right) - 2 Y_D (m_Q^2 + m_D^2 + \mu_1^2 + A_D^2 m^2) \quad (I.5c)$$

$$\frac{dm_L^2}{dt} = (3 \tilde{\alpha}_2 M_2^2 + \tilde{\alpha}_1 M_1^2) - Y_L (m_L^2 + m_E^2 + \mu_1^2 + A_L^2 m^2) \quad (I.5d)$$

$$\frac{dm_E^2}{dt} = (4 \tilde{\alpha}_1 M_1^2) - 2 Y_L (m_L^2 + m_E^2 + \mu_1^2 + A_L^2 m^2) \quad (I.5e)$$

In the case of the first two generations the Yukawa contributions in Eqs. (I.5) are negligible. Concerning the Higgs doublets, one has

$$\frac{d\mu_1^2}{dt} = (3 \tilde{\alpha}_2 M_2^2 + \tilde{\alpha}_1 M_1^2) - 3 Y_b (m_Q^2 + m_D^2 + \mu_1^2 + A_b^2 m^2) - Y_\tau (m_L^2 + m_E^2 + \mu_1^2 + A_\tau^2 m^2) \quad (I.6a)$$

$$\frac{d\mu_2^2}{dt} = (3 \tilde{\alpha}_2 M_2^2 + \tilde{\alpha}_1 M_1^2) - 3 Y_t (m_Q^2 + m_U^2 + \mu_2^2 + A_t^2 m^2) \quad (I.6b)$$

where the masses m_Q^2 , m_D^2 , m_U^2 , m_L^2 and m_E^2 in Eqs. (I.6) correspond to the third generation.

Finally, one has, for the trilinear scalar couplings corresponding to the third generation

$$\frac{dA_t}{dt} = \left(\frac{16}{3} \tilde{\alpha}_3 \frac{M_3}{m} + 3 \tilde{\alpha}_2 \frac{M_2}{m} + \frac{13}{9} \tilde{\alpha}_1 \frac{M_1}{m} \right) - 6 Y_t A_t - Y_b A_b \quad (I.7a)$$

$$\frac{d A_b}{dt} = \left(\frac{16}{3} \tilde{\alpha}_3 \frac{M_3}{m} + 3 \tilde{\alpha}_2 \frac{M_2}{m} + \frac{7}{9} \tilde{\alpha}_1 \frac{M_1}{m} \right) - Y_t A_t - 6 Y_b A_b - Y_\tau A_\tau \quad (I.7b)$$

$$\frac{d A_\tau}{dt} = \left(3 \tilde{\alpha}_2 \frac{M_2}{m} + 3 \tilde{\alpha}_1 \frac{M_1}{m} \right) - 4 Y_\tau A_\tau - 3 Y_b A_b \quad (I.7c)$$

In all these equations μ' has been neglected and m is the soft scalar mass at the GUT scale. The gauge coupling constants at M_X verify :

$$\alpha_3(0) = \alpha_2(0) = \frac{5}{3} \alpha_1(0) \equiv \alpha_{sum} \quad (I.8)$$

APPENDIX II

We collect here some formulae relevant for Sections 4 and 5 of the text. These involve the evolution equations for m_1^2, m_2^2, m_3^2 and A_1^2, A_2^2, A_3^2 and the conditions for $SU(2) \times U(1)$ breaking. The functions $\vec{G}(t)$ appearing in Eq. (40) are :

$$G_1(t) = \left(\frac{16}{3} \tilde{\alpha}_3 M_3^2 + \frac{16}{9} \tilde{\alpha}_1 M_1^2 \right) - \gamma_t (3 m_5^2 - m_4^2 + 2 m^2 A_t^2) \quad (\text{II.1a})$$

$$G_2(t) = \left(\frac{16}{3} \tilde{\alpha}_3 M_3^2 + \frac{4}{9} \tilde{\alpha}_1 M_1^2 \right) - \gamma_b (2 m_7^2 - m_4^2 - 2 m_6^2 + 2 m^2 A_b^2) \quad (\text{II.1b})$$

$$G_3(t) = (4 \tilde{\alpha}_1 M_1^2) - \gamma_\tau (4 m_7^2 - 2 m_6^2 + 2 m^2 A_\tau^2) \quad (\text{II.1c})$$

and the functions $\vec{G}_A(t)$ appearing in Eq. (45) can be obtained from Appendix I :

$$G_{A1}(t) = \tilde{\alpha}_{\text{sum}} \frac{M}{m} \left(\frac{16}{3} z_3^2 + 3 z_2^2 + \frac{13}{15} z_1^2 \right) \quad (\text{II.2a})$$

$$G_{A2}(t) = \tilde{\alpha}_{\text{sum}} \frac{M}{m} \left(\frac{16}{3} z_3^2 + 3 z_2^2 + \frac{7}{15} z_1^2 \right) \quad (\text{II.2b})$$

$$G_{A3}(t) = \tilde{\alpha}_{\text{sum}} \frac{M}{m} (3 z_2^2 + \frac{9}{5} z_1^2) \quad (\text{II.2c})$$

where z_i is defined

$$z_i \equiv (1 + \beta_i t)^{-1} \quad (\text{II.3})$$

As we commented in Section 4, the solution for $\vec{m}^2(t)$ can be written in the form

$$m_i^2(t) = M^2 \tilde{a}_i + M A m \tilde{b}_i + m^2 \tilde{c}_{1i} + A^2 m^2 \tilde{c}_{2i} \quad (\text{II.4})$$

The coefficients have the following complicated expression :

$$\tilde{a}_j = \tilde{\alpha}_{\text{sum}} \int_0^t dt' C_{j\ell}(t, t') \left(B_{c\ell}(t') - \gamma_\ell(t') \eta(t') - 2 \tilde{\alpha}_{\text{sum}} B_{b\ell}^2(t') \gamma_\ell(t') \right) \quad (\text{II.5})$$

$$\tilde{b}_j = -4\tilde{\alpha}_{sum} \int_0^t dt' C_{j\ell}(t, t') B_{a\ell}(t') B_{b\ell}(t') Y_\ell(t') \quad (II.6)$$

$$\tilde{c}_{1j} = Q_j(t) - \int_0^t dt' C_{j\ell}(t, t') Y_\ell Y_\ell(t') \quad (II.7)$$

$$\tilde{c}_{2j} = -2 \int_0^t dt' C_{j\ell}(t, t') B_{a\ell}^2(t') Y_\ell(t') \quad (II.8)$$

where there is a sum over the ℓ index, and the functions used in Eqs. (II.5) to (II.8) are the following

$$Q_j(t) \equiv \sum_k R_{jk}^{-1}(t) \quad (II.9a)$$

$$C_{j\ell}(t, t') = \sum_k R_{jk}^{-1}(t) R_{k\ell}(t') \quad (II.9b)$$

$$B_{a\ell}(t') = \sum_k S_{\ell k}^{-1}(t') \quad (II.9c)$$

$$B_{b\ell}(t') = \sum_k S_{\ell k}^{-1}(t') \int_0^{t'} dt'' S_{kj}(t'') G_{Aj}(t'') \frac{m}{\tilde{\alpha}_{sum} M} \quad (II.9d)$$

$$B_{c\ell}(t') = \begin{pmatrix} \frac{16}{3} z_3^3 + \frac{16}{15} z_1^3 \\ \frac{16}{3} z_3^3 + \frac{4}{15} z_1^3 \\ \frac{12}{5} z_1^3 \end{pmatrix} \quad (II.9e)$$

$$\eta(t) = -8f_3(t) + 6f_2(t) - \frac{8}{5}f_1(t) \quad (II.9f)$$

$\vec{\gamma}$ is a constant vector $\vec{\gamma} = (-1, -2, -1)$ and R and S are the 3×3 matrices

$$R(t) \equiv \exp \int_0^t dt' \eta(t') \quad ; \quad S(t) \equiv \exp \int_0^t dt' N(t') \quad (II.9g)$$

where m and N are the matrices defined in Eqs. (41) and (46) in the text. Equations (II.5) to (II.9) allow the calculation of the $SU(2) \times U(1)$ breaking condition in formulae (53) and (54) of the text.

APPENDIX III

We give here the complete expression for the $\tilde{q}$ and $\tilde{l}$ masses including the contribution from the $D/2$ term in the potential. We obtain for the first two generations (negligible Yukawas) the masses :

$$m^2(\tilde{U}_L) = m^2 + 2M^2 \left(\frac{4}{3} \tilde{\alpha}_3 f_3 + \frac{3}{4} \tilde{\alpha}_2 f_2 + \frac{1}{36} \tilde{\alpha}_1 f_1 \right) + \cos 2\theta M_Z^2 \left(-\frac{1}{2} + \frac{2}{3} \sin^2 \theta_w \right) \quad (\text{III.1a})$$

$$m^2(\tilde{D}_L) = m^2 + 2M^2 \left(\frac{4}{3} \tilde{\alpha}_3 f_3 + \frac{3}{4} \tilde{\alpha}_2 f_2 + \frac{1}{36} \tilde{\alpha}_1 f_1 \right) + \cos 2\theta M_Z^2 \left(\frac{1}{2} - \frac{1}{3} \sin^2 \theta_w \right) \quad (\text{III.1b})$$

$$m^2(\tilde{U}_R) = m^2 + 2M^2 \left(\frac{4}{3} \tilde{\alpha}_3 f_3 + \frac{4}{9} \tilde{\alpha}_1 f_1 \right) - \cos 2\theta M_Z^2 \left(\frac{2}{3} \sin^2 \theta_w \right) \quad (\text{III.1c})$$

$$m^2(\tilde{D}_R) = m^2 + 2M^2 \left(\frac{4}{3} \tilde{\alpha}_3 f_3 + \frac{1}{9} \tilde{\alpha}_1 f_1 \right) + \cos 2\theta M_Z^2 \left(\frac{1}{3} \sin^2 \theta_w \right) \quad (\text{III.1d})$$

$$m^2(\tilde{E}_L) = m^2 + 2M^2 \left(\frac{3}{4} \tilde{\alpha}_2 f_2 + \frac{1}{4} \tilde{\alpha}_1 f_1 \right) + \cos 2\theta M_Z^2 \left(\frac{1}{2} - \sin^2 \theta_w \right) \quad (\text{III.1e})$$

$$m^2(\tilde{N}_L) = m^2 + 2M^2 \left(\frac{3}{4} \tilde{\alpha}_2 f_2 + \frac{1}{4} \tilde{\alpha}_1 f_1 \right) - \cos 2\theta \frac{M_Z^2}{2} \quad (\text{III.1f})$$

$$m^2(\tilde{E}_R) = m^2 + 2M^2 (\tilde{\alpha}_1 f_1) + \cos 2\theta M_Z^2 \sin^2 \theta_w \quad (\text{III.1g})$$

For the third generation, the Yukawa couplings are not negligible in general and one obtains in terms of m_1^2, m_2^2, m_3^2 as calculated in the text :

$$m^2(\tilde{t}_R) = m_1^2 - \cos 2\theta M_Z^2 \left(\frac{2}{3} \sin^2 \theta_w \right) \quad (\text{III.2a})$$

$$m^2(\tilde{b}_R) = m_2^2 + \cos 2\theta M_Z^2 \left(\frac{1}{3} \sin^2 \theta_w \right) \quad (\text{III.2b})$$

$$m^2(\tilde{\tau}_R) = m_3^2 + \cos 2\theta M_Z^2 (\sin^2 \theta_w) \quad (\text{III.2c})$$

$$m^2(\tilde{t}_L) = \frac{1}{2} m_1^2 + \frac{1}{2} m_2^2 - \frac{M^2}{2} (-3\tilde{\alpha}_2 f_2 + \tilde{\alpha}_1 f_1) + \cos 2\theta M_Z^2 \left(-\frac{1}{2} + \frac{2}{3} \sin^2 \theta_w \right) \quad (\text{III.2d})$$

$$m^2(\tilde{b}_L) = \frac{1}{2} m_1^2 + \frac{1}{2} m_2^2 - \frac{M^2}{2} (-3\tilde{\alpha}_2 f_2 + \tilde{\alpha}_1 f_1) + \cos 2\theta M_Z^2 \left(\frac{1}{2} - \frac{1}{3} \sin^2 \theta_w \right) \quad (\text{III.2e})$$

$$m^2(\tilde{\tau}_L) = \frac{1}{2} m_3^2 + \frac{1}{2} m^2 + \frac{M^2}{2} (3\tilde{\alpha}_2 f_2 - \tilde{\alpha}_1 f_1) + \cos 2\theta M_Z^2 \left(\frac{1}{2} - \sin^2 \theta_w \right) \quad (\text{III.2f})$$

$$m^2(\tilde{\nu}_\tau) = \frac{1}{2} m_3^2 + \frac{1}{2} m^2 + \frac{M^2}{2} (3\tilde{\alpha}_2 f_2 - \tilde{\alpha}_1 f_1) - \cos 2\theta \frac{M_Z^2}{2} \quad (\text{III.2g})$$

In all these formulae, $\tilde{\alpha}_i$ stands for its value at M_X , i.e., $\tilde{\alpha}_i(0)$, and the $f_i(t)$ should be evaluated at the appropriate t . M_Z^2 is the square of the Z boson mass and θ_W is the weak mixing angle.

Notice that one should add to formulae (III.1) and (III.2) a contribution equal to the square of the mass of the corresponding fermionic partner. This is especially important for the $\tilde{t}$ squarks. There are also $\Delta I_W = \frac{1}{2}$ masses connecting left-handed to right-handed squarks as discussed in Section 7.

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FIGURE CAPTIONS

- Fig. 1 Diagrams inducing μ' and μ_3^2 terms. a) and b) in a model with a missing partner mechanism ; c) and d) in a model with a heavy singlet.
- Fig. 2 The m_b/m_t ratio as a function of the t quark mass.
- Fig. 3 The two mechanisms for radiative $SU(2) \times U(1)$ breaking. a) with a negative μ_2^2 b) with a $\sin 2\theta(M_W) \approx 1.0$.
- Fig. 4 Gaugino masses at M_X versus $\sin 2\theta(M_W)$ for various values of the t quark Yukawa. The smaller numbers indicate the corresponding value of the t quark mass. This is for $A = 1$, $m = 100$ GeV.
- Fig. 5 The same as in Fig. 4 but in terms of $\sin 2\theta$ measured at the GUT scale.
- Fig. 6 The same as in Fig. 5 but with $A = 2$.
- Fig. 7 Gaugino masses at M_X versus t quark mass for various values of $\sin 2\theta(M_X)$. This is for $A = 1$, $m = 100$ GeV.

- Figure 1 -

- Figure 2 -

- Figure 3 -

- Figure 4 -

- Figure 5 -

- Figure 6 -

- Figure 7 -